PUZZLES

1. Dr. Frank N. Stein of the CSE faculty is teaching a course in fuzzy logic this semester. The eminent AI guru is notorious for his difficult tests, so the students have begged him repeatedly for a multiple choice quiz. Finally, with a devious smile, he agrees. On the next test, he asks the first question in Swahili, which no one can read. However, the following answer choices are in English.

   (a) All of the below

   (b) None of the below

   (c) All of the above

   (d) One of the above

   (e) None of the above

   (f) None of the above

   Select the correct answer and submit only the corresponding letter.

2. A bored ME named Jason sits in his TTh 11:00 a.m. – 12:20 p.m. class checking his watch, which is not digital. As he waits impatiently for class to end, Jason formulates the following problem. From exactly noon, how long will it take the minute hand and the hour hand of his watch to be precisely 90 degrees apart for the first time? State your answer in <u>minutes</u> rounded off to three decimal places.

3. An EE named Nguyen Li likes to study with scented candles burning. She has two new candles of different scents that have equal lengths but burn at different rates. One is consumed uniformly in four hours, the other uniformly in five hours. If she lights them at the same time, in how many <u>hours</u> will one candle be exactly three times as long as the other? State your answer as a reduced fraction.

4. The nation of Griddonesia consists of eighty-one equally-spaced islands represented by intersections of the lines in the grid below. Each island is connected to all its adjacent islands by horizontal and vertical bridges. There are no diagonal bridges.
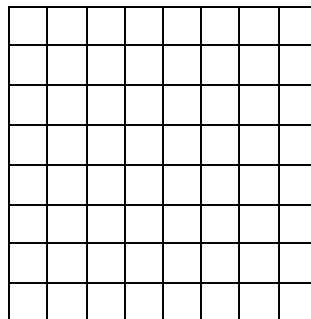
Griddonesia has a presidential election this year. In the nation's presidential politics there are exactly two parties, the Yins and the Yangs. In a presidential election, each eligible Griddonesian can vote for either the Yin or the Yang candidate. For each island, the candidate receiving the most popular votes on the island gets that island's one electoral vote. The candidate with the most electoral votes then becomes president. For this year's election, each island in Griddonesia has exactly 1001 eligible voters who might possibly vote.

(a) The Yin candidate is the incumbent female president. To the nearest tenth, what is the largest percentage of popular votes that she can receive and still lose the election?

(b) The male Yang candidate intends to campaign by car. He will begin and end at the center island with no interim stops there. Using only the bridges, he will proceed from island to island without going to any island more than once (other than the center island). What is the maximum number of islands (with the center one counted exactly once) on which the Yang challenger can campaign during this trip?

5. A small nanotech laboratory is housed in a 38-feet long, 20-feet wide, and 10-feet high rectangular room whose walls are kept "clean" by a tiny dust-eating robot. One morning the dustbug, as it's called, sits halfway up a 20×10 end wall, 1 foot from the closest 38×10 side wall. On the opposite 20×10 end wall, halfway up and 1 foot from the other 38×10 side wall, lies a speck of dust. What is the shortest distance in feet that the dustbug can crawl along the room's surfaces to reach this dust? Round off to two decimal places.

6. Five biomedical engineering students decide to meet in the lobby of Nedderman Hall at noon to discuss a class project. Each student, independent of the others, is equally likely to arrive between 11:52 a.m. and 12:04 p.m. What is the probability that at least 3 of the students arrive by noon? Express your answer as a reduced fraction.

7. A materials science student named Chen Feng has developed a new alloy called tico from the elements titanium and cobalt. He stacks 1000 one-inch cubes of tico into a perfect ten-inch cube. Obviously this stack forms 1000 one-inch cubes and 1 ten-inch cube. How many cubes of any size are contained in the $10 \times 10 \times 10$ stack?

8. Civil engineers George and his wife Laura give a dinner for 5 other married couples. At least one person in each invited couple is acquainted with either George or Laura (or both). During the introductions, no one shakes hands with someone he or she has previously met (including his or her own spouse). After the introductions, Laura realizes that each of the other 11 people shook a different number of hands. Furthermore, no one shook the same person's hand more than once, and no one shook his or her own hand. How many hands did George shake?

9. An orbiting astronaut, an AE graduate from UTA named Naresh, simultaneously fires two projectiles A and B along two straight-line trajectories forming an angle of 100 degrees. Assume that the projectiles instantaneously attain a constant velocity, with A traveling twice as fast as B. If they are separated by a distance of 334 miles after 4 minutes, how fast is B traveling in miles per hour to the nearest tenth?

10. One spring afternoon an environmental engineering student named Praveena takes her dog and flightless kiwi bird to Square Park, which has 100-meter sides. When she is ready to leave, it so happens that Praveena, her two pets, and a stray kitten are standing at the four corners of Square Park as shown in the figure below. Praveena sees her dog's reaction to the kitten and begins running directly toward her dog. At that exact instant, the dog runs directly toward the kitten, the kitten runs directly toward the kiwi, and the kiwi runs directly toward Praveena for protection. Assume that Praveena, the dog, the kitten, and the kiwi instantaneously attain the same constant running speed. Hence, they reach the center of Square Park at precisely the same time, each following a curved path. How many meters does each run? Round off to two decimal places.

kitten   dog

100

kiwi   Praveena
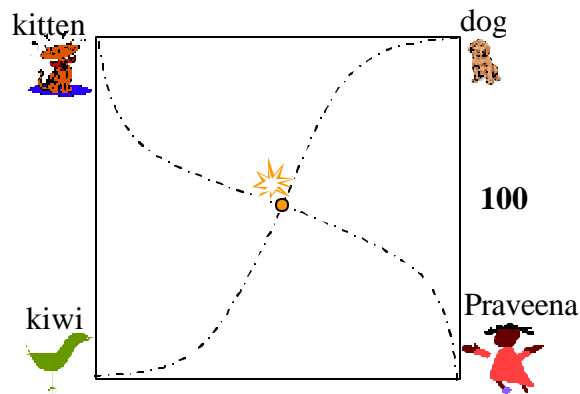
11. A CSE student has developed a program to randomly generate (x, y) points in the first quadrant of a Cartesian coordinate system. Determine the probability that such a randomly generated point lies below the curve $y = e^x$. In other words, what portion of the first quadrant lies below $y = e^x$? Round off to three decimal places.

12. Two fraternal twins Bob and Sue, both IE students, inherit a ranch from their West Texas grandfather. Both have taken engineering economy, so they decide to do some low-tech wheeling and dealing. They sell a herd of cattle and receive as many dollars for each animal as there are cattle in the herd. Using all the proceeds of the sale, they buy a flock of sheep at $10 a head and then a less expensive lamb with the rest of the money (less than $10). Finally, the twins divide up the sheep and the lamb between them. To equalize the twins' net monetary gains, Bob gets an extra sheep, while Sue gets both the lamb and her brother's calculator. What is the value of this calculator in dollars rounded off to the nearest cent?

13. (Remember, it's a dirty dozen.) A chemical engineer is taking an EE course in information theory, where he's currently studying codes. For a homework assignment he numerically encrypts a seven-word sentence in the following table.

5	88	23	8
53	95	16	8
62	18	73	60
16	1	18	15

Decode this message. If your answer is correct, you should know immediately.

ANSWERS

1. (e) All other answers give a contradiction.

2. 16.364. The time t satisfies t(6°/minute) – t(0.5°/minute) = 90°.

3. 40/11 hours. Two equations in two unknowns yield the slower candle burning 8/11 of its length. Multiply that by 5 hours.

4. (a) 99.9 %. She gets all 1001 votes in 40 islands and loses 0-1 on the other 41.

    (b) 80. The total number of bridges up from the center must equal the number down, and the number right must equal the number left. Hence an even number of bridges must be traversed. Since 81 bridges are needed to go through all 81 islands and end on the center, this many islands cannot be reached. However, it is easy to find a way to reach 80.

5. 50.00 feet. Unfold the room into the two-dimensional unfoldings that provide a surface for a line between the starting and ending points. For each, use the Pythagorean theorem to find the straight line between the two points. The shortest of the distance is the hypotenuse of a triangle with legs (5 + 20 + 5) and (1 + 38 + 1). The hypotenuse is then 50 feet. The dustbug can walk at angles from the end wall across a corner of the nearest side wall across the ceiling across a corner of the other side wall to the dust in a "straight" line of 50 feet.

6. 64/81. The probability of a student being on time is 2/3. Add the probabilities $10(2/3)^3 \ (1/3)^2 + 5(2/3)^4 \ (1/3)^1 + 1(2/3)^5 \ (1/3)^0$.

7. $3025 = 10^3 + 9^3 + 8^3 + \ldots + 2^3 + 1^3$.

8. 5. Number all except Laura as 0,1, … ,10 (the number of hands they shook). By elimination, 10 is married to 0, 9 to 1, etc.; and both Laura & George shook 5 apiece.

9. 2099.5 mph. Use the law of cosines for the distance, then divide by the time in hours.

10. 100.00 meters. The four runners always run at right angles to each other at some speed s meters per second. Hence, their positions always represent the four corners of a diminishing square that takes 100/s seconds to become a single point. The distance each runs is s(100/s) = 100. One can also integrate a parametric curve for arc length.

11. 1.000. Assume the randomly generated point lies within a square of side t with two sides along the axes and a vertex at the origin. Then by integration, the area within the square and below the curve is t2 + t – 1 – t(ln t). Divide by the total area t^2 and let t→∞ using l'Hospital's rule.

12. $2.00. Let n be the number of cattle. Then n^2 is the number of dollars from the sale. The number of 10's in n^2 is the number of sheep they bought. Since the sheep could not be divided equally, there was an odd number of 10's in n^2. There are an infinite number of choices for such an n^2: 16, 36, 196, 256, 576, 676, 1156, 1296, … , all of which end in the digit 6. This fact, which makes the answer unique without knowing n, could be proved or simply inferred from enough values of n. Thus the lamb costs $6, and Michael must compensate Sue with $2. He now has $8 from the sheep minus the calculator, and she has $8 from the lamb plus the calculator.

13. "Bravo, I am so smart and sharp." As hinted by the nature of question 7 and by the student being a chemical engineer, each number represents the number of an element in the periodic table. Put the symbols in the table to give the following.

B	Ra	V	O
I	Am	S	O
Sm	Ar	Ta	Nd
S	H	Ar	P