PUZZLES

1. On a biomedical engineering examination a student named Syed computes in feet and inches the maximum distance that a certain artificial heart design could pump blood upward against gravity. Unfortunately, in recording this distance on his examination paper, he reverses the numbers for feet and inches. As a result, his recorded answer is only 30% of the computed length, which was less than 10 feet with no fractional feet or inches. What length did Syed <u>compute</u> in feet and inches?

2. A female IE student named Aruna jogs north across a bridge used for both a road and a train track. Three-eighths across the bridge, she hears a train coming north toward the bridge from behind her. Aruna calculates that if she keeps running, she will reach the north end of the bridge at the same instant as the train. She also calculates that if she turns around and runs back south, she will reach the south end of the bridge at the same instant as the train. If Aruna jogs at a constant speed of 8 mph, what is the speed of the train to the nearest two decimal places?

3. An ME named Eduardo uses some three-dimensional CAD/CAM software to draw a right circular cone with a height of 3 feet and with a base of radius 1 foot. Next he inscribes a cube in the cone so that the face of the cube is contained in the base of the cone. What is the length of the cube's side in feet to the nearest three decimal places?

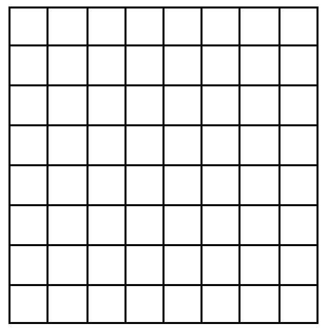
4. The UTA student chapter of SWE (Society of Women Engineers) has noted that the proportion of the IMSE faculty members at UTA who are female is greater than the proportion of all engineering faculty members at UTA who are female. Let P denote the proportion of female IMSE faculty to female engineering faculty at UTA, and let Q denote the proportion of all IMSE faculty members to all engineering faculty at UTA. With no further information or numerical data, exactly one of the following statements is true.

   (a) $P > Q$
   (b) $P < Q$
   (c) $P = Q$
   (d) The relation between P and Q cannot be determined.

   Select the correct answer and submit only the corresponding letter.

5. A fabrication engineer named Dave lies a lot. In fact, he tells the truth only one day a week and always on the same day. One day he said, "I lie on Mondays and Thursdays." The next day he said, "Today is either Thursday, Saturday, or Sunday." And then the following day he said, "I lie on Wednesdays and Fridays." On what day of the week does Dave tell the truth?

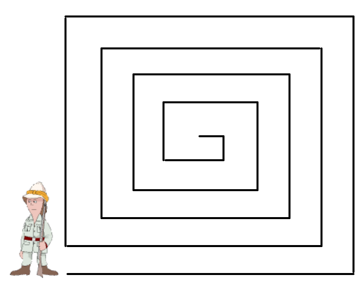
6. The nation of Griddonesia consists of eighty-one equally-spaced islands represented by intersections of the lines in the grid below, where north is up and east is right as on a standard map. Each island is connected to all its adjacent islands by horizontal and vertical bridges exactly one-mile long. There are no diagonal bridges.



A native environmental engineer named ♍♋♓⌘♌ lives on the northwest island and must drive to the southeast island to assess the damage from an oil spill. The shortest driving distance on bridges between the northwest and southeast islands is obviously 16 miles. How many different such shortest routes could she take?

7. A CE graduate student named Masoud has a grant to study construction methods of the ancient Egyptians. One day in the Valley of the Kings he discovers an odd structure whose stone walls form a square spiral as indicated in the figure below, which is obviously not to scale. The sides of the outside square measure 102 feet by 102 feet. The interior spiral path is 2 feet wide and spirals through the entire structure. To the nearest tenth of a foot, what is the distance along the exact middle of the path from the entrance to the center of the structure?

102 feet



102 feet

8. One day Masoud, the CE of problem 7, stops for lunch in the center of the square spiral. Then he discovers that he brought water to drink but nothing to eat. So Masoud offers to pay his two Bedouin assistants, Al-Fahl and Gamal, to share their lunches with him. Al-Fahl has five loaves of bread and Gamal has three loaves, all identical. The two Bedouins agree to divide up their eight loaves equally among the three men. After eating, Masoud lays down 8 one-piaster coins. To be fair, how many coins should Gamal receive?

9. Four EE students have formed a string quartet called the Ohms. One dark night they are warming up for a concert in Trinity River Park in Fort Worth. Suddenly the four musicians realize that it's almost time for the concert, and they must all cross a small bridge over the river to reach the stage. At most two people can cross this bridge at one time, and any such group crossing the river must have a flashlight. The four musicians have only one flashlight, however, and it must be carried back and forth across the bridge. Moreover, each EE walks at a different speed, and a pair must walk together at the rate of the slower one. The time required for each musician to cross the bridge is given as follows, where they are identified by their instruments:

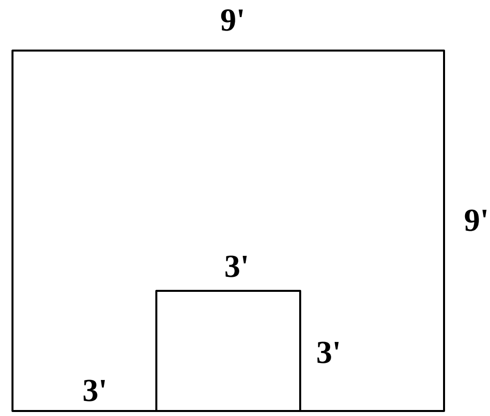
   - Cello – 1 minute to cross
   - First violin – 2 minutes to cross
   - Second violin – 5 minutes to cross
   - Viola – 10 minutes to cross.

   What is the minimum time in minutes required for all the Ohms to cross the bridge?

10. A CSE named Olga uses a computer simulation program to generate three numbers in the interval (0,1), where each number is equally likely to have any value in the interval and is independent of the other two. What is the probability that three lines with lengths in feet equal to these three numbers would form a nondegenerate triangle (one with nonzero area)? Express your answer as a reduced fraction.

11. An Arlington High School senior named Jamal is a finalist for two scholarships at UTA – a university-wide scholarship worth $15,000 per year for four years and an engineering scholarship worth $10,000 per year for four years. The recipients for each type of scholarship are decided independently. However, a student can win only one scholarship and must take the university award if offered both types. There are 15 finalists for 8 engineering scholarships, and 3 finalists for 1 university scholarship. Exactly one other student besides Jamal is a finalist for both the university and engineering scholarships. If all the finalists for each type are equally likely to win, to the nearest three decimals what is the probability that Jamal wins one of these scholarships?

12. A materials science graduate student named Hsu Wen has taken several CSE courses offered by Dr. Frank N. Stein. For this reason Hsu has asked the eminent AI guru to be on his Ph.D. supervisory committee. As part of the written portion of Hsu's comprehensive examination, Dr. Stein e-mails Hsu a drawing on Friday afternoon with the following instructions. By Stein's return on Monday, Hsu must produce the minimum integer number of cubic feet (i.e., 1, 2, 3, …) of composite material required to cast an object formed by plane surfaces with both a top and front view as shown below. Then Dr. Stein flies off to an applied probability workshop in Las Vegas. Determine the integer number of cubic feet of composite material that Hsu must produce by Monday.



13. (Remember, it's a dirty dozen.) A petite five-foot AE named Kim weighs 95 pounds normally. During the past two week, however, she has gained weight from eating pizza at midnight while studying. A revolutionary way to lose weight occurs to her. She would weigh less standing on a scale at the equator than standing on the same scale at the geographic north pole (i.e., on the earth's axis of rotation). If Kim weighs exactly 100 pounds at the north pole, to the nearest tenth of a pound how much would she weigh at the equator? Assume the earth is a perfect sphere of radius 6370 kilometers.

ANSWERS

1. 9 feet, 2 inches. Let x = computed feet, y = computed inches. Then 0.30(12x + y) = 12y + x in inches. Thus x = (9/2)y. Let y take on 0, 1, 2, …,11 since y must be an integer. Only y = 2 yields a nonzero integer number of feet x under 10. Thus he records 2 feet, 9 inches on his test.
2. 32.00 mph. From the two possibilities, it is apparent that Aruna can run one-fourth the bridge's length in the same time that the train can go the entire length. Thus the train's speed is four times Aruna's.
3. 0.961 feet. Consider the plane containing both the axis of the cone and two opposite vertices of the cube's bottom face. Let s be the length of the cube's side. Then the cross section of the cone and the cube in this plane consists of a rectangle of sides s and $s\sqrt{2}$ inscribed in an isosceles triangle of base 2 and height 3, where the $s\sqrt{2}$ side of the rectangle lies on the base of the triangle. Similar triangles now yield $s/3 = (1 – s\sqrt{2}/2)/1$. Thus $s = (9\sqrt{2} - 6)/7$.
4. (a) Let f = IMSE female faculty, t = IMSE total faculty, F = COE female faculty, T = COE total faculty. Then $f/t > F/T$, so $P = f/F > Q = t/T$.
5. Thursday. One of the statements: "I lie on Mondays and Thursdays" and "I lie on Wednesdays and Fridays" must be false. Hence, Dave must tell the truth on either Monday, Wednesday, Thursday, or Friday. All except Thursday yield a contradiction to Dave's statements.
6. 12,870. There are 16 bridges to traverse - 8 east and 8 south in some order. Then the number of shortest paths is the simply the number of ways to select 8 of 16 to go south (and hence 8 to go east). This number is simply 16!/8!8! = 12,870. Incidentally, the number 2^{16} = 65,532 doesn't account for situations when there are not two choices, such the case from the northeast corner.
7. 5201.0. Any foot forward results in 2 square feet of area along the path. The distance to the end of the spiral path is $(102)^2/2$ = 10,404/2 = 5202 feet. But the center of the structure is 1 foot from the end.
8. 1 coin. Al-Fahl ate 8/3 loaves and contributed 7/3 loaves. Gamal 8/3 loaves and contributed 1/3 loaf. Hence, Al-Fahl gets 7 coins, and Gamal gets 1 coin.
9. 17 minutes. C + FV = 2. Return C = 1. SV + V = 10. Return FV = 2. C + FV = 2. Alternately, C + FV = 2. Return F = 2. S + V = 10. Return C = 1. C + FV = 2.
10. 1/2. Denote the lengths by x, y, z. To form a nondegenerate triangle, the largest side must be less than the sum of the other two. There are 6 mutually exclusive orderings of the sides. To simplify, we ignore any equality in an ordering such as $x < y < z$ since its probability is zero. Now by the law of total probability

$$P[x,y,z \text{ form } \Delta] = \Sigma P[x,y,z \text{ form } \Delta \,|\, \text{ordering of } x,y,z]\, P[\text{ordering of } x,y,z],$$

where the sum is over the six possible orderings. By the symmetry involved, each

P[x,y,z form Δ| ordering of x,y,z] is equal and each P[ordering of x,y,z] = 1/6. In particular, choose x < y < z and compute

$$P[x,y,z \text{ form } \Delta \mid x < y < z] = \int_0^1\int_x^1\int_0^{x+y} dzdydz = 1/2.$$

Substitution now yields the result.

11. 0.702. Define events as follows. Let S = Jamal wins a scholarship, U = Jamal wins the university scholarship, $\tilde{U}$ = Jamal does not win it, C = the other common finalist wins the university scholarship, $\tilde{C}$ = the other common finalist does not win it. We condition on U since the university scholarship will always be taken by a student. From the law of total probability

$$P(\text{S}) = P(\text{S} \mid \text{U})\text{P(U)} + P(\text{S} \mid \tilde{U})\text{P}(\tilde{U}) = 1(1/3) + P(\text{S} \mid \tilde{U})(2/3).$$

In this equation, compute

$$P(S \mid \tilde{U}) = \frac{P(S\tilde{U})}{P(\tilde{U})} = \frac{P(S\tilde{U}C) + P(S\tilde{U}\tilde{C})}{P(\tilde{U})}$$

$$= \frac{P(S \mid \tilde{U}C)P(\tilde{U}C) + P(S \mid \tilde{U}\tilde{C})P(\tilde{U}\tilde{C})}{P(\tilde{U})}$$

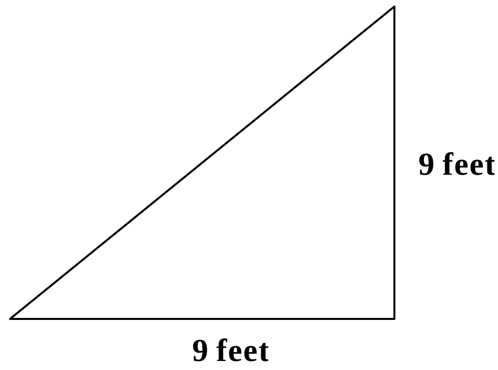
$$= \frac{P(S \mid \tilde{U}C)P(C \mid \tilde{U})P(\tilde{U}) + P(S \mid \tilde{U}\tilde{C})P(\tilde{C} \mid \tilde{U})P(\tilde{U})}{P(\tilde{U})}$$

$$= P(S \mid \tilde{U}C)P(C \mid \tilde{U}) + P(S \mid \tilde{U}\tilde{C})P(\tilde{C} \mid \tilde{U})$$

= (8/14)(1/2) +(8/15(1/2) = 116/210.

Thus P(S) = 221/315 = 0.702.

12. 1 cubic foot. Consider the object shown below, where a flat sheet of composite material has been bent to form a triangle in the side view. The hypotenuse portion has a hole in it as shown in the problem's figure. By making the sheet sufficiently thinner than the width of a line, less than 1 cubic foot of composite material would be required. Obviously zero cubic feet would not work.



13. 99.7 pounds. From the physics of circular motion, $mrw^2 = mg - N$, where w is the angular velocity, r is the earth's radius, and N is the net normal force exerted by the scale. Then $N = mg - mrw^2 = mg[1 - (r/g)(2pf)^2] = (0.997)mg = 99.7$ pounds for the frequency $f = 1/24$ revolution per hour.