PUZZLES

1. An EE graduate student named Siriwat works in the Nanotech Center. He has developed and patented a miniature robotic Nanobot® for NASA to use in exploring the surface of Mars during a future mission. To test its traction, he places it at the top of the side of a vertical metal cylinder exactly 1.2 meters long, with radius $20/\pi$ centimeters. The Nanobot® then moves down the cylinder in a perfect uniform spiral for three complete revolutions until it reaches the bottom. What is the length of its path down the cylinder to the nearest two decimal places?

2. A CSE student named Sara uses a computer simulation program to randomly generate five points on the surface of a sphere with radius $10/\pi^2$ feet. What is the maximum number of points that are <u>guaranteed</u> to lie in <u>some</u> closed hemisphere (one that includes the great circle dividing the surface of the sphere into two equal areas)?

3. Bioengineering professor Dr. Mi Yin has developed a lie-detecting device that is 100% accurate. In testing her invention, she learned that AE's under 21 years old always tell the truth whereas AE's 21 or older always lie. Furthermore, ME's under 21 always lie, whereas ME's 21 or older always tell the truth. Suppose a group of three AE's and three ME's tell us the following facts.

   <u>Aran</u>: Bimal and Chris are equally truthful. Bimal is the youngest of all.

   <u>Bimal</u>: Fahid is the oldest of us all. Devi is the youngest ME.

   <u>Chris</u>: Erika is the oldest ME. Fahid is an AE.

   <u>Devi</u>: There is at least one ME here younger than Aran.

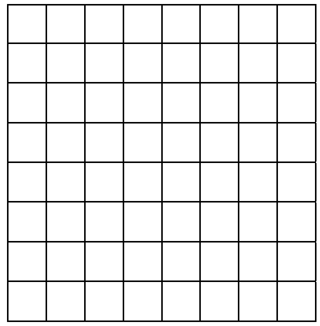
   <u>Erika</u>: Fahid is less truthful than me.

   <u>Fahid</u>: If you asked Devi, she would tell you that Chris is an ME.

   (a) List the six students in increasing age by the first letter of their names.

   (b) How many of the six are under 21 years old?

4. The nation of Griddonesia consists of eighty-one equally-spaced islands represented by intersections of the lines in the following grid, where north is up and east is right as on a standard map. Each island is connected to all its adjacent islands by horizontal and vertical bridges exactly one-mile long. There are no diagonal bridges. A native environmental engineer named ♍♋♓⌘♌ lives on the northwest (upper left) island. One Sunday he takes his girlfriend on a leisurely drive to visit friends on the southeast (lower right) island. Obviously a route with shortest driving distance across bridges is 16 miles. What is the distance of a route between the two islands with the longest driving distance across bridges without crossing any bridge twice?



5. Two CE students play a game on a round table of radius 4 feet at the University Center. One puts a $1 dollar coin of radius 1 inch anywhere on the table, followed by the other putting another $1 dollar coin anywhere on the table without touching or moving the first coin in any way. Thereafter the players alternate in a similar fashion, placing a $1 coin anywhere on the table without touching or moving a previous coin. The game ends when one player cannot fit his coin on the table without violating this condition. The other player then wins all the coins. Submit only the letter below corresponding to the correct answer under the assumption that neither player will run out of $1 dollar coins.

   (a) There is no strategy for either player to always win.
   (b) The player going first can always win with a suitable strategy.
   (c) The player going second can always win with a suitable strategy.
   (d) There is not enough information to select (a) – (c) with certainty.

6. Dr. Frank N. Stein of the CSE faculty is teaching a course in decision-making this semester. For the first quiz, the eminent AI guru gives an oral examination designed as follows. There are 4 questions of which everyone must attempt the first question. If this first question is answered incorrectly, a student receives a numerical grade of 0 on the quiz and cannot proceed. Otherwise, he receives a 50. In this case, for the remaining 3 questions the student has the option of keeping his/her numerical grade from the previous question or attempting the next question for a higher grade. If the student misses any optional question, he/she receives the consolation grade for that question and cannot continue. The probabilities of correctly answering the three optional questions, the grade received for a correct response, and the consolation grade are given below.

Optional Question	Probability	Grade	Consolation Grade
first	0.6	69	40
second	0.5	80	55
third	0.4	100	75

   Suppose a student's objective is to maximize the expected numerical grade over the entire oral examination. If the student correctly answers the compulsory and first optional question, to the nearest decimal place what is the expected grade the student will achieve on the rest of the examination if he/she proceeds optimally?

7. An EE graduate student named Pratana loves to do the daily 9×9 Sudoku puzzle in the *Shorthorn*. She even makes up 4×4 Sudoku puzzles for her precocious 5-year-old daughter Jennifer, where a completed 4×4 Sudoku puzzle is shown below. It is a 4×4 grid such that every row, every column, and every 2×2 subgrid in bold lines contains the digits 1 through 4. How many such different completed 4×4 Sudoku puzzles are possible?

2	**3**	**1**	**4**
1	**4**	**2**	**3**
4	**1**	**3**	**2**
3	**2**	**4**	**1**

8. IMSE professor Dr. Maria Savant teaches an 8:00 a.m. statistics class on MWF. On the first day of Engineering Week, she makes the following observation. The number x of students in class wearing glasses and the number y in class not wearing glasses satisfy the relationship $x^2(y+4) = y^2$ at 8:00 a.m., before all students have arrived, and at 8:20 a.m., after all students have arrived.

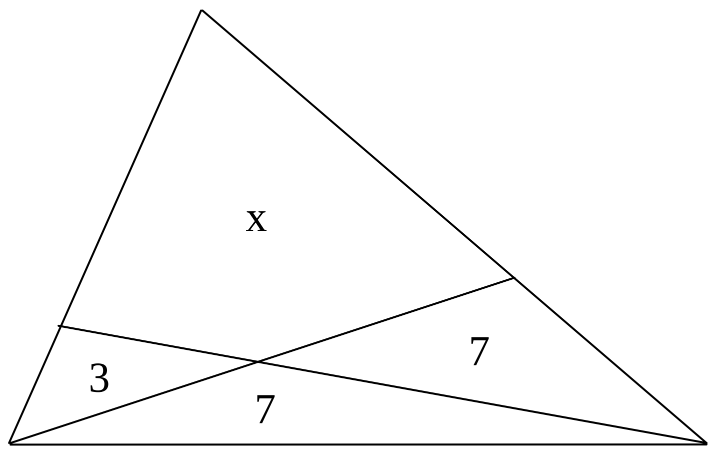
   (a) How many students are in class at 8:00 a.m.?

   (b) How many students are in class at 8:20 a.m.?

9. An EE named Dirk plays basketball every afternoon at the Rec Center. One day he practices shooting free throws. He hits the first shot and misses the second. Thereafter, the probability that he hits shot $n = 3, 4, 5, \dots$, equals the proportion of shots he has hit in the first $n - 1$. What is the probability that he hits exactly 50 of his first 100 shots? Express your answer as a reduced fraction.

10. A ME named Brooke works as a movie stunt driver during the summer. On her first movie, which is being filmed on the west coast of Mexico, she is asked to drive a motorcycle horizontally off a 50-meter-high cliff. At what velocity must the motorcycle leave the cliff top so it lands in the bay below, 90 meters from the base of the vertical cliff, where the cameras are filming in a boat? Express your answer in miles per hour to the nearest mile. Neglect air resistance.

11. A materials science student named Carlos needs to cut the large triangular piece of composite material below into three triangles of areas 3, 7, and 7 square feet, plus a quadrilateral with area x. To the nearest two decimal places, find x.



12. An ME student named Luke drives west on I-30. On this stretch of I-30, he always drives downhill at 72 m.p.h., level at 63 m.p.h., and uphill at 56 m.p.h. Luke takes 4 hours to travel from town Alpha, Texas, to Beta, Texas. But the return trip on I-30 takes 4 hours and 40 minutes. Find the distance in miles between Alpha and Beta to the nearest decimal place. Assume both sides of I-30 have the same grade, plus such idealizations as the car's constant velocity with instantaneous changes in speed when the road changes.

13. (Remember, it's a dirty dozen.) During Christmas break an EE student named Ryan goes on a skiing trip to Keystone, Colorado. The morning after his arrival he awakes to a snowfall. Let s(t) denote the depth in inches of the snow t hours after 8 a.m., when the snow is falling at one inch per hour. Suppose that this rate <u>increases</u> uniformly two inches per hour. However, the snow is also melting at a rate in inches per hour equal to the depth of snow. To the nearest three decimal places, find an initial depth of snow d at 8 a.m. such that the depth of snow is the same at 9 a.m.

ANSWERS

1. 1.70 meters. The circumference of the circle of the cylinder top is 0.40 meters. Hence the Nanobot® moves 1.2 meters down and 1.2 meters laterally. Its path length is then the hypotenuse of an isosceles right triangle with equal sides of 1.2 meters. Thus by the Pythagorean theorem the path length is 1.70 meters.
2. 4. Take two of the points and draw a great circle through them forming two closed hemispheres. Then at least two of the remaining three points lie in one of the two closed hemispheres so formed. But all three remaining points are not.
3. (a) BADCEF (b) 3.
4. 128 miles.
5. (b). The first player places a coin exactly in the center of table. Then the second player must put a coin somewhere else. Thereafter, the first player puts his coin on a spot 180 degrees from the other player's coin and the same distance from the center. At some point there will be room for any more coins on the second player's turn.
6. 70.0. The complete optimal policy is given below.

   Max{80, 0.4(100)+0.6(75) = 85} = 85   Take optional question 3 if possible.

   Max{69, 0.5(85)+0.5(55) = 70} = 70   Take optional question 2 if possible.

   Max{50, 0.6(70)+0.6(40) = 66} = 66   Take optional question 1 if possible.
7. 384 = 4! × (2! × 2!) × (2! ×2!) × 1. For a 9×9 Sudoku grid, the possible number of completed puzzles is 6,670,903,752,021,072,936,960 according to the web. The number of starting standard Sudoku puzzles is much, much greater.
8. (a) 0 (b) 15.

   Solve $y^2 - x^2y - 4x^2 = 0$ as a quadratic in terms of x. Then $y = \frac{x}{2}\left(x \pm \sqrt{x^2 + 16}\right)$.

   Since y is an integer, the radicand must be a perfect square. Hence, $x = 0$ or ± 3. Thus all solutions are (0,0), $(\pm 3, -3)$, $(\pm 3, 12)$. The only nonnegative integer pairs are (0,0) and (3,12).
9. 1/99. An outline of a proof is as follows. In general, after n shots here, the probability of making any number of shots from 1 to n – 1 is 1/(n – 1). It's obviously true for

   n = 2. Assume it's true for n, and show it's true for n + 1. Then let n = 100.
10. 63 miles per hour. Standard kinematic equations for projectile motion give 28.2 meters per second in the original units, which must be converted.
11. 18.00 square feet. Break the quadrilateral into two triangles by connecting the top corner of the large triangle to the intersection of the 3 inner triangles. Then find the areas of the new triangles and add them.

12. 273.0 miles. Let the total distance traveled downhill, on the level, and uphill, from Alpha to Beta be x, y, and z, respectively. Hence the trip from Alpha to Beta takes $x/72 + y/63 + z/56 = 4$. From Beta to Alpha the time is $x/56 + y/63 + z/72 = 14/3$. We seek the value of $x + y + z$, not the values of the individual variables. Multiplying both equations by the least common multiple of denominators 56, 63, and 72, we obtain

$$7x + 8y + 9z = 4 \cdot 7 \cdot 8 \cdot 9$$
$$9x + 8y + 7z = (14/3) \cdot 7 \cdot 8 \cdot 9.$$

Adding these equations yields

$$16(x + y + z) = (26/3) \cdot 7 \cdot 8 \cdot 9.$$

Therefore $x + y + z = 273$.

13. 2.164 inches. The snowfall is described by the differential equation

$$\frac{ds}{dt} = -s(t) + 2t + 1.$$

For an initial depth d at $t = 0$, $s(t) = (d+1)e^{-t} + 2t - 1$. Set $s(0) = s(1)$ and solve to give $d = 2.164$ inches.