PUZZLES

1. Some UTA engineering students decide to have a parade during Engineering Week. When the students form ranks of 3 abreast, two are left over. In ranks of 5, four students are left over. In ranks of 7, six are left over. In ranks of 11, ten are left over. What is the least possible number of students in the parade?

2. Each letter *a, b, c, d, e, f* in the following multiplication problems denotes a unique <u>nonzero</u> integer from 1 to 9. Thus any permutation of these letters forms a distinct six-digit number. Determine the six-digit number *abcdef*, where

$$fabcde = 5 \bullet abcdef$$

$$efabcd = 4 \bullet abcdef$$

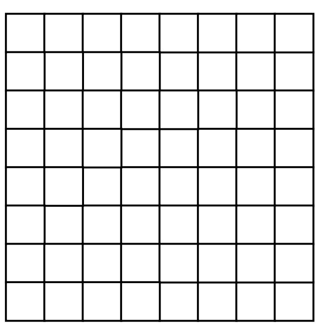
$$defabc = 6 \bullet abcdef$$

$$cdefab = 2 \bullet abcdef$$

$$bcdefa = 3 \bullet abcdef.$$

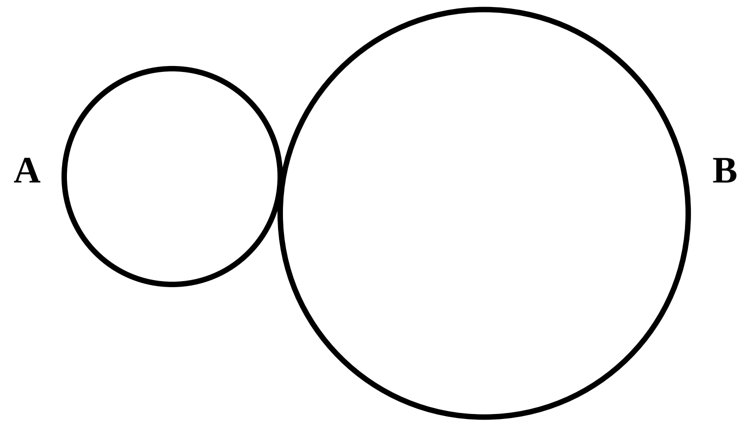
3. Any AE knows that jet engines operate independently in flight and that a plane makes a successful flight if and only if at least half of its engines do not fail. Let q be the probability that any engine fails in flight. For what value $q \neq 0,1$ are a four-engine plane and a two-engine plane equally like to make a successful flight?

4. The nation of Griddonesia consists of eighty-one equally-spaced islands represented by intersections of the lines in the grid below. Each island is connected to all its adjacent islands by horizontal and vertical bridges. There are no diagonal bridges.



As might be expected, the governor of each island wishes that he or she were governor of an adjacent island (one connected by a single horizontal or vertical bridge) instead of his or her own. What is the total number of different ways in which each of the eighty-one governors could swap islands with an adjacent governor?

5. In the figure below, the radius of circle A is 5 inches and the radius of circle B is 10 inches. Starting from the position shown, circle A rolls clockwise exactly <u>four</u> times around circle B, which remains stationary, and ends with its center in the original position. How many rotations does circle A make?



6. At his oral dissertation defense, a CSE named Mac is being interrogated by his supervising professor, Dr. Frank N. Stein. The redoubtable AI guru reminds Mac that "P and/or Q" denotes *inclusive or* (either P or Q or possibly both), while "P or else Q" denotes *exclusive or* (either P or Q but not both). Then Dr. Stein hands Mac a sheet of paper. On one side of the sheet are the two statements:
   (a) *There is a knife and/or a fork on the table, or else a fork and a spoon on the table.*
   (b) *Both statements on the other side of this sheet are true.*

   On the other side of the sheet are the two statements:
   (c) *There is a fork and a spoon on the table.*
   (d) *Both statements on the other side of this sheet are false.*

   Answer Dr. Stein's question for Mac: If you assign truth or falsity to each statement (a) – (d), what is the maximum possible number of true statements for which (a) – (d) are mutually consistent?

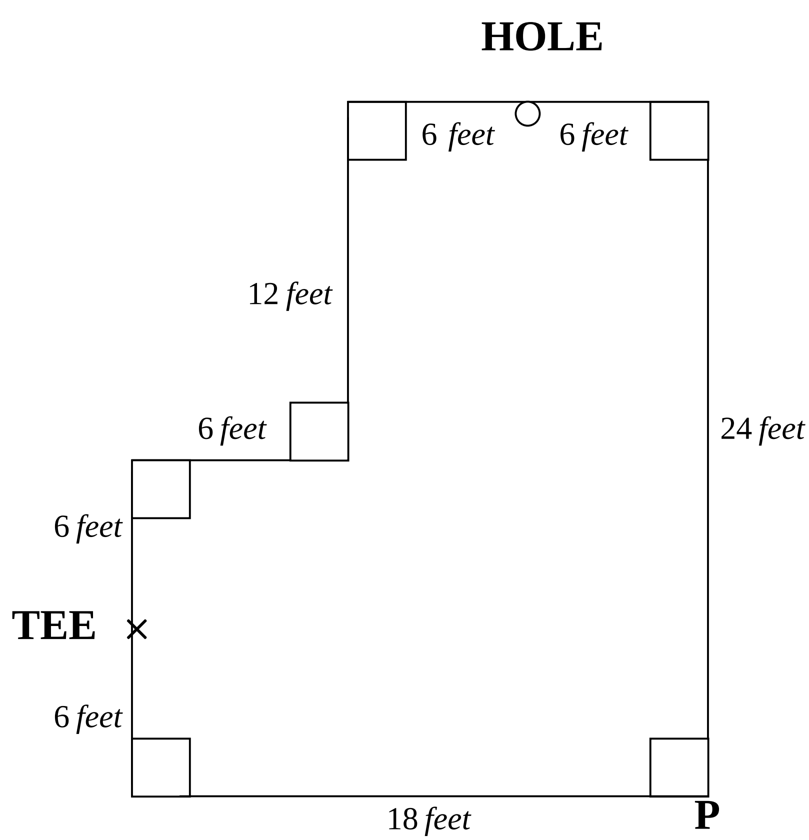
7. Cowtown Chips Inc. (CCI) donates ten boxes, each containing 1000 identical chips, to the EE Department for use in labs. Supposedly each chip weighs 5 grams. However, CCI immediately calls to say that a shipping error has occurred. In <u>at least</u> one box, all chips weigh 6 grams. If you can open the boxes, remove chips as needed, and use <u>only</u> a digital scale (not a two-pan balance), what is the minimum number of weighings that can identify <u>all</u> boxes of 6-gram chips?

8. Consider the following game played on a 3×3 checkerboard with a stack of 9 checkers. Two players alternate turns. On a given turn, a player takes the one or more checkers that he intends to play from those remaining in the stack. The player then places them on vacant squares, provided these vacant squares (not necessarily adjacent) lie in the same horizontal row or the same vertical row. The winner is the player who places the final (ninth) checker on the board.

A female IE student named Zhuo Yin and a male CSE student named Wen Yang play this game. Yang goes first. From the choices below, select the correct statement about Yin and Yang. Submit only the letter corresponding to your answer.

(a) With a suitable strategy, Yin can always win the game regardless of Yang's strategy.
(b) With a suitable strategy, Yang can always win the game regardless of Yin's strategy.
(c) Neither player has a strategy that guarantees a win regardless of the other's strategy.
(d) It is never possible to finish the game in three plays.

9. A female ME student named Maria has been so busy with school that she gave up regular golf and only plays miniature golf. But she can now use physics to analyze her shots. The first hole of her favorite course, which has a level putting surface, is diagrammed below. Determine the point where Maria's tee shot should strike the 24-foot wall so that the ball next banks off the 12-foot wall (without hole or tee) and then makes a hole-in-one. Express your answer in feet above corner P.



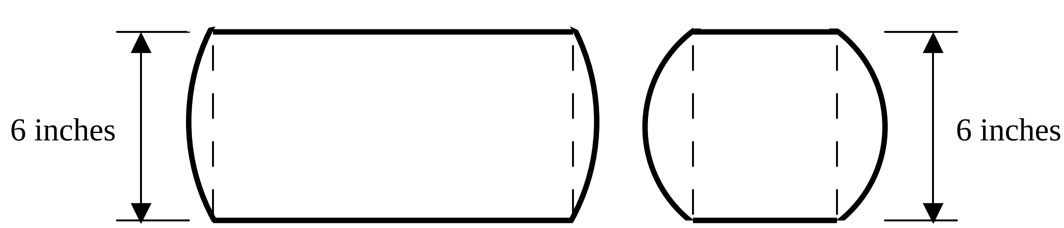
10. Jacques, a French materials science student, sneaks a bottle of wine into his lab and fills beaker A two-thirds full of wine. Next he fills an identical beaker B two-thirds full of water. Then he transfers exactly one-sixth of A's contents into B and thoroughly mixes the wine into the water. Finally Jacques transfers enough solution from B into A until both beakers are again two-thirds full. Determine the ratio of the amount of water in beaker A to the amount of wine in beaker B.

11. A circular wave pool was constructed on the Fox studio lot for filming the movie *Titantic*. The pool is perfectly cylindrical and has a radius of 50 feet and a height of 8 feet. It is filled with water to a depth of 4 feet. Leonardo, an actor with an ME mentality, floats a small barge carrying a 1000 cubic-foot solid iceberg in the pool and then measures the depth of water. Next he dumps the iceberg into the water, lets it melt, and measures the depth again (without removing the barge). This experiment is conducted at a temperature for which water weighs 62.40 pounds per cubic foot and for which a cubic foot of ice melts into 0.93 cubic feet of water. How much would the water level change on the second measurement? Express your answer in <u>millimeters</u> to four decimal places, and specify whether the water level is up or down.

12. An environmental engineer named Naresh and his CE friend Ahmed decide to explore the notoriously polluted Styx River. Both Naresh and Ahmed have motorboats. They start on opposite sides of the river and drive their respective boats toward each other at constant (but different) speeds in a direction perpendicular to the straight shorelines. When their boats pass each other the first time, they are 700 yards from the east shoreline. When the boats reach shore, they turn around and start toward each other again. When they pass a second time, they are 300 yards from the west shore. Afterward, they dock Naresh's boat and take a leisurely trip in Ahmed's. They set the cruise control to a speed of 10 mph in still water, travel 45 miles upstream against the current, and then return 45 miles back downstream. This entire trip takes 12 hours. Under such idealized assumptions as constant speeds and instantaneous turnarounds,

    (a) Calculate the width of the Styx River in yards.
    (b) Calculate the speed of the current in miles per hour.

13. (Remember, it's a dirty dozen.) A biomedical engineer named Chad bores through the centers of two solid spheres, one of radius 10 inches and the other of radius 5 inches, and leaves perfect cylindrical holes 6 inches deep as shown in the side views below. Determine the ratio of the <u>remaining</u> material in the large sphere to the <u>remaining</u> material in the small sphere.



ANSWERS

1. 1154. Note that $x+1 \equiv 0$ mod 3, mod 5, mod, 7, mod 11. Thus x+1 is divisible by 3, 5,7, 11. The least number so divisible is $3\times5\times7\times11$. So x+1 =1155 and x = 1154.

2. *abcdef* = 142857. Note that a = 1 immediately from the first equation. Incidentally, 142857 is the only number known to have this cyclic property.

3. q = 1/3. We have $1 - [q^4 + 4q^3(1-q)] = 1 - q^2$, so $3q^4 - 4q^3 + q^2 = 0$.

4. 0 ways. Color the intersections red and black like a checkerboard, starting with a red in the upper left. Then there are 41 reds and 40 blacks. But each governor on a red intersection wants to be on a black one, so the swap is not possible.

5. 12 times. In addition to 8 rotations from the circumferences, circle A rotates once more on each revolution around B. Roll one quarter around another to get the idea.

6. 1. The key is that (a) and (c) are inconsistent. If (c) is true, the second proposition in (a) is true. The *exclusive or* then dictates that the first proposition in (a) must be false. But by (c) the first proposition in (a) is true because of its *inclusive or*.

7. 1 weighing. Number the boxes 1 to 10. Take 1 chip from box 1, 2 from box 2, 4 from box 3, 8 from box 4, 2^{n-1} chips from box n,…, 512 chips from box 10. These chips are supposed to weigh a total of 1023 grams. Subtract 1023 from the actual weight. Then observe that there is a unique correspondence between the possible weight differences and the possible combination of heavy boxes.

8. (a)

9. 15 feet

10. 1

11. 0 millimeters. The iceberg displaces a volume of water equal to its weight while in the barge. The melted iceberg (now water) in the pool displaces its volume, which weighs the same as the iceberg. Note that if a solid steel ball had been in the barge, the water level would have gone down.

12. (a) 1800 yards (b) 5 mph

13. 1. Let the radius of the sphere be R and the depth of the cylinder be 2d. For $d \leq R$, triple integrations in cylindrical coordinates immediately (and unexpectedly) establish that the remaining volume is $(4/3)\pi d^3$, independent of R. For the problem here, both remaining volumes equal 36π cubic inches.