PUZZLES

1. IMSE professor Dr. Maria Savant grows plants for a hobby. One evening she goes to eBay and buys some oncidium orchids at $15 each, monkey puzzle tree seeds at $1 each (http://www.victorialodging.com/monkey_puzzle_tree.htm), and iris bulbs at $0.25 each. Dr. Savant buys exactly 100 items (with at least one orchid, one seed, and one bulb) and spends exactly $100. How many bulbs did she buy?

2. Two EE students named Jose and Carlos meet in the lobby of Nedderman Hall to discuss a lab experiment. To determine who writes the report Jose suggests the following game, where the loser does the report. He places 40 pennies on a table. Then each player in turn removes 1, 3, or 5 pennies. The winner is the player who removes the last penny. Carlos agrees to the game, so Jose lets him go first. Select the correct statement from the choices below. Submit only the letter corresponding to your answer.

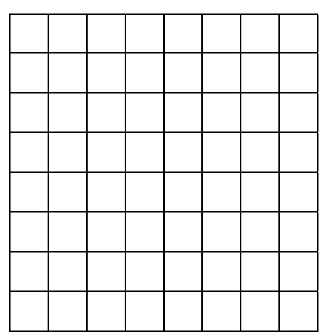
   (a) Jose can always win the game regardless of Carlos' strategy.

   (b) Carlos can always win the game regardless of Jose's strategy.

   (c) Neither player has a strategy that guarantees a win.

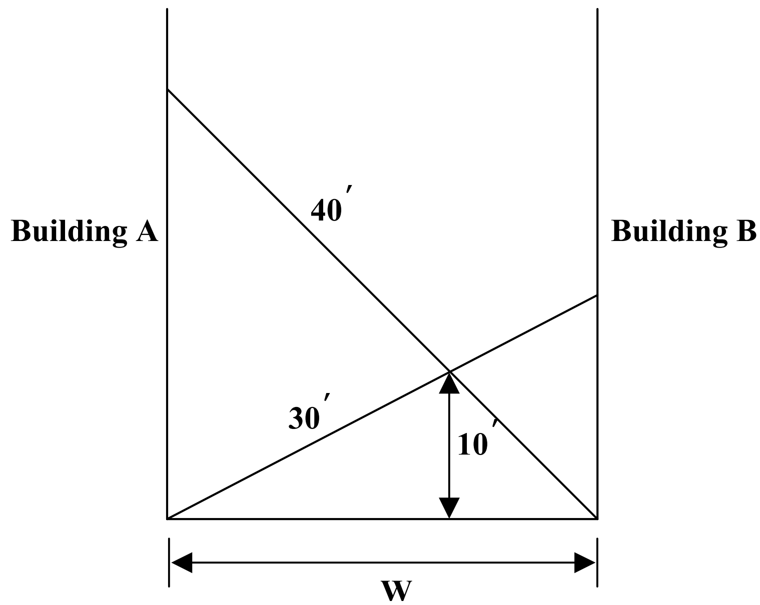
   (d) There is insufficient information to answer (a), (b), or (c).

3. The nation of Griddonesia consists of eighty-one equally-spaced islands represented by intersections of the lines in the grid below. These lines represent horizontal and vertical bridges exactly one-mile long that connect the islands.



   A Griddonesian environmental engineer named ♍♋♓⌘♌ has designed a small flying robot for the continuous monitoring of air-pollution levels on the islands. The Robird® is programmed as follows. After taking a pollution reading on an island, it is <u>equally likely</u> to fly to <u>any other</u> island for the next reading. The process then repeats automatically. If the Robird® starts on the center island, what is the probability that after three flights, it returns to the center island? Express your answer as a reduced fraction.

4. An ME named Jacob pays his UTA tuition by painting houses. The rear of his apartment building A is separated from another apartment building B by an alley of width W. Jacob stores his two longest ladders in the alley by laying them against opposite walls as shown below. Jacob's ladders intersect 10 feet above the alley. Find the width W of the alley in feet to the nearest three decimal places.



5. Six IE classmates decide to spread a rumor during Engineering Week about one of their professors and humorously (or perhaps not) dub themselves the "Rumor Tumors." On the first day of Engineering Week each of the six IE's tells it to six other friends with no duplications. Each of these six new Rumor Tumors is instructed to tell the rumor on each remaining day of Engineering Week, beginning the day after he/she hears it, to six more people who have not previously heard it. The original six IE classmates do likewise. Each new person who hears the rumor is given the same instructions. At the end of the seven days of Engineering Week, how many people know the rumor?

6. Beth, an EE, is doing lightbulb research. She installs three incandescent lightbulbs in three corners of a lab on the fifth floor of Nedderman Hall. Next she wires three switches, one per bulb, in the first-floor lobby. The switches are not labeled as to which switch controls a particular bulb, but all are installed in the off position. Beth then grabs a CSE student named Gunther and asks him to determine the switch that operates each lightbulb. No help or equipment is allowed. What is the minimum number of trips from lobby to lab required by Gunther to determine the switch corresponding to each lightbulb?

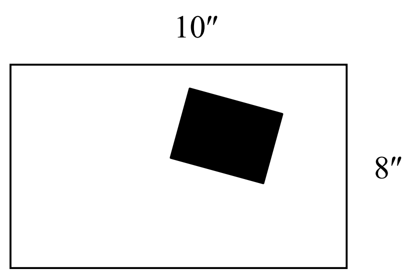
7. The American Psychiatric Association recently recognized as a valid psychiatric diagnosis the condition known as Nervous Examination Response Disorder (NERD) in which a student "freezes up" on examinations and performs badly. A biomedical engineering graduate student named Ramya has developed an

objective neurological test for NERD as part of her Ph.D. dissertation. Validating her test on UTA engineering students reveals that one in a hundred suffers from NERD and that her test has a 5% error rate for false negatives, 2% for false positives. To the nearest two decimal places, what is the probability that a student testing positively for NERD actually has the disorder?

8. Two CSE students, a female Sumalee and a male Xiao Hu, drive their respective cars $D > 0$ miles west on I-30. While driving, Sumalee averages 60 mph on her first pD miles for some $0 < p < 1$ and 70 mph on her last $(1 – p)D$. In addition, she stops for gas for exactly 15 minutes during her first pD miles. Similarly, Xiao Hu averages 60 mph on his first qD miles for some $0 < q < 1$ and 70 mph on his last $(1 – q)D$, but he stops for gas for exactly 15 minutes during his last $(1 – q)D$ miles. Determine all corresponding values of p and q for which Sumalee and Xiao Hu covcr the D miles in exactly the same time.

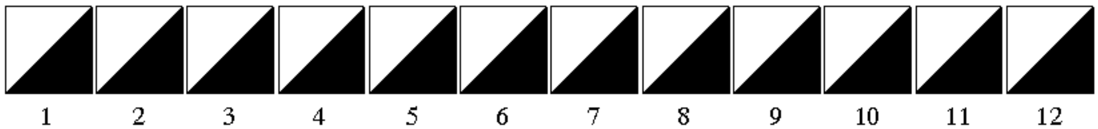
9. An ME student named Satish buys one Lotto ticket every Saturday and always chooses the cash-value option rather than 25 equal payments (one immediately and then yearly on the anniversary of the first payment). The cash-value option is the amount invested at 5% by the lottery commission that would yield the series of 25 payments and end with a zero balance. During Engineering Week, Satish learns that he has won the jackpot. Assume that the Internal Revenue Service automatically withholds 35% in taxes from each payment. To the nearest tenth, what percent of the jackpot amount will Satish receive (after taxes) as the cash value option?

10. A materials science graduate student named Tanya has designed a knife made from a new material called überium that is harder than diamond. She tests it on a thin 8″ × 10″ rectangular sheet of stainless steel with a smaller rectangular hole of unknown dimensions cut out of its interior at an unknown angle, as indicated in the figure below (not drawn to scale). Using only the knife, a pencil, and three standard 8 ½″ × 14″ pieces of legal paper, what is the minimum number of perfectly straight cuts that she can make with the knife through the top surface of the stainless steel sheet such that these cuts divide the sheet into two separate pieces of equal area of stainless steel? Any change in the direction of the knife constitutes an additional cut.



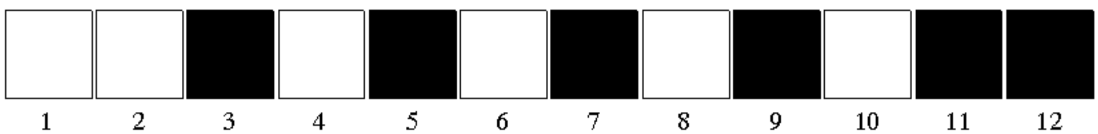
11. Dr. Frank N. Stein of the CSE faculty is teaching a course in quantum computing this spring semester. On the first test, the eminent AI guru gives a problem involving cellular automata. Consider the following cellular automaton, where the 12 squares are 12 quantum objects in superposition having values both 0 and 1

(i.e., qubits). Black represents the value 0 and white the value 1, so each square shows both colors. Furthermore, each object is entangled, or correlated, with exactly one other object. In other words, if an object collapses to value 0, its correlated object must collapse to value 1 and vice versa. Finally, no odd-numbered object is entangled with another odd-numbered object.



Suppose the 12 qubits are measured, and the superposition randomly collapses to the following cellular automaton.

To this cellular automaton, apply the rule: if a square is correlated with an adjacent square, switch colors; otherwise do not. The result is shown below.



What square is correlated with square 2?

12. An EE named Farid is taking a course in information theory, where he's currently studying codes. For a homework assignment Farid designs a code by letting 10 English letters be denoted by numbers as follows: 0 – I, 1 – E, 2 – U, 3 – A, 4 – V, 5 – N, 6 – R, 7 – F, 8 – P, 9 – S. If the English word "is" is numerically encrypted as $2\frac{2}{21}$ and "run" as $3\frac{5}{8}$, then what English word used in mathematics does $\pi + 1$ encrypt?

13. (Remember, it's a dirty dozen.) During Christmas break an IE student named Elisa goes on a skiing trip to Keystone, Colorado, which has one snowplow for the road from the town to the ski slopes. It plows at a rate inversely proportional to the depth of the snow on the road. On Elisa's first morning there, snow starts falling on the clear road at a constant rate. At 9:00 a.m. the snowplow starts plowing the road. It travels one mile in the first hour, then one-half mile in the second hour. At what time did the snow begin? Express your answer to the nearest minute in the form 8:12 a.m., for example.

ANSWERS

1. 56. Let x = the number of orchids, y = the number of seeds, and z = the number of bulbs. Then 15x + 1y + 0.25z = 100 and x + y + z + 100, where x, y, and z must be positive integers. Let x = 1 and solve to find that y and z are not both positive integers. Next let x = 2, and solve with the same result. For x = 3, one obtains y = 41, and z = 56. Letting x = 4, 5, or 6, however, again yields that y and z are not both positive integers.
2. (a). Each player takes an odd number of pennies per play. After Carlos plays first, there will be an odd number of pennies left. After Jose plays second there will be an even number of pennies left. The situation repeats. Hence, Carlos can never take the last penny since there will always be an even number.
3. 79/6400. Compute directly, or let p_n denote the probability that the robird returns to the center island after n flights, where $p_0 = 1$. From the law of total probability

   $p_{n+1} =$ P(on center after n + 1 flights) =

   P(on center after n + 1 | on center after n)P(on center after n) +

   P(on center after n + 1 | not on center after n)P(not on center after n) =

   0 + (1/80)(1 - p_n), n = 0, 1, 2, … .

   Thus $p_1 = 0$, $p_2 = 1/80$, $p_3 = 79/6400$.
4. 26.033. From similar triangles, $W = \frac{10W}{\sqrt{30^2 - W^2}} + \frac{10W}{\sqrt{40^2 - W^2}}$. Solve this equation in various ways (including MatLib, Mathcad, or Mathematica).
5. $7^7 \times 6 = 4{,}941{,}258$.
6. 1. Designate the switches as 1, 2, and 3. In the lobby, turn on switch 2 for ten minutes and then turn it off. Immediately turn switch 1 on and go to the lab. The lightbulb turned on is controlled by switch 1. The lightbulb off and warm is controlled by switch 2. The lightbulb off and cold is controlled by switch 3.
7. 0.32. Let TP denote the event "test positive," N denote "has NERD," and $\tilde{N}$ be its complement. Compute directly, or use Bayes' Theorem to write

$$P(N \mid TP) = \frac{P(TP \mid N)P(N)}{P(TP \mid N)P(N) + P(TP \mid \tilde{N})P(\tilde{N})}$$
$$= \frac{(0.95)(0.01)}{(0.95)(0.01) + (0.02)(0.99)}$$
$$\cong 0.324.$$

8. p = q for all 0 < p < 1. Solve $\frac{pD}{60} + \frac{(1-p)D}{70} + \frac{1}{4} = \frac{qD}{60} + \frac{(1-q)D}{70} + \frac{1}{4}$.

9. 38.5%. Let V be the jackpot value. Then he would receive V/25 immediately and then 24 more such annual payments. Now 0.04V in k years is worth $\frac{0.04V}{(1.05)^k}$ now.

Thus the present cash value after taxes is

$$(0.65)(0.04V)\sum_{k=0}^{24}\frac{1}{(1.05)^k} = (0.026)(14.7987)V \cong 0.3848V.$$

Dividing by V and rounding to the nearest tenth of a percent give the answer.

10. 1 cut. The diagonal of the 8″ × 10″ sheet is less than 14″. Place one piece of paper under the stainless steel such that the hole is completely filled with paper. Connect two opposite corners of the 8″ × 10″ sheet with a 14″ side of a second piece of paper. Then connect the other two opposite corners with a 14″ side of the third piece. Mark the intersection either on the stainless steel or the underlying paper, as the case may be. This intersection is the center of the 8″ × 10″ sheet. Next do the same with the rectangular hole, marking the center of the hole on the underlying sheet. Draw a line on the stainless steel that connects the two centers. Cutting along this line divides the both the sheet and hole into two equal pieces and gives the desired result. If the two centers coincide, any line drawn through the point would work.

11. Square 5. The given information produces a unique set of correlations.

12. "Even." The number $2\frac{2}{21} = 2.095\ldots$, and $3\frac{5}{8} = 3.6250\ldots$. The integer portion represents the number of letters, and the decimal digits (unrounded) gives the letters. Since $\pi + 1 = 4.14159\ldots$, it yields "even."

13. 8:23 a.m. Let b = the depth of snow at 9:00 a.m., and a = its rate of increase. Let t denote hours after 9:00 a.m., so t = 0 is 9:00 a.m. Hence, the snow depth at time t is at + b for positive constants a,b with the snowfall starting at –b/a. For positive constant c, the plow's rate of progress is ds/dt = c/(at + b), where s(t) is the distance traveled. Thus s(t) = (c/a) ln (1 + at/b). Note s(2) = 1.5 miles = 1.5s(1), so

$$(c/a)\ln(1 + 2a/b) = 1.5(c/a)\ln(1 + a/b).$$

Or,

$$\ln(1 + 2a/b) = 1.5\ln(1 + a/b) = \ln(1 + a/b)^{1.5}.$$

Taking the exponential function of both ends gives

$$1 + 2a/b = (1 + a/b)^{3/2}.$$

We seek – b/a, so let x = a/b, and solve

$$(1 + 2x)^2 = (1 + x)^3.$$

Then the starting time is 9:00 a.m. – (1/x converted to minutes).

The only relevant solution to the equation yields the exact answer

$8{:}(90 - 30\sqrt{5})$ a.m. $\cong 8{:}22.92$ a.m.