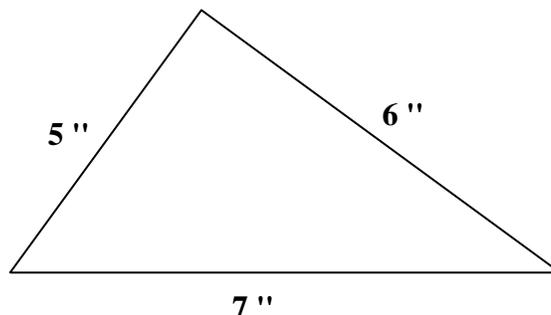


PUZZLES

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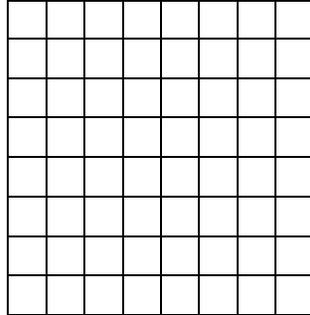
1. An EE student named Praveena is doing lightbulb research and gets the following lightbulb data for UTA engineering students. She finds that 2 EE's, 3 ME's, and 1 AE change 12 lightbulbs in 6 hours, while 3 EE's, 2 ME's, and 2 AE's change 8 lightbulbs in 4 hours. From this information, what is the maximum number of lightbulbs that 1 AE and 5 ME's can change in four hours? Assume that rate of changing lightbulbs by the three majors is additive.
2. A materials science graduate student named Hsia Tseng is working on a project funded by NASA. The objective is to construct a composite beam that will be used in a space station. Everywhere along its length the 9-foot-long beam must have the same cross-sectional shape shown in the following figure with dimensions in inches. In addition, NASA requires that the density of the beam in pounds per cubic foot must be a quadratic function of the form $d(x) = ax^2 + bx + c$, where x is the distance along the beam in yards from one end. At this end, the density must be 100 pounds per cubic foot as measured on the earth's surface in Houston. This density must increase to 500 pounds per cubic foot 2 yards from this low-density end and to 1000 pounds per cubic foot at the other end. If Hsia Tseng produces such a beam, to the nearest two decimal places how many pounds will it weigh at Houston?



3. Five biomedical engineering students meet in the lobby of Nedderman Hall to discuss a class project. What is the probability that at least two were born on the same day of the week (i.e., Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, or Sunday)? Express your answer as a reduced fraction.
4. IMSE professor Dr. Maria Savant enjoys keeping fit outdoors. Starting at noon one Saturday with her iPod, she runs up Mount Arlington at a constant speed, where she camps overnight. At noon the next day she runs halfway down along precisely same path at a speed exactly half as fast as she ran up. She then reverses instantaneously, retraces her path up, and returns to the top at twice the previous day's speed. At how many points on the path up was she at exactly the same place at exactly the same time on the two days? Submit only the letter corresponding to your answer.

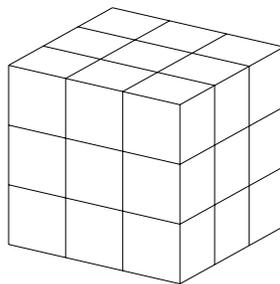
- (a) 0
- (b) 1
- (c) 2
- (d) None of the above.

5. The nation of Griddonesia consists of eighty-one equally-spaced islands represented by intersections of the lines in the grid below. These lines represent horizontal and vertical bridges exactly one-mile long that connect the islands.



Tragically, an earthquake in the ocean floor causes a tsunami that hits Griddonesia, and all the bridges are damaged. A Griddonesian civil engineering student named $\mathfrak{M}\mathfrak{D}\mathfrak{X}\mathfrak{K}\mathfrak{Q}$ returns home from UTA to help with the cleanup and reconstruction.

- (a) What is the minimum number of bridges that must be repaired so $\mathfrak{M}\mathfrak{D}\mathfrak{X}\mathfrak{K}\mathfrak{Q}$ can drive by car from every island to every other island?
 - (b) Suppose the monetary damage suffered on each island is the average of the closest islands to it along the original bridges. If the northwest (upper left) island has \$100 million damage, what amount of damage did the southeast (lower right) island suffer?
6. An ME graduate student named Hsu Wen working at ARRI (the Automation & Robotics Reseach Institute) is developing a miniature robotic “robo-rooter” for manufacturing purposes. As an experiment, Hsu stacks twenty-seven cubical wooden blocks as a single large cube three blocks high, wide, and long as shown below.

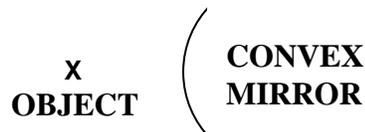


The robo-rooter inside the center cube is programmed to burrow through all twenty-seven blocks without visiting the same block twice. It will bore only into an adjacent block sharing a plane face, not into adjacent blocks that share only an edge or a corner. In how many ways can the robo-rooter accomplish this task?

6. Five female IE students – Ashley, Basheera, Chanya, Devi, and Elena – take an English course on Shakespeare taught by Dr. Reed Moore. On the third period, he gives a pop quiz on *All's Well That Ends Well* consisting of two multiple-choice questions (A, B, or C) and three true-or-false (T or F) questions. Which IE student gets the most correct answers if no two of the five have the same number of correct answers and if their answers are as follows?

| QUESTION | 1 | 2 | 3 | 4 | 5 |
|----------|---|---|---|---|---|
| Ashley | A | A | T | T | T |
| Basheera | B | B | T | F | T |
| Chanya | A | B | T | T | F |
| Devi | B | C | T | T | F |
| Elena | C | A | F | T | T |

8. The outside right-hand mirror of a new car purchased in the United States has the familiar warning: CAUTION: OBJECTS ARE CLOSER THAN THEY APPEAR. This warning appears because a convex mirror as below is used there unlike the plane mirror on the driver's side. Without the caveat, the brain would interpret the image as if the convex mirror were a plane one.



A convex mirror's advantage is that it provides a wider field of view than a plane mirror. Suppose in a convex mirror with radius 30 centimeters an AE student named Jamal sees the image of an SUV that is 2 meters tall and 20 meters behind the mirror. If he saw this same image in a plane mirror, to the nearest meter how far away would he interpret it to be?

9. An EE graduate student named Sara working in the Nanotech Center challenges Ryan, another EE there, to gamble on the following game. With their backs to each other, Sara and Ryan place a penny on their palms with either heads or tails up, then

turn and show their coins simultaneously. If both show heads, then Sara pays Ryan \$3. If both show tails, then Sara pays Ryan \$1. If the pennies do not match, then Ryan pays Sara \$2. Ryan agrees to play and states that he will choose heads and tails randomly (i.e., each is equally likely). If Sara uses her optimal strategy, on the average how much would she win or lose in a series of 300 games? Assume that neither player runs out of money.

10. Dr. Frank N. Stein of the CSE faculty is teaching a course in discrete mathematics this semester. On the very first day of class the eminent AI guru mentions Fermat's Last Theorem, which was stated by the French mathematician Fermat in 1630 but not proved until 1993. Fermat's Last Theorem states that the equation $x^n + y^n = z^n$ has no positive integer solutions x, y, z for $n = 3, 4, 5, \dots$. On the other hand, for $n = 2$, there is the obvious solution $3^2 + 4^2 = 5^2$. Thus the redoubtable Dr. Stein notes that the following statement is false. For all combinations of $k = 2, 3, 4, \dots$ and $n = 1, 2, 3, \dots$, there exist positive integers x_1, x_2, \dots, x_{k+1} satisfying

$$x_1^n + x_2^n + \dots + x_k^n = x_{k+1}^n.$$

In particular, this equation is false for $k = 2$ and $n = 3$ by Fermat's Last Theorem. On the in-class portion of his first quiz, Dr. Stein partially states Fermat's Lost (not Last) Theorem, as he terms it. There exist positive integers x_1, x_2, \dots, x_{k+1} satisfying

$$x_1^{1/n} + x_2^{1/n} + \dots + x_k^{1/n} = x_{k+1}^{1/n}.$$

He gives the four possible choices below as conditions for Fermat's Lost Theorem to be valid. Submit only the letter below corresponding to the correct answer.

- (a) Fermat's Lost Theorem is true for all combinations of $k = 2, 3, 4, \dots$ and $n = 1, 2, 3, \dots$.
 - (b) Fermat's Lost Theorem is true for some but not all combinations of $k = 2, 3, 4, \dots$ and $n = 1, 2, 3, \dots$.
 - (c) Fermat's Lost Theorem is false for all combination of $k = 2, 3, 4, \dots$ and $n = 1, 2, 3, \dots$.
 - (d) None of the above.
11. As a take-home problem on the first quiz of the discrete mathematics course of problem 10, Dr. Frank N. Stein gives the following question. Consider a function $f(n)$ defined on the positive integers $n = 1, 2, 3, \dots$ as the number of ones required to write out all numbers from 1 to n (including both 1 and n). For example, $f(15) = 8$. Note that $f(1) = 1$. What is the next largest positive integer n such that $f(n) = n$? Write your answer in the form 4,578,116,131, for example, if you obtain that number.
12. As another take-home problem on his first quiz in the discrete mathematics course of problems 10 and 11, Dr. Frank N. Stein gives the following question. Suppose that in a CSE class with $n \geq 2$ students, each student is given a different part of the directions for finding the room on the UTA campus where the final examination will be given.

Each student's part is needed to determine the exact room. Any two students are allowed to call each other any number of times on their cell phones (no conference calls are allowed) and exchange all the information they know about the location. After each such call, each of the two students knows all parts of the directions that the other student knew before the call. What is the smallest number of calls required so that all n students know all directions for finding the location of the final examination? Hint: break your answer into the cases $n = 2$, $n = 3$, and $n \geq 4$.

13. (Remember, it's a dirty dozen.) Two ME students named Carlos and Jose enjoy souping up their respective cars and drag racing them against each other at a local drag strip. In their most recent race, each accelerates at a uniform rate from a standing start. Carlos covers the last one-fourth of the distance in 3 seconds, while Jose covers the last one-third in 4 seconds.
- (a) Who won this race?
 - (b) To the nearest three decimal places, by how many seconds did the winner cross the finish line ahead of loser?

ANSWERS

12. The equations have no unique solution. Maximizing $20M + 4A$ subject to the given information with $E, M, A \geq 0$ yields for $M = 0.5$ and $A = 0.5$.
- 367.42 pounds. By Heron's formula, the area of the cross section is $6\sqrt{6}$ square inches. Solving for the values of a, b, c from the three equations obtained from $d(x)$ at $x = 0, 2, 3$ gives that $d(x) = 100(x^2 + 1)$. Then the average density along the length is $(1/3)\int_0^3 100(x^2 + 1)dx = 400$ pounds per cubic foot. The total weight is thus $9(400)(6/144)\sqrt{6} = 367.42$ pounds.
- 2041/2401.
- (b). She is at the same point at the same time $2/3$ of the way up the mountain path.
- (a) 80 (b) \$100 million
- 0.
- Devi with four correct answers.
2686. Use the mirror equation with the sign convention for convex mirrors to get a magnification of 0.00745. But the magnification for a plane mirror is 1. Hence, the distance is $20/0.00745$ meters. The number is so large because the radius is so small.
- \$150 by choosing tails all the time.
- (a).
- 199, 981 using a computer.
- 1 for $n = 2, 3$ for $n = 3, 2n - 4$ for $n = 4, 5, 6, \dots$
- (a) Jose (b) 0.594 seconds - Use the equations of accelerated motion and find the completion times.