UTA EE5350 PhD Diagnosis Exam (Fall 2011)
Digital Signal Processing

Instructions:

- Verify that your exam contains 7 pages (including the cover sheet).
- Some space is provided for you to show your work. Only if more space is needed, you may show your work on the back of the exam sheet.
- The point values listed on this exam serve only as a guideline. The Department reserves the right to make modifications to the weighting of the problems.
- You may use a calculator.

I Choose to work on Problems _____ and _____ (Choose only 2 from the 3 problems).

Problem	Possible Points	Scores
1	50	
2	50	
3	50	
Total Score (Choose 2 Problems)	100	

Problem 1: [50 pts] A causal IIR digital filter has the transfer function

$$H(z) = \frac{2 - 7z^{-1}}{1 - 7z^{-1} + 10z^{-2}}$$

(a) [15 pts] Give the poles of H(z), and its region of convergence. Are the poles inside the H(z)'s region of convergence ?

(b) [10 pts] Using the partial fraction expansion, or another method, find the causal impulse response h(n), as the sum of two exponentials.

(c) [15 pts] Give a causal difference equation that calculates h(n) in terms of $\delta(n)$.

(d) [10 pts] Give the first two terms of h(n) (h(0) and h(1)) that result when long division is used to find the causal h(n).

Problem 2: [50 pts] The analog signal $x_a(t)$ is ideally sampled at a rate of $2\pi/T$ radians/sec., producing the discrete time signal x(n), where T is the sampling period in seconds. We want to decimate x(n) with an integer decimation rate N_1 , so that the resulting signal y(n) has a sampling period of $N_1 \cdot T$. Assume that this decimation requires us to lowpass filter x(n) before sub-sampling. In sub-sampling, we will delete consecutive groups of $(N_1 - 1)$ samples and keep every N_1th sample.

(a) [10 pts] If the sampling period of y(n) is to be $N_1 \cdot T$, what is its sampling rate in radians/sec ?

(b) [15 pts] If y(n) is not aliased, what is the maximum cut-off frequency of x(n) in radians/second ?

(c) [10 pts] Multiplying your answer in part (b) by T, find the required cut-off frequency w_o in radians for the lowpass decimation filter h(n).

(d) [15 pts] Give the impulse response h(n) in terms of n and N_1 .

Problem 3: [50 pts] Let L_p be the space of signals x(n) such that

$$\left(\sum_n | x(n) |^p \right) < \infty$$

where p is a finite, positive integer.

(a) [25 pts] Show that $L_1 \subset L_2$. (Hint: How many numbers x(n) satisfy $|x(n)| \geq 1$?)

(b) [25 pts] Show by a counter example that the converse is not true in general, i.e. give x(n) such that $x(n) \in L_2$ but $x(n) \notin L_1$.