

UTA EE5362 PhD Diagnosis Exam (Fall 2011)

Instructions:

- Verify that your exam contains 10 pages (including the cover sheet).
- Some space is provided for you to show your work. If more space is needed, show your work on the back of the exam sheet.
- The point values listed on this exam serve only as a guideline. The Dept reserves the right to make modifications to the weighting of the problems.
- Calculator is okay.

I Choose to work on Problems _____ and _____ (Choose only 2 from the 3 problems).

	Problem	Points	Scores
1	Channel Coding	50	
2	Signal Detection	50	
3	Orthogonal Modulation	50	
	Total Score (choose 2 problems)	100	

1 (50 Points) Channel Coding

For the $[4,1]$ linear block code \mathbf{C} having generator matrix

$$G = [1 \ 1 \ 1 \ 1]$$

- (a) (15 points) Set up the entire standard array for the code \mathbf{C} . The array must contain all possible binary 4-tuples as well as all possible syndromes. The entire row of your standard array must list all the codewords. The leftmost column should contain the coset leaders and the rightmost column should list the syndromes.

- (b) (10 points) When the transmitted codeword is $\mathbf{c} = [0000]^t$, given that the received vector

$$\mathbf{r} = [1011]^t$$

what is the decoded codeword? What is the residue error? How many message bits are in error?

(c) (15 Points) When the code is used only for the purposes for correcting error, what is the probability of decoding error when the crossover probability of the binary symmetric channel (BSC) is ε ?

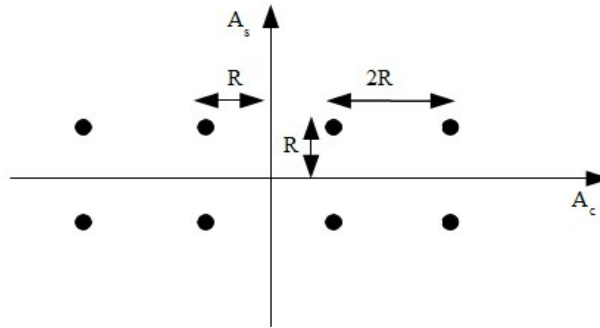
(d) (10 Points) What is the probability of undetected error if the code is used solely for the purpose of error detection error when the crossover probability of the BSC is ε ?

2 Signal Detection (50 points)

Consider a set of equally probable 8-ary QAM signals that are transmitted through an AWGN channel whose noise has a flat power spectral density that is equal to $N_0/2$ W/Hz. Suppose that the received signal can be expressed as

$$r(t) = A_c \cos(2\pi f_c t) + A_s \sin(2\pi f_c t) + n_t, \quad 0 \leq t \leq T.$$

The constellation of the QAM symbols (A_c, A_s) is shown below.



- i) Find the average power of the transmitted signals.

- ii) Assume that the symbol $(3R, R)$ is transmitted, then use the Gaussian $Q(\cdot)$ function to express the probability of the symbol detection error at the receiver.
- iii) Assume that the symbol (R, R) is transmitted, then use the Gaussian $Q(\cdot)$ function to express the probability of the symbol detection error at the receiver.

iv) Find the average symbol detection error probability.

3. Orthogonal Modulation (50 Points)

Consider a 4-FSK digital modulation technique where information is conveyed via discrete shifts in the carrier frequency. The received signal is modeled by 4 equal probability hypotheses ($t \in [0, T]$):

$$H_m : r(u, t) = \sqrt{\frac{2E}{T}} \sin[(2\pi f_c + \Delta_m)t] + n(u, t) \quad m = 0, 1, 2, 4.$$

where $n(u, t)$ is the standard AWGN (i.e., $K_n(\tau) = \frac{N_0}{2} \delta_D(\tau)$). Assume that

$0 < \frac{\Delta_m}{2\pi} \ll f_c$ so that the narrowband assumption is valid.

(a) (10 Points) Give a choice of Δ_m ($m = 0, 1, 2, 3.$) for which the signals are orthogonal.

(b) **(10 Points)** Based on the choice of Δ_m in (a), give a set of orthonormal basis function $\phi_i(t)$ ($i = 0,1,2,3.$), then determine the equivalent vector model for the decision problems in terms of

$$R(u, i) = \int_0^T r(u, t) \phi_i(t) dt$$

specifically, the equivalent model is of the form

$$H_m : \mathbf{R}(u) = \mathbf{S}_m + \mathbf{N}(u)$$

Determine \mathbf{S}_m and the pdf of $\mathbf{N}(u)$, $f_{\mathbf{N}(u)}(z)$.

- (c) **(10 Points)** Determine optimal decision rule for the four hypotheses, and draw the diagram of correlation receiver.

- (d) **(10 Points)** Based on this decision rule, suppose it's BFSK (i.e., only H_0 and H_1), what is the probability of CORRECT decision given H_0 and $R(u,0) = r(0)$, i.e., $P_2(C | H_0, r(0))$?

- (e) **(10 Points)** Based on the decision rule derived in (c) and result obtained in (d), derive the probability decision error for this 4-FSK modulation, $P(\varepsilon)$.