SOLUTIONS

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Ph.D. Diagnostic Exam

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Digital Signal Processing

Problem 1: [50 pts] A causal IIR digital filter has the transfer function

$$H(z) = \frac{2-7z^{-1}}{1-7z^{-1}+10z^{-2}}$$

(a) [15 pts] Give the poles of H(z), and its region of convergence. Are the poles inside the H(z)'s region of convergence ?

(b) [10 pts] Using the partial fraction expansion, or another method, find the causal impulse response h(n), as the sum of two exponentials.

(c) [15 pts] Give a causal difference equation that calculates h(n) in terms of δ(n).

(d) [10 pts] Give the first two terms of h(n) (h(0) and h(1)) that result when long division is used to find the causal h(n).

(a) $H(z) = \frac{2-7z^{-1}}{(1-5z^{-1})(1-2z^{-1})}$ SO POLES ARE $z = 2, 5$

3 pts 3 pts

R.O.C. $|z| > 5$ 4 pts OR $|z| > 2$ 2 pts

NO, POLES NOT IN R.O.C. 5 pts

(b) $h(n) = [5^n + 2^n]\, u(n)$

4 PTS 4 PTS 2 PTS

(c) $h(n) = 2\delta(n) - 7\delta(n-1) + 7h(n-1) - 10h(n-2)$

(d) $h(0) = 2$, $h(1) = 7$

5 pts 5 pts.

Problem 2: [50 pts] The analog signal $x_a(t)$ is ideally sampled at a rate of $2\pi/T$ radians/sec., producing the discrete time signal x(n), where T is the sampling period in seconds. We want to decimate x(n) with an integer decimation rate N_1 , so that the resulting signal y(n) has a sampling period of $N_1 \cdot T$. Assume that this decimation requires us to lowpass filter x(n) before sub-sampling. In sub-sampling, we will delete consecutive groups of $(N_1 - 1)$ samples and keep every N_1th sample.

(a) [10 pts] If the sampling period of y(n) is to be $N_1 \cdot T$, what is its sampling rate in radians/sec ?

(b) [15 pts] If y(n) is not aliased, what is the maximum cut-off frequency of x(n) in radians/second ?

(c) [10 pts] Multiplying your answer in part (b) by T, find the required cut-off frequency w_o in radians for the lowpass decimation filter h(n).

(d) [15 pts] Give the impulse response h(n) in terms of n and N_1 .

(a) $\dfrac{2\pi}{\text{SAMPL. PER.}} = \dfrac{2\pi}{N_1 \cdot T}$ 2 2 3 3

(b) $\dfrac{2\pi}{N_1 \cdot T} > 2 \times \dfrac{w_c}{T}$ $\div 2$: $\dfrac{w_c}{T} < \dfrac{\pi}{N_1 \cdot T}$ 5 5 5

SAMPLING RATE YIELDING y(n), FROM x(n)

TWICE HIGHEST FREQUENCY OF x(n)

(c) $\dfrac{\pi}{N_1 \cdot T} \times T = \dfrac{\pi}{N_1}$ 5 5

(d) $h(n) = \dfrac{\sin\left(\frac{\pi}{N_1} n\right)}{\pi \cdot n}$ 5 3 2 5

Problem 3: [50 pts] Let L_p be the space of signals x(n) such that

$$\left(\sum_n |x(n)|^p\right) < \infty$$

where p is a finite, positive integer.

(a) [25 pts] Show that $L_1 \subset L_2$. (Hint: How many numbers x(n) satisfy $|x(n)| \geq 1$?)

(b) [25 pts] Show by a counter example that the converse is not true in general, i.e. give x(n) such that $x(n) \in L_2$ but $x(n) \notin L_1$.

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(a) FOR L_1 SEQUENCE $x(n)$, $S_1 = \sum_n |x(n)|^p$

FOR ALL $x(n)$'S $\ni |x(n)| \geq 1$. $S_2 = \sum_n |x(n)|^p$ 5

FOR ALL $x(n)$'S $\ni |x(n)| < 1$.

5

(1) $|S_1| < \infty$ SO $S_1 < \infty \ \forall$ FINITE P
CARDINALITY

5 5

(2) S_2 DECREASES AS P INCREASES . $S_1 + S_2 < \infty \ \forall p \geq 1$.

(b) $x(n) = \frac{1}{n} u(n-1)$. $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$ FOR $P=1$, VIA P-SERIES

20 + 5

$\sum_{n=1}^{\infty} \frac{1}{n^2} < \infty$ FOR $P \geq 2$.