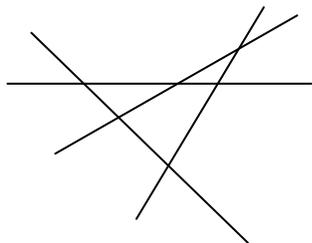


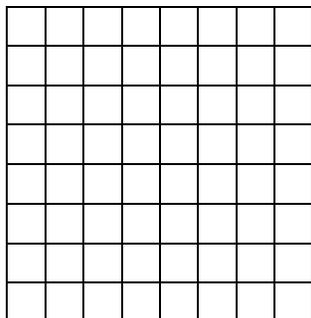
PUZZLES

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1. Ninety UTA engineering students attend a political rally for the upcoming March 4 Texas primary. Pizza, soda, and cookies are served at the rally. Three students have pizza, soda, and cookies; 24 have pizza; 5 have pizza and soda; 33 have soda; 10 have soda and cookies; 38 have cookies; and 8 have pizza and cookies. How many of these 90 engineering students have neither pizza, soda, nor cookies?
2. An IE student named Carlos lives in Kalpana Chawla Hall. From his closet he pulls out three shoe boxes labeled for their contents: BB containing two black sneakers, BW containing one black and one white sneaker, and WW containing two white sneakers. Carlos knows that his roommate Jose has switched the labels so that each box is now incorrectly labeled. If Carlos takes one shoe at a time from any box without looking inside, what is the minimum number of shoes that he must remove to determine the contents of each shoe box?
3. A materials science graduate student named Davana configures four straight wires of the same length to cross each other a maximum number of times, which is 6 in the diagram below. What is the maximum number of crossings for 20 such wires?



4. The nation of Griddonesia consists of eighty-one equally-spaced islands represented by intersections of the lines in the following grid, where north is up and east is right as on a standard map. Each island is connected to all its adjacent islands by horizontal and vertical bridges exactly one-mile long. There are no diagonal bridges. A Griddonesian environmental engineer named $\mathfrak{M}\mathfrak{D}\mathfrak{X}\mathfrak{K}$, a UTA graduate, seeks the following information. In the representation of Griddonesia below, how many different squares are there with sides either 1, 2, 3, 4, 5, 6, 7, or 8 miles in length?



5. Dr. Frank N. Stein of the CSE faculty is teaching a graduate course in logic this semester. To emphasize the importance of deductive reasoning, the eminent AI guru begins his first lecture with the following imaginary scenario. Suppose Dr. Stein had his laptop stolen by one of five students in the class: Linda, Judy, David, Tom, or Michelle. When questioned by the campus police, each of these students makes 3 statements.

Linda: (a) I didn't steal the laptop.
(b) I have never stolen anything in my life.
(c) Tom stole it.

Judy: (d) I didn't steal the laptop.
(e) I have my own laptop.
(f) Michelle knows who stole it.

David: (g) I didn't steal the laptop.
(h) I didn't know Michelle before I enrolled at UTA.
(i) Tom stole it.

Tom: (j) I didn't steal the laptop.
(k) Michelle stole it.
(l) Linda is lying when she says that I stole the laptop.

Michelle: (m) I didn't steal the laptop.
(n) Judy stole it.
(o) David has known me since elementary school.

Later each of these students admits that two of his/her statements were true and that one was false. Who stole the Dr. Stein's laptop?

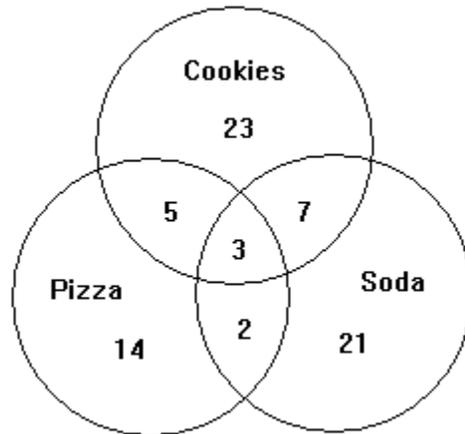
6. An AE student named Evita hears about the discovery of the new dwarf planet Neo, which is larger than Eris. In a thought experiment, she assumes that Neo is a perfectly smooth sphere with a nano-thin wire stretched tightly against the surface around the equator. This wire is then lengthened one yard and raised the same distance off Neo's surface around the equator. To the nearest decimal, how high in inches will the wire be raised off the surface of Neo?

7. EE professor Dr. Max Short receives a grant that is a whole number of dollars represented by a five-digit number in which no two digits are the same. This number of dollars satisfies the following conditions: (i) the sum of the first digit and second digit equals the third digit; (ii) the third digit times 3 equals the fourth digit; (iii) the second digit times 2 equals the first digit; and (iv) the grant is a prime number of dollars. What is the dollar amount of the grant?
8. IMSE professor Dr. Maria Savant attends an applied probability conference at the MGM Grand Hotel in Las Vegas and plays roulette one evening. Her strategy is to play \$1 on red each time, then double her bet each time she loses until she wins. Since 18 of the 38 spaces on the roulette wheel are red, the probability is $\frac{9}{19}$ that she wins any bet.
 - (a) To the nearest tenth, what is her expected loss on each bet in cents without considering any previous losses?
 - (b) What are her expected net winnings if she continues this betting system until she wins for the first time?
9. Frustrated by a failed lab experiment one afternoon, bioengineering professor Dr. Mi Yin needs some exercise. After changing into his running attire at the MAC, he goes outside. First he runs on a level road, then comes to a hill and runs to the top. When he reaches the top of the hill, he turns around and runs back exactly the same way, stopping at his starting point. Dr. Yin runs at 8 miles an hour on level ground, 6 miles an hour uphill, and 12 miles an hour downhill. If he runs for exactly two hours, how far did Dr. Yin run in miles, to the nearest tenth of a mile?
10. The CSE student named Tom of problem 5 has a nightmare after the first class with Dr. Frank N. Stein. He dreams that Dr. Stein gives him 10 white balls and 10 black balls, identical except in color, plus two boxes. In the dream Dr. Stein tells him to distribute the 20 balls between the two identical boxes, denoted I and II. Dr. Stein will then randomly choose a box and then a ball from the chosen box. If this chosen ball is white, Tom gets an A. But if the ball is black, Tom gets an F.
 - (a) In this dream, how should Tom distribute the balls between I and II to maximize his chances of getting an A? Give your answer in form: I - 4 white, 4 black ; II – 6 white, 6 black. Give only one of the two symmetric answers
 - (b) What is the maximum probability that Tom will make an A. Express your answer as a decimal to the nearest hundredth.
11. Two ME students, a female Sumalee and a male Xiao Hu, play a game with three piles of identical poker chips. Pile I has 1 chip, pile II has 2 chips, and pile III has 3 chips. The rules of the game state that (a) each player in turn either takes exactly one chip or takes all chips from any single pile with remaining chips, (b) that the player taking the last chip loses, and (c) Sumalee plays first. Determine Sumalee's initial play to guarantee that she will win the game. Give your answer in the form 3-III, for example, to denote that she initially takes all 3 chips from pile III.

12. A CSE named AI has developed the equation $m^{n+1} - (m+1)^n = 2001$ to predict the number of delegates m that Hillary Clinton will win in a certain state primary and the number n that Barack Obama will win. Find the unique positive integers m and n that satisfy this equation. Give your answer in the form (m,n) .
13. (Remember, it's a dirty dozen.) In a class lab experiment a civil engineering student named Alisha lets water flow into an initially empty tank at the rate of 1 gallon per second. However, water also leaves at the rate of 1 gallon per second per 100 gallons in the tank. To the nearest tenth of a second, how long in seconds will it take Alisha to get 50 gallons of water into the tank?

ANSWERS

1. 15 students. Subtract the 75 students below from 90.



2. 1 shoe. If Carlos selects one shoe from the box labeled BW (without loss of generality, we will assume this shoe is white), he knows that the box contains only that color of shoe. Therefore, since both of the remaining boxes are known also to be mislabeled, the WW box must contain black shoes, and the BB box must be the mix.
3. Two wires can cross once, an additional wire can cross both of these, a fourth wire can cross the given three, and so forth. Adding lines in this manner, we determine the maximum number of crossings for 20 wires to be given by $1 + 2 + 3 + \dots + 19 = 190$.
4. 204. There are one row and one column of squares with side length 8. There are two rows and two columns of squares of side length 7. Continuing in this way, we determine the total number of squares to be given by $1^2 + 2^2 + 3^2 + \dots + 8^2 = 204$.
5. Judy. We first examine Tom's case. As he is essentially saying the same thing with his first and third statements, we conclude these statements are true. Thus,

David's third statement is a lie and his second is true. David did not know Michelle before he enrolled at UTA. Thus, Michelle's third statement is a lie, and we conclude she must be telling the truth when she names Judy as the thief.

6. 5.7 inches. We assume the radius of the planet to be r , the circumference then given by $2\pi r$. We increase this value by 36 inches to yield $2\pi r + 36$. This being the new circumference, we divide by 2π to attain the new radius, $r + 36/(2\pi)$. The wire is thus raised above the surface of the planet by $36/(2\pi) \approx 5.7$ inches.
7. 21,397. The first digit is two times the second, so neither can be zero. Also their sum yields the third digit, a number that, when multiplied by three, is still a single digit. This can only be true for a number of the form 2139x, x being the last unknown digit. In order to be prime, a multi-digit number cannot end with a 0, 2, 4, 5, 6, or 8, and since 1, 3, and 9 have already been taken, the last digit must be 7. We can then verify that 21397 is in fact prime..
8. (a.) 52.6¢. The probability that she loses any bet is given by $1 - 9/19 = 10/19$. We simply multiply this \$1

(b.) \$1. Considering her system, the sum of all losses since the last win is given by $2b - 1$, where b is the amount of the current bet. A win results in leaving the table with $2b$, thus earning net winnings of \$1.
9. 16 miles. Regardless of the size of the hill, since Dr. Yin spends twice as long running up as running down, his average rate in this region will be given by $(1/3)*12\text{mph} + (2/3)*6\text{mph} = 8\text{mph}$. Since this is the same as his normal rate, Dr. Yin averages 8 mph for the entire trip, and thus covers $2\text{h}*8\text{mph} = 16$ miles.
10. (a) A-1 white, 0 black; B-9 white, 10 black OR the reverse. Tom maximizes his chances by giving himself complete certainty in one of the boxes with a probability of 0.5 of being chosen and maximizing the odds of white in the other. He thus puts only 1 white ball in one box and the rest of the balls in the other.

(b) 0.74 The maximum probability is thus given by the law of total probability to be $0.5*1.0 + 0.5*(9/19) \approx 0.74$.
11. 1-II. If Sumalee takes one chip from the second stack, this leaves Xiao Hu with the following options: take one chip from the stack containing three, take the entire stack, or take one of the two single chips. If he takes the first option, Sumalee takes another chip off of this stack leaving three forced moves left in the game (an odd number, Xiao Hu loses). If he takes the second option, Sumalee takes one of the two remaining chips, forcing Xiao Hu to end the game. The same is true if he takes the third option, for Sumalee can simply take the stack of three, leaving the last chip to Xiao Hu.

12. (13,2). A simple C program, employing two “for” loops to test combinations of the integer values yields a unique result of $m = 13$ and $n = 2$.
13. 69.3 seconds. This problem resolves into a first order linear differential equation of the form

$$dV/dt = 1 - \frac{V}{100}, \text{ where}$$

V is the volume of water in the tank and t is the time in seconds. The general solution is given by

$$V = 100 - \frac{C}{e^{t/100}}$$

As the tank is initially empty at $t = 0$, the constant C becomes 100. At $V = 50$ gallons, we can solve for t as equal to $100(\ln 2) \approx 69.3$ seconds.