PUZZLES

1. Each letter in the addition problem below represents a unique integer from 0 to 9. Determine the number that each letter represents.

$$\begin{array}{r} \text{SEND} \\ \underline{\text{MORE}} \\ \text{MONEY} \end{array}$$

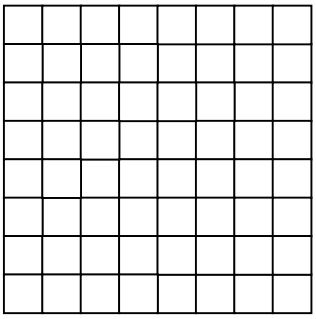
2. Michelle, an environmental engineer, works at her father's organic farm on weekends. One summer morning Michelle takes 100 pounds of cucumbers to the Dallas Farmer's Market. The cucumbers are 99% water by weight at the start of the trip. Unfortunately Michelle's pickup truck breaks down on the way, and the cucumbers dry in the hot sun for hours. When Michelle finally arrives at the market, the cucumbers are 98% water by weight. How much do the cucumbers weigh now?

3. A jeweler makes necklaces from silver chains costing him $100 each and standard-sized diamonds, emeralds, rubies, and sapphires. The costs to the jeweler (including the chain) of some necklaces with different stones are given as follows:

   - 1 diamond and 2 rubies - $3100
   - 4 sapphires and 1 diamond - $2100
   - 3 emeralds, 1 sapphire, and 1 diamond - $1500.

   An IE named Susan wants a necklace with exactly one of each gem. How much would this necklace cost the jeweler?

4. The nation of Griddonesia consists of eighty-one equally-spaced islands represented by intersections in the grid below. Each island is connected to all its neighboring (ie., closest) islands by horizontal and vertical bridges. There are no diagonal bridges.



A rowdy native born on the central island becomes a local rock star calling himself ∝. He had never left the central island until one day he is banished (forced to leave and forbidden to return). Thereupon ∝ crosses a bridge to a neighboring island, continues his wild ways, and is soon banished again. This pattern continues until one day ∝ has nowhere else to go. He has been banished from each neighboring island and must finally settle down. Then ∝ realizes that he has been on the minimum possible number of Griddonesia islands. How many islands has ∝ been on, including the one on which he is trapped?

5. Elisa, an EE, canoes upstream while her material science boyfriend Ryan sleeps in the rear of the boat. As they pass a big tree, an overhanging branch knocks Ryan's baseball cap into the water. He awakes five minutes later and discovers his cap is missing. Elisa instantly turns the canoe around and paddles downstream at the same rate relative to the water. They pick up the floating hat one mile downstream from the tree. How fast is the stream moving?

6. An adventurous biomedical engineering student named Marco wants to cross an Australian desert on foot over Christmas break. The trek will begin in the outback at the aboriginal village of Kwazi and take six days. However, one man can only take enough food and water to last himself four days. Fortunately, Marco can hire porters from Kwazi to assist him. Each porter costs $100 per day that he is away from the village. What is the least possible cost to Marco of hiring enough porters so he can cross the desert and the porters can return safely to Kwazi?

7. A computer scientist has two sons named Chip and Hal. After the father's death, as specified in his will, the executor of the estate assigns two positive integers to the sons - one to Chip and one to Hal. Neither son knows the other's number. Chip and Hal are then told that the two integers could possibly be equal and that the product of the two integers is either 8 or 16. The sons are also informed that the first one to determine his brother's number will inherit their father's entire estate. Immediately thereafter, the following conversation transpires between the two brothers.

   Chip: I have no idea what your number is.
   Hal: Same here.
   Chip: You'll have to give me a hint.
   Hal: Same here.

   Determine Hal's number.

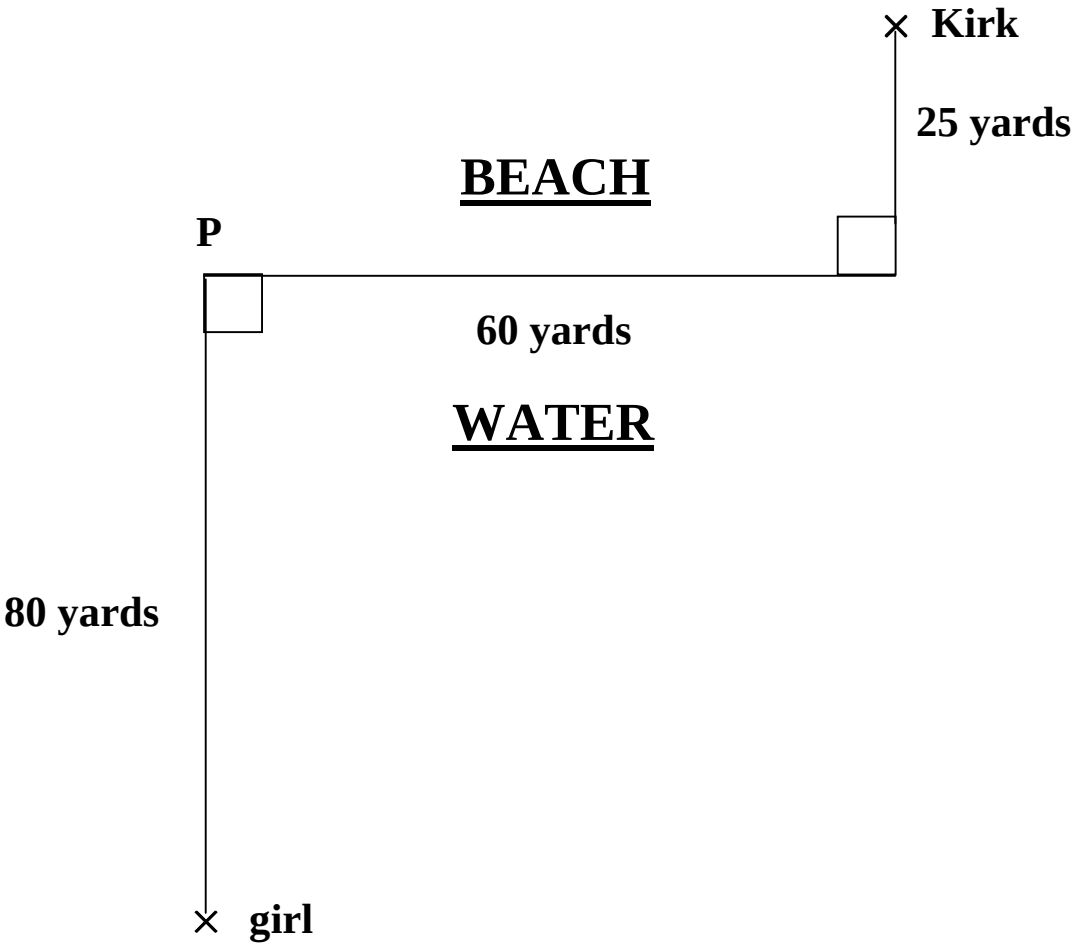
8. A vain ME named Brad is admiring himself in the mirror when he notices that the mirror is exactly long enough to show his entire reflection. If Brad's height is 72 inches, how long is the mirror?

9. Consider three piles of poker chips. Pile A has one chip, pile B has two chips, and pile C has three chips. These three piles are to be used in a game played by an IE and a CE. The rules of the game are as follows.

    - Each player, in turn, takes either one chip or all the chips from any pile with remaining chips.
    - The player who takes the last chip loses the game.
    - The IE goes first.

    What play by the IE (specify the pile and the number of chips taken) will guarantee that he wins?

10. An EE named Kirk lifeguards in the summer at Waikiki Beach. One day as he sits atop his lifeguard chair and twirls a whistle around his finger, a teenaged girl screams. Instantly Kirk spots the beautiful young lady. Her raft is losing air quickly, and she is yelling that she can't swim. Kirk knows that his running speed of 5 yard per second on the sand is exactly twice his swimming speed. To reach the girl as quickly as possible, where along the perfectly straight shoreline should he enter the water? Refer to the figure below, and express your answer (to the nearest tenth) in yards to the right of point P.



11. A CSE named Jason is hiking back to camp with his dog Byte at the rate of two miles per hour. When they are ten miles from camp, Byte begins running back and forth to their tent. Because of the steep grade, Byte runs ten miles per hour toward camp and only six miles per hour back toward Jason. How far does Byte run before Jason reaches the tent?

12. A ten-pound weight is attached to one end of a twenty-one-foot rope. The rope is passed over a circular frictionless pulley with a diameter of $2/\pi$ feet that is suspended thirty feet above the ground. A ten-pound monkey is next placed at the other end of the rope exactly opposite the weight to balance it. The monkey then begins climbing at the constant rate of ten feet per minute with respect to the rope. After fifteen seconds what is the direction and speed of the weight with respect to the ground?

13. (Remember, it's a dirty dozen.) Clark Kent has retired from the Daily Planet and now emcees the new TV show *Doorway to A Million* on which a biomedical engineer named Ana is a contestant. Clark shows Ana five identical closed doors. Behind one of the doors is the grand prize of a million dollars in cash. Behind each of the other four doors is a Superman poster - the consolation prize. Ana gets the prize behind the door that she chooses.

    She selects door 1.

    "Is that your final choice?" asks Clark.

    Before Ana can respond, however, Clark opens door 2 to reveal a Superman poster that he has located using his x-ray vision. He tantalizes Ana by stating that the grand-prize door has been randomly selected and then offering her another option. She can either stick to door 1 or switch to door 3. What is the probability, expressed as a reduced fraction, that Ana wins the million dollars if she switches to door 3?

ANSWERS

1. 9567
   1085
   10652

2. 50 pounds

3. $2400

4. 8

5. 6 miles per hour

6. $600

7. 4

8. 36 inches

9. 1 chip from pile B

10. 32.1 yards

11. 40 miles

12. 5 feet per minute up

13. 4/15

    This problem is a version of a well-known paradox. The answer can be verified by simulation. Clark has selected a door that does not hide the money. Intuitively, his selection adds further information beyond the apparent one-in-four chance. Analytically, from the law of total probability,

    P[switch correct] = P[switch correct | guess wrong] P[guess wrong] +
    P[switch correct | guess correct] P[guess correct]

    = (1/3)(4/5) + 0 = 4/15.