Apply sticker ID

Fall 2011

EE 5340 Semiconductor Device Theory

In the table below, check which two problems you wish to be graded. Problems not selected will not be graded. If all three boxes happen to be marked, only the first two problems will be graded. If you need extra space to complete a problem **do not** write on the back. Place the ID number and the problem number on all additional sheets used. There should be 9 pages following this sheet.

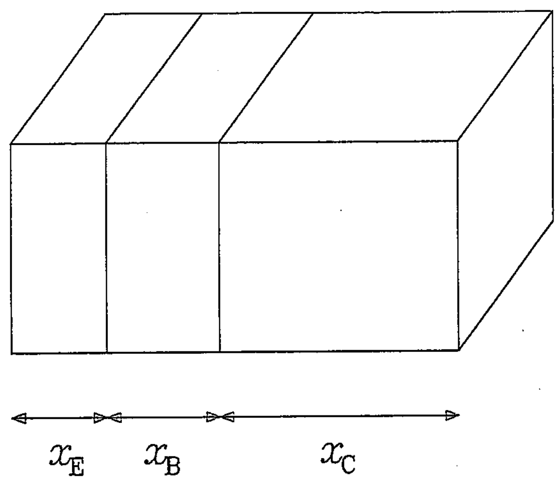
Designation TABLE

Problem Number	**Check Problems to be Graded**	Left Blank	Instructor Grade
1		blank	/50
2		blank	/50
3		blank	/50

Semiconductor Device Theory EE 5340
Diagnostic Exam

For all problems, solve numerically to at least 3 significant figures of accuracy. The answer is not correct if it does not have units.

1. A certain *npn* bipolar transistor has an emitter size of $x_E = 1\ \mu$m a base size of $x_B = 0.8\ \mu$m, and a collector size of $x_C = 4\ \mu$m. The emitter dopant density is 6×10^{17} cm^{-3}, the base is 5×10^{16} cm^{-3}, and the collector is 2×10^{14} cm^{-3}. If the minority carrier lifetime in all the transistor sections is 4 ns, what is the expected short circuit current gain, β, for this transistor.



Semiconductor Device Theory EE 5340
Diagnostic Exam

2. An aluminum contact is joined on one side of a thin piece of SiO_2 insulator and a p-type piece of silicon is joined to the other side. This forms an MOS device. The semiconductor has a doping concentration of $1.5 \times 10^{16}\ \mathrm{cm}^{-3}$.

a. Draw the energy band diagram and label the critical energy levels.

b. Determine the flat band voltage and again draw the energy level diagram when the flat band voltage is applied.

Semiconductor Device Theory EE 5340
Diagnostic Exam

3. In a *pn* abrupt junction, one side has a doping concentration of $N_a = 2 \times 10^{17}$ cm^{-3} and the other side has $N_d = 5 \times 10^{15}$ cm^{-3}.

a. Integrate Poisson's equation to get the electric field as a function of distance on the n side and again on the p side. Make use of the known boundary conditions on the electric field.

b. Knowing the expression for the electric field, determine the internal built-in voltage drops on the n side and on the p side. The reference potential is at the junction where it is assumed $\phi(0) = 0$.

c. If the maximum electric field (in magnitude) is -4000 V/cm, what is the total depletion length under zero bias.

In the following equations, E stands for the electric field and $\mathcal{E}$ stands for the energy. Physical constants that might be useful. Here, N_0 is number of Si atoms, and m_o is the mass of an electron.

$n_i = 1.45 \times 10^{10} \quad \text{cm}^{-}3$	$\mathcal{E}_g = 1.124 \quad \text{eV}$	$\epsilon_r = 11.7 \quad \text{for Si}$
$\epsilon_r = 3.9 \quad \text{for SiO}_2$	$N_0 = 5 \times 10^{22} \quad \text{cm}^{-3}$	$N_c = 2.8 \times 10^{19} \quad \text{cm}^{-3}$
$N_v = 1.04 \times 10^{19} \quad \text{cm}^{-3}$	$V_T = 25.843 \quad \text{mV}$	$\chi = 4.05 \quad \text{V Si}$
$\chi = 1.0 \quad \text{V SiO}_2$	$\epsilon_0 = 8.854^{-14} \quad \text{F/cm}$	$k = 1.38 \times 10^{-23} \quad \text{J/K}$
$\Phi_M = 4.1 \quad \text{V Al}$	$\Phi_M = 5.3 \quad \text{V Pt}$	$\Phi_M = 4.75 \quad \text{V Au}$
$m_o = 9.11 \times 10^{-28}$	$q = 1.602 \times 10^{-19} \quad \text{C}$	

Basic Semiconductors

$$n = \frac{N_d - N_a}{2} + \left[\left(\frac{N_d - N_a}{2} \right)^2 + n_i^2 \right]^{1/2}$$

Fermi-Dirac distribution function

$$f_D(\mathcal{E}) = \frac{1}{1 + \exp[(\mathcal{E} - \mathcal{E}_f)/kT]}$$

Maxwell-Boltzmann distribution function

$$f_M(\mathcal{E}) = \exp[(\mathcal{E} - \mathcal{E}_f)/kT]$$

$$n = N_c \exp\left[-\frac{(\mathcal{E}_c - \mathcal{E}_f)}{kT} \right] = n_i \exp\left[\frac{(\mathcal{E}_f - \mathcal{E}_i)}{kT} \right]$$

$$p = N_v \exp\left[\frac{(\mathcal{E}_f - \mathcal{E}_v)}{kT} \right] = n_i \exp\left[\frac{(\mathcal{E}_i - \mathcal{E}_f)}{kT} \right]$$

quasi-Fermi level

$$\mathcal{E}_{fn} = \mathcal{E}_i + kT \ln(n/n_i)$$

$$\mathcal{E}_{fp} = \mathcal{E}_i - kT \ln(p/n_i)$$

$$e^{-(\phi_{fn} - \phi_{fp})q/kT}$$

Drift plus diffusion current:

$$J_n = q\mu_n nE + qD_n \frac{dn}{dx}$$

$$J_p = q\mu_p nE - qD_p \frac{dp}{dx}$$

Metal-Semiconductor Contacts

$$q\Phi_B = q(\Phi_M - \chi) = \mathcal{E}_c - \mathcal{E}_f$$

$$= q\phi_i + \mathcal{E}_c - \mathcal{E}_f$$

Work functions for a semiconductor and a metal.

$$q\Phi_{\mathrm{S}} = q\chi + (\mathcal{E}_{\mathrm{c}} - \mathcal{E}_{\mathrm{f}})$$
$$q\Phi_{\mathrm{M}} = q\phi_{\mathrm{i}} + q\Phi_{\mathrm{S}}$$
$$\phi_{\mathrm{i}} = \Phi_{\mathrm{M}} - \chi - (\mathcal{E}_{\mathrm{c}} - \mathcal{E}_{\mathrm{f}})/q$$
$$Q_{\mathrm{S}} = qN_{\mathrm{d}}x_{\mathrm{d}} = \sqrt{2qN_{\mathrm{d}}\epsilon_{\mathrm{s}}(\phi_{\mathrm{i}} - V_{\mathrm{a}})}$$

Capacitance per unit area

$$\left|\frac{\partial Q_{\mathrm{s}}}{\partial V_{\mathrm{a}}}\right| = C = \sqrt{\frac{q\epsilon_{\mathrm{s}}N_{\mathrm{d}}}{2(\phi_{\mathrm{i}} - V_{\mathrm{a}})}}$$
$$\frac{1}{C^2} = \frac{2(\phi_{\mathrm{i}} - V_{\mathrm{a}})}{q\epsilon_{\mathrm{s}}N_{\mathrm{d}}}$$
$$J_{\mathrm{n}} = J_{\mathrm{s}}\left[e^{qv_{\mathrm{a}}/kT} - 1\right]$$
$$J_{\mathrm{s}} = \frac{q^2 D_{\mathrm{n}} N_{\mathrm{c}}}{kT}\sqrt{\frac{2qN_{\mathrm{d}}(\phi_{\mathrm{i}} - V_{\mathrm{a}})}{\epsilon_{\mathrm{s}}}}e^{-q\Phi_{\mathrm{B}}/kT}$$

Debeye length:

$$L_{\mathrm{D}} = \sqrt{\frac{\epsilon_{\mathrm{s}}kT}{q^2 n_{\mathrm{s}}}}$$
$$x_{\mathrm{d}} = \sqrt{2}L_{\mathrm{D}}\left(e^{q|\phi_{\mathrm{i}}|/2kT} - 1\right)$$
$$q\phi_{\mathrm{n}} = \mathcal{E}_{\mathrm{c}} - \mathcal{E}_{\mathrm{f}}$$
$$|\phi_{\mathrm{i}}| = \phi_{\mathrm{n}} + \chi - \Phi_{\mathrm{M}}$$

PN Junctions

$$E_{\mathrm{c}} = -\frac{d\phi}{dx} = -\frac{kT}{q}\frac{1}{n}\frac{dn}{dx} = \frac{kT}{q}\frac{1}{p}\frac{dp}{dx}$$
$$d\phi = \frac{kT}{q}\frac{dn}{n}$$
$$n = n_{\mathrm{i}}\exp\left(\frac{q\phi}{kT}\right)$$
$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon_{\mathrm{s}}}(p - n + N_{\mathrm{d}} - N_{\mathrm{a}}) = \frac{q}{\epsilon_{\mathrm{s}}}\left(2n_{\mathrm{i}}\sinh\frac{q\phi}{kT} + N_{\mathrm{a}} - N_{\mathrm{d}}\right)$$

Quasi-neutrality for gradual doping

$$E_{\mathrm{x}} = -\frac{kT}{q}\frac{1}{N_{\mathrm{d}}}\frac{dN_{\mathrm{d}}}{dx} \quad \text{or} \quad \frac{kT}{q}\frac{1}{N_{\mathrm{a}}}\frac{dN_{\mathrm{a}}}{dx}$$

Since

$$n = n_{\mathrm{i}}e^{q\phi/kT}$$
$$\phi_{\mathrm{n}} = \frac{kT}{q}\ln\frac{N_{\mathrm{d}}}{n_{\mathrm{i}}}$$

$$\phi_\mathrm{p} = -\frac{kT}{q}\ln\frac{N_\mathrm{a}}{n_\mathrm{i}}$$

$$\phi_\mathrm{i} = \phi_\mathrm{n} - \phi_\mathrm{p} = \frac{kT}{q}\ln\left(\frac{N_\mathrm{d}N_\mathrm{a}}{n_\mathrm{i}^2}\right)$$

For np heterojunctions:

$$\phi_\mathrm{i} = \chi_2 - \chi_1 + \frac{\mathcal{E}_\mathrm{g2}}{q} - \frac{kT}{q}\ln\frac{N_\mathrm{c1}N_\mathrm{v2}}{N_\mathrm{d1}N_\mathrm{a2}}$$

Even when $\chi_1 \neq \chi_2$:

$$\mathcal{E}_\mathrm{c2} - \mathcal{E}_\mathrm{c1} = \mathcal{E}_\mathrm{g2} - kT\ln\frac{N_\mathrm{c1}N_\mathrm{v2}}{N_\mathrm{d1}N_\mathrm{a2}}$$

Avalnche Breakdown:

$$\alpha \approx KE\exp\left(-\frac{B}{E}\right)$$

$$M = \frac{1}{1-(|V_\mathrm{R}|/BV)^n} \qquad 2 < n < 6$$

$$E_\mathrm{max} = \left(\frac{2qN_\mathrm{a}|V_\mathrm{R}|}{\epsilon_\mathrm{s}}\right)^{1/2}$$

Currents in pn Junctions

The continuity equations for electrons and holes are:

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_\mathrm{n}(x)}{\partial x} + (G_\mathrm{n} - R_\mathrm{n})$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_\mathrm{p}(x)}{\partial x} + (G_\mathrm{p} - R_\mathrm{p})$$

SHR recombination

$$U = \frac{N_\mathrm{T}v_\mathrm{th}\sigma_\mathrm{n}\sigma_\mathrm{p}(pn - n_\mathrm{i}^2)}{\sigma_\mathrm{n}\left(n + n_\mathrm{i}e^{(\mathcal{E}_\mathrm{f}-\mathcal{E}_\mathrm{i})/kt}\right) + \sigma_\mathrm{p}\left(p + n_\mathrm{i}e^{(\mathcal{E}_\mathrm{i}-\mathcal{E}_\mathrm{t})/kt}\right)}$$

When $\sigma_\mathrm{o} = \sigma_\mathrm{n} = \sigma_\mathrm{p}$:

$$U = \frac{N_\mathrm{T}v_\mathrm{th}\sigma_\mathrm{o}(pn - n_\mathrm{i}^2)}{p + n + 2n_\mathrm{i}\cosh(\mathcal{E}_\mathrm{f} - \mathcal{E}_\mathrm{i})/kT}$$

$$\tau_x = \frac{1}{N_\mathrm{T}v_\mathrm{th}\sigma_x}$$

Across a depleted pn junction, excess minority carriers are:

For low level injection:

$$-U = G_\mathrm{p} - R_\mathrm{p} = -\frac{p'}{\tau_\mathrm{p}}$$

$$\frac{\partial p_\mathrm{n}}{\partial t} = D_\mathrm{p}\frac{\partial^2 p_\mathrm{n}}{\partial x^2} - \frac{p_\mathrm{n} - p_\mathrm{no}}{\tau_\mathrm{p}}$$

$$p_\mathrm{n}'(x) = Ae^{-(x-x_\mathrm{n})/L_\mathrm{p}} + Be^{(x-x_\mathrm{n})/L_\mathrm{p}}$$

where

$$L_{\mathrm{p}} = \sqrt{D_{\mathrm{p}}\tau_{\mathrm{p}}} \qquad \text{and } L_{\mathrm{n}} = \sqrt{D_{\mathrm{n}}\tau_{\mathrm{n}}}$$

$$J_{\mathrm{p}}(x) = \frac{qD_{\mathrm{p}}n_{\mathrm{i}}^2}{N_{\mathrm{d}}L_{\mathrm{p}}}\left(e^{qV_{\mathrm{a}}/kT} - 1\right) \times e^{(x-x_{\mathrm{n}})/L_{\mathrm{p}}}$$

$$J_{\mathrm{n}}(x) = \frac{qD_{\mathrm{n}}n_{\mathrm{i}}^2}{N_{\mathrm{a}}L_{\mathrm{n}}}\left(e^{qV_{\mathrm{a}}/kT} - 1\right) \times e^{(x+x_{\mathrm{p}})/L_{\mathrm{n}}}$$

For the long-base diode:

$$J_{\mathrm{t}} = qn_{\mathrm{i}}^2\left(\frac{D_{\mathrm{p}}}{N_{\mathrm{d}}L_{\mathrm{p}}} + \frac{D_{\mathrm{n}}}{N_{\mathrm{a}}L_{\mathrm{n}}}\right)\left(e^{qV_{\mathrm{a}}/kT} - 1\right)$$

For the short-base diode:

$$J_{\mathrm{t}} = qn_{\mathrm{i}}^2\left(\frac{D_{\mathrm{p}}}{N_{\mathrm{d}}W_{\mathrm{B}}'} + \frac{D_{\mathrm{n}}}{N_{\mathrm{a}}W_{\mathrm{E}}'}\right)\left(e^{qV_{\mathrm{a}}/kT} - 1\right)$$

$$\frac{J_{\mathrm{t}}}{J_{\mathrm{g}}} = \frac{2n_{\mathrm{i}}}{x_{\mathrm{d}}}\left[\frac{L_{\mathrm{n}}}{N_{\mathrm{a}}} + \frac{L_{\mathrm{p}}}{N_{\mathrm{d}}}\right]$$

Bipolar Transistors

For an *npn* trasistor:

For short base and $n(x)$ varies linearly in the base:

$$J_{\mathrm{n}} = J_{\mathrm{s}}\left[\exp\left(\frac{qV_{\mathrm{BC}}}{kT}\right) - \exp\left(\frac{qV_{\mathrm{BE}}}{kT}\right)\right]$$

where

$$J_{\mathrm{s}} = \frac{qD_{\mathrm{n}}n_{\mathrm{i}}^2}{x_{\mathrm{B}}N_{\mathrm{aB}}}$$

or where

$$Q_{\mathrm{B}} = q\int_0^{x_{\mathrm{B}}} p\,dx$$

$$J_{\mathrm{s}} = \frac{q^2n_{\mathrm{i}}^2\tilde{D}_{\mathrm{n}}}{Q_{\mathrm{B}}}$$

$$\alpha_{\mathrm{T}} = 1 - \frac{|I_{\mathrm{rB}}|}{|I_{\mathrm{nE}}|} = 1 - \frac{x_{\mathrm{B}}^2}{2\tau_{\mathrm{n}}\tilde{D}_{\mathrm{n}}} = 1 - \frac{x_{\mathrm{B}}^2}{2L_{\mathrm{n}}^2}$$

$$\begin{aligned}\gamma &= \frac{1}{1 + |I_{\mathrm{pE}}|/|I_{\mathrm{nE}}|} \\ &= \frac{1}{1 + (x_{\mathrm{B}}N_{\mathrm{aB}}\tilde{D}_{\mathrm{pE}})/(x_{\mathrm{E}}N_{\mathrm{dE}}\tilde{D}_{\mathrm{nB}})} \\ &= \frac{1}{1 + (GN_{\mathrm{B}}\tilde{D}_{\mathrm{pE}})/(GN_{\mathrm{E}}\tilde{D}_{\mathrm{nB}})}\end{aligned}$$

$$\alpha = \gamma\alpha_{\mathrm{T}}$$

$$I_{\mathrm{E}} = -I_{\mathrm{ES}}\left(e^{qV_{\mathrm{BE}}/kT} - 1\right) + \alpha_{\mathrm{R}}I_{\mathrm{CS}}\left(e^{qV_{\mathrm{BC}}/kT} - 1\right)$$

$$I_{\mathrm{C}} = -I_{\mathrm{CS}}\left(e^{qV_{\mathrm{BC}}/kT} - 1\right) + \alpha_{\mathrm{F}}I_{\mathrm{ES}}\left(e^{qV_{\mathrm{BE}}/kT} - 1\right)$$

Bipolar Transistor Models

$$\alpha_{\mathrm{F}} = \frac{\tau_{\mathrm{BF}}}{\tau_{\mathrm{F}} + \tau_{\mathrm{BF}}}$$
$$\alpha_{\mathrm{R}} = \frac{\tau_{\mathrm{BR}}}{\tau_{\mathrm{R}} + \tau_{\mathrm{BR}}}$$
$$\frac{Q_{\mathrm{F0}}}{\tau_{\mathrm{F}}} = \frac{Q_{\mathrm{R0}}}{\tau_{\mathrm{R}}}$$

Include Early effect and high-level injection via Q_{BT}

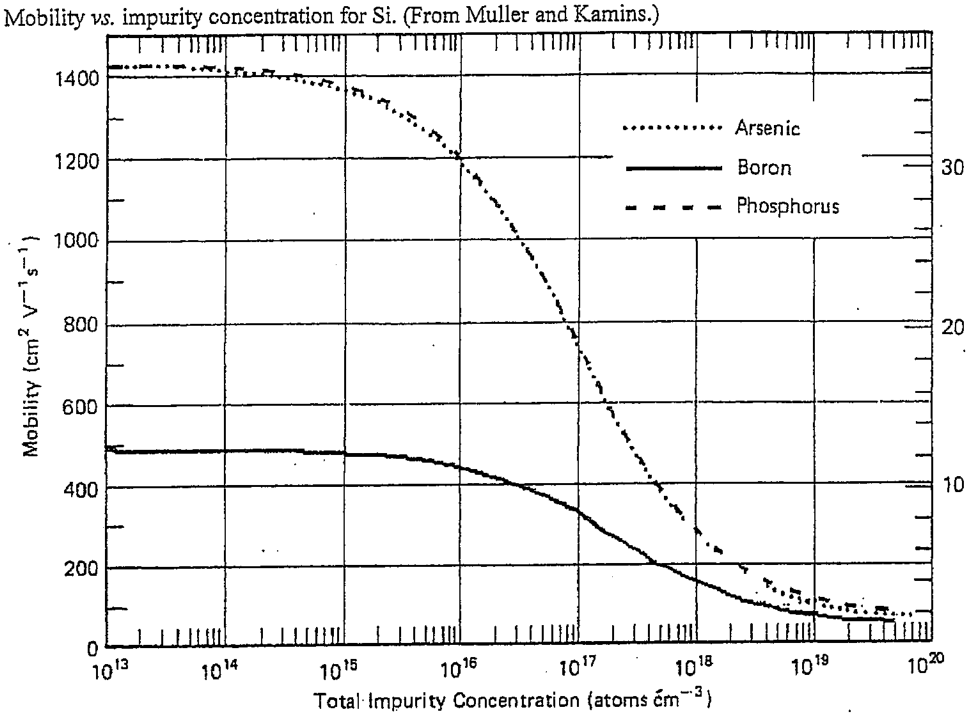
$$I_{\mathrm{S}} = \frac{q^2 A_{\mathrm{E}}^2 n_{\mathrm{i}}^2 \tilde{D}_{\mathrm{n}}}{Q_{\mathrm{BT}}}$$
$$Q_{\mathrm{BT}} = qA_{\mathrm{E}} \int_0^{x_{\mathrm{B}}} p(x)\ dx \qquad \text{Total charge}$$
$$Q_{\mathrm{B0}} = qA_{\mathrm{E}} \int_0^{x_{\mathrm{B}}} N_{\mathrm{a}}(x)\ dx \qquad \text{Built in charge}$$
$$q_1 = 1 + \frac{V_{\mathrm{BE}}}{|V_{\mathrm{B}}|} + \frac{V_{\mathrm{BC}}}{|V_{\mathrm{A}}|}$$
$$q_2 = \frac{I_{\mathrm{S}}}{I_{\mathrm{KF}}}\left(e^{V_{\mathrm{BE}}/V_{\mathrm{T}}} - 1\right) + \frac{I_{\mathrm{S}}}{I_{\mathrm{KR}}}\left(e^{V_{\mathrm{BC}}/V_{\mathrm{T}}} - 1\right)$$
$$q_{\mathrm{b}} = q_1 + \frac{q_2}{q_{\mathrm{b}}} = \frac{q_1}{2} + \frac{\sqrt{q_1^2 + 4q_2}}{2}$$

$$V_{\mathrm{t}} = V_{\mathrm{FB}} + V_{\mathrm{C}} + 2|\phi_{\mathrm{p}}| + \frac{1}{C_{\mathrm{ox}}}\sqrt{2\epsilon_{\mathrm{s}} q N_{\mathrm{a}}(2|\phi_{\mathrm{p}}| + V_{\mathrm{C}} - V_{\mathrm{B}})}$$
$$= V_{\mathrm{FB}} + V_{\mathrm{C}} + 2|\phi_{\mathrm{p}}| - \frac{Q_{\mathrm{d}}}{C_{\mathrm{ox}}}$$

For strong invrsion, $\phi_{\mathrm{s}} = |\phi_{\mathrm{p}}| + (V_{\mathrm{C}} - V_{\mathrm{B}})$

$$Q_{\mathrm{n}} = -C_{\mathrm{ox}}(V_{\mathrm{G}} - V_{\mathrm{t}})$$

28. Mobility *vs.* impurity concentration for Si. (From Muller and Kamins.)



29. Critical electric field *vs.* doping concentration for Si. (From Muller and Kamins.)

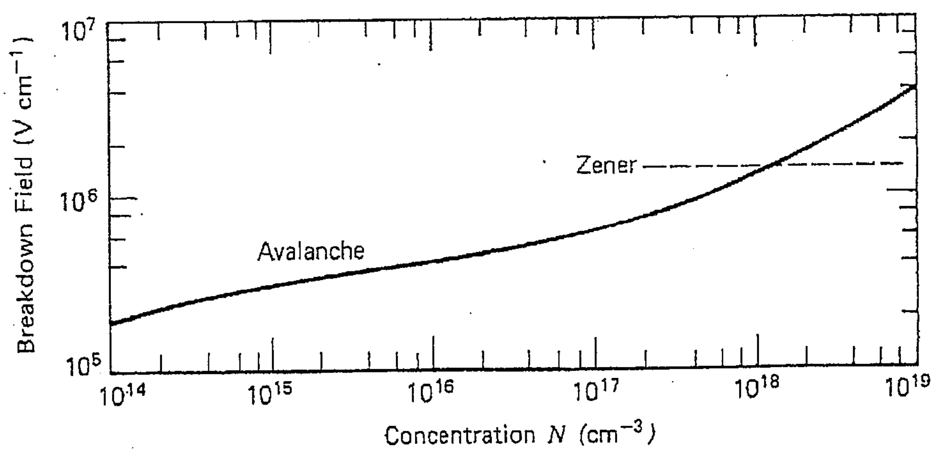


Figure 4.12 The critical electric fields for avalanche and Zener breakdown in silicon as functions of dopant concentration.[1,2,3]