# PUZZLES

1. In the election for UTA Student Congress president, where a plurality of votes wins, 5,219 total votes were cast for the four candidates A, B, C, D. The number of votes for the winning candidate C, IMSE student Zack Cohen, exceeds his opponents A, B, and D by 22, 30, and 73 votes, respectively. List the four candidates and the number of votes obtained by each in the format: A – w votes, B – x votes, C – y votes, D – z votes.

2. An environmental engineering graduate student named Ravi is studying global warming. He has determined that the optimal location to place a sensor for measuring the change in average global temperature over time must satisfy the equations

$$\frac{1}{x} + \frac{1}{2y} = (x^2 + 3y^2)(3x^2 + y^2)$$

$$\frac{1}{x} - \frac{1}{2y} = 2(y^4 - x^4),$$

where x and y are in miles, the origin of the right-handed coordinate system is the magnetic north pole with y south along the prime meridian, and the earth is assumed flat in a circle of radius 5 miles about this origin. What is the <u>sum</u> in miles of the optimal coordinates x* and y* solving the above equations? Express your answer as a two-decimal real number to a fractional power, i.e., $(2.75)^{1/3}$, to ensure that your solution was obtained analytically without computer software.

3. An AE student named Svetlana lives in Kalpana Chawla Hall. She places a map of the United States flat on the floor of her dormitory room. Suppose the scale of this map is 1 to 50 million. How many points on this map will directly lie above their corresponding actual locations in the United States?

   (a) None.
   (b) Exactly 1.
   (c) An infinite number.
   (d) There is not enough information to select (a) – (c) with certainty.

4. EE professor Dr. Max Short is concerned about his 403(b) plan in which he has all his retirement money invested in mutual funds. He wishes to approximate the average annual net return required for the current value of his retirement plan to double in exactly eight years, excluding any future contributions to it. For his estimate, he assumes that the average annual net return over these eight years is achieved at the end of each year and that this new total is then compounded the following year in the same way. To the nearest tenth of a percent, calculate the average annual net interest rate necessary to double the present value of his account in exactly eight years.

5. The nation of Griddonesia consists of eighty-one equally-spaced islands represented by intersections of the lines in the following grid, where north is up and east is right

as on a standard map. Each island is connected to all its adjacent islands by horizontal and vertical bridges exactly one-mile long. There are no diagonal bridges. A native nautical engineer student named ♍︎♋︎♓︎♈︎♌ plans to enter the famous Griddonesia Madcap Regatta on April 1, in which each contestant <u>must</u> sail his/her sailboat under <u>all</u> 112 "interior" bridges not forming any part of the grid's perimeter. In doing so, the rules require that each contestant must enter and exit the island waters as shown by the arrows below. In addition, he/she must make only <u>right-angle</u> turns and must sail under the Sun Bridge (designated by the sun symbol), which is Griddonesia's only suspension bridge and the world's third longest. Otherwise, a contestant may use any route and even sail outside the grid. The winner is the contestant who sails a valid regatta course in the least time. With his engineering background, ♍︎♋︎♓︎♈︎♌ intends to maximize his chance of winning.

(a) What is the fewest number of right-angle turns required to sail a valid regatta course in the shortest distance from the starting to ending bridges?

(b)  How many distinct such shortest routes exist?



end  begin
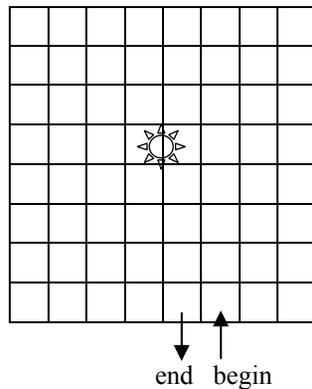
6.  Dr. Frank N. Stein of the CSE faculty is teaching a course in logic this semester. To begin his lecture in the first class, the eminent AI guru proposes the following list of statements numbered 1 to 100. Statement n says, "Exactly n-1 other statements are true." As your answer, submit only the letter corresponding to the correct description of these 100 statements.

(a) Each statement contradicts itself.

(b) All statements are true, and none is false.

(c) All statements are false, and none is true.

(d) Exactly one statement is true, and the rest are false.

(e) Exactly one statement is false, and the rest are true.

(f) Exactly half the statements are true, and half are false.

7. A ME named David places five spherical plastic ball bearings of various sizes in a conical metal funnel. Each ball bearing touches an adjacent ball bearing at exactly one point, and the entire circumference of a ball bearing touches the funnel wall. The smallest ball bearing has radius 8mm., while the largest radius has 18mm. What is the radius of the middle ball bearing?

18

8
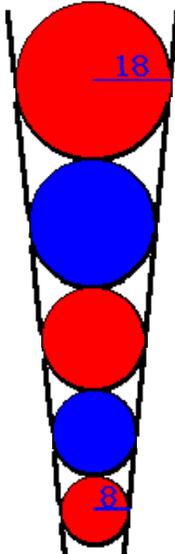
8. IMSE professor Dr. Maria Savant is the last of 100 people in line to board a chartered flight for a skiing trip to Keystone, Colorado. There are 100 seats on the plane, each assigned to exactly one passenger. Before boarding, Dr. Savant proposes an experiment to the other passengers, who agree to it. Then the first person in line to board the plane follows her instructions and takes a random seat that has a one-in-a-hundred chance of being his assigned seat. Thereafter, each passenger takes his/her assigned seat if it is empty. If not, the passenger takes a seat at random. Finally, Dr. Savant boards and takes the only remaining empty seat. What is the probability that she sits in her assigned seat? Express your answer as a reduced fraction.

9. An civil engineering student named Bret has a set of 5 identical coins, where coin $C_n$, $n = 1, \ldots, 5$, is biased so that its probability of landing heads is $1 / (2n + 1)$. If Bret tosses these five coins on a table, what is the probability that the number of heads is odd? Express your answer as a reduced fraction.

10. Bioengineering professor Dr. Mi Yin is studying in the lab how bats navigate using sonar. From a stationary source, she directs a 5000-Hz sound wave at a flat piece of metal moving on tracks toward the source at the constant speed of 3.50 meters per second. What is the frequency of the reflected sound wave to the nearest hertz?

11. A materials science student named Antara has a board of composite material as shown below. The board is obtained from a rectangle twice as long as it is wide by removing at an angle a piece from the upper right corner, beginning at a vertical distance from the lower right corner equal to the rectangle's width. What is the minimum number of pieces into which she can cut this board and then rearrange the pieces to form a perfect square?
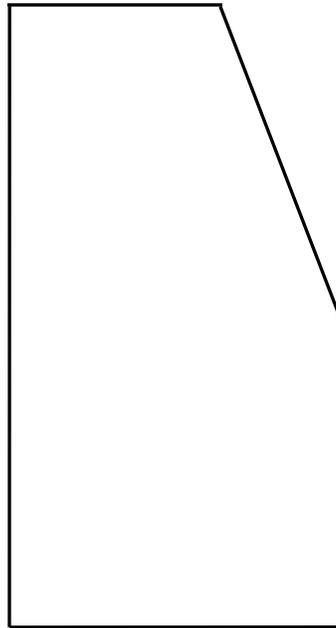
12  A CSE graduate student named Alex loves to do the daily *Shorthorn* cryptarithm, a type of puzzle in which most or all of the digits in an arithmetic operation such as a sum are replaced by letters or other symbols. The objective is to determine what numbers the symbols represent. Sitting at a table in the University Center, Alex turns to the following multiplication cryptarithm in the *Shorthorn*, where each asterisk represents a prime number 2, 3, 5, or 7.

```
        * * *
      ×   * *
      ─────────
      * * * *
    * * * *
    ───────────
    * * * * *
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Determine the unique solution to this puzzle, where each of the prime numbers 2, 3, 5, or 7 can obviously be used more than once. For your answer, give only the five-digit integer final product for the above multiplication.

13.  (Remember, it's a dirty dozen.) Three EE students named Jorge, Carlos, and Miquel take a trip of forty miles using a souped-up two-person Segway electric scooter. Since it can carry only two of them at a time, the third student must walk. In particular, Jorge walks at six miles per hour, Carlos walks at four miles per hour, and Miquel walks at three miles per hour. The Segway travels at forty miles per hour for any pair of students riding it. What is the shortest time for <u>all</u> three students to complete the trip with the most efficient combination of two students riding the Segway and one student walking? Assume constant speeds for the Segway and three students at the above rates, instantaneous stops and turns of the scooter, and instantaneous exchanges of one rider for another. State your answer in hours to the nearest tenth.

# ANSWERS

1. A – 1314, B – 1306, C – 1336, D – 1263. Add the winning margins to the total vote and divide by the number of candidates. The quotient will be the winning number of votes, from which the others can be obtained.

2. $(3.00)^{1/5}$ miles. Add and subtract the two given equations to give an equivalent pair. Multiply one by x and the other by y to obtain $3 = (x + y)^5$ and $1 = (x - y)^5$. Solve to get $x^* = (3^{1/5} + 1)/2$ and $y^* = (3^{1/5} - 1)/2$. The answer is found by adding $x^*$ and $y^*$.

3. (b) Exactly 1. Some reflection will convince you that there is one. But the difference in scale between the map and the actual room dictates no more than one.

4. 9.1%. Let P be the present value of his account. Then he seeks the interest rate i for which $(1 + i)^8 P = 2P$. Or, $(1 + i)^8 = 2$. Then $i = \exp[(\ln 2) / 8] - 1 = 0.0905$. Note that this value is very close to 9% and that $8 \times 9 = 72$. This result is a consequence of the "rule of 72," which states that for reasonable interest rates, then the number of years n for an amount P to double is the interest rate i for which $n \times i = 72$. Similarly, his investment would approximately double in 6 years at 12% interest. At this rate, it follows that his original money would increase eightfold in 18 years since it would double three times.

5. (a) 14   (b) 1.

6. (a). If statement 1 is true, then the other statements require it to be false. If statement 1 is false, then the other statements require it to be true. Similar reasoning applies to each statement in succession.

7. 12 mm. Consider two adjacent ball bearings, of radii a < b. We will show that b/a is a constant, whose value depends only upon the slope of the funnel wall.

   The spheres are in contact with each other, and therefore the vertical distance between their centers is b + a.

   The spheres are also in contact with the funnel wall. Since the slope of the funnel wall (in cross section) is a constant, the two green triangles are similar. Hence the horizontal distance from the center of each sphere to the funnel wall is bc and ac, respectively, where c = sec(x) is a constant dependent upon the slope of the funnel wall with x the angle the funnel wall makes with the vertical.
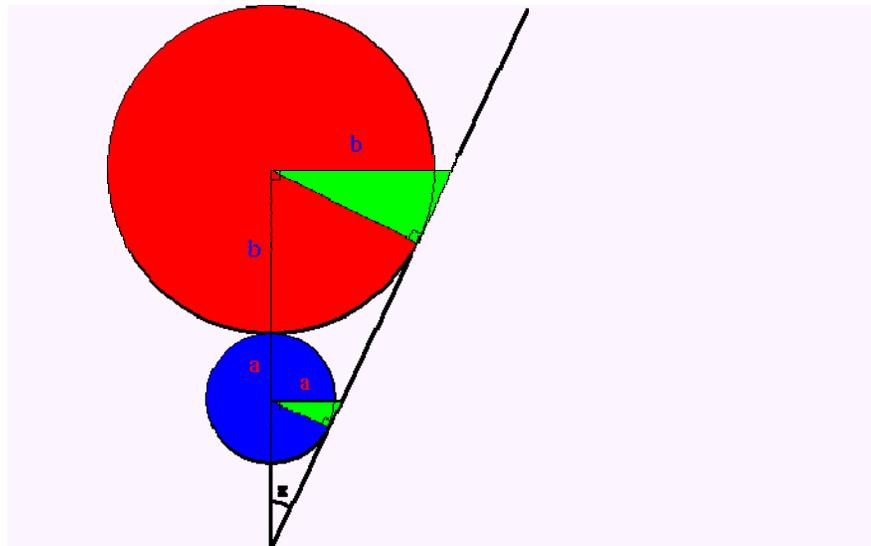
Let the slope of the funnel wall be m. Then $m = (b + a) / [(b - a)c]$.
Rearranging, $b/a = (mc + 1)/(mc - 1)$. Hence the ratio of the radii of adjacent spheres is a constant, dependent only upon the slope of the funnel wall. Let this constant be k. In this case, we have $18 = 8k^4$, so $k^2 = 3/2$.

Therefore the radius of the middle ball bearing is $8 \cdot (3/2) = 12$mm.

8. 1/2. Without loss of generality, suppose that the 100 seats are arranged in a straight line and numbered 100, …, 1 from front to rear of the plane, that the entry door is at the front, and that seat n is assigned to the $n^{th}$ passenger in line. Hence, the first passenger in line is assigned seat 1 at the rear of the plane, and Dr. Savant is assigned seat 100 at the front. If the first passenger takes any seat 2, …, 99, then it can be shown by taking cases that Dr. Savant ends up getting her assigned seat if and only if seat 1 is taken before seat 100 by a displaced passenger. Since a displaced passenger picks a seat randomly from the empty seats, the probability that seat 1 is taken before seat 100 equals 1/2. Hence, using the law of total probability,

P(Savant gets assigned seat) = P(Savant gets her assigned seat | first passenger in line takes seat 1) P(first passenger takes seat 1) + P(Savant gets her assigned seat | first passenger takes seat 100) P(first passenger takes seat 100) + P(Savant gets her assigned seat | first passenger takes some seat 2, …, 100) P(first passenger takes some seat seat 2, …, 99) = (1)(1/100) + 0(1/100) + (1/2)(98/100) = 1/2.

9. 5/11. In general, let $P_N$ denote the probability for the positive integer N, where N = 5 in the problem. Then $P_1 = 1/3$, $P_2 = 2/5$, $P_3 = 3/7$, $P_4 = 4/9$, and $P_5 = 5/11$.

10. 5104 Hz. There are two Doppler shifts in this situation. The metal acts as a moving observer and detects a sound wave of 5051 Hz according to the standard frequency equation. Then the metal acts as a moving source and reflects this 5051-Hz sound wave at 5104 Hz.

11. 3. Draw a line from the middle E of the left side to the middle D of line BC. Then draw a perpendicular from corner F to line DE. Cut along these straight lines. Then turn over piece A, and fit the three pieces together to form a perfect square.

B

**A**

D

E

C

F
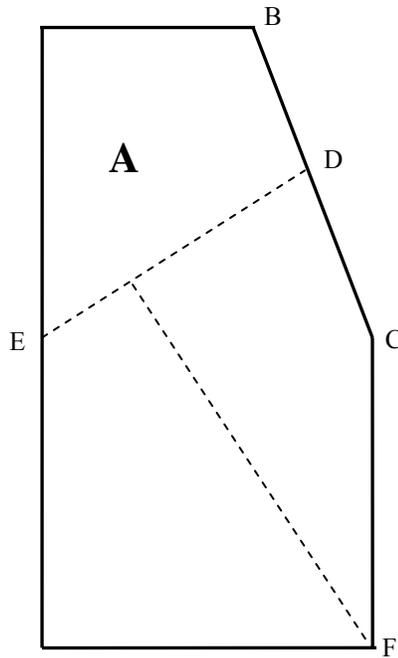
12. 25575.

13. 2.3 hours. Miquel, the slowest walker, always rides the Segway. He and Jorge, the fastest walker, ride it for 31.04 miles, while Carlos walks. Then Jorge dismounts, and Miquel rides back to pick up Carlos at a spot 5.63 miles from the start. Miquel and Carlos turn the Seqway around and ride it the rest of the trip, arriving at the same time as Jorge.z