Algebraic Derivation of IS-LM Eqilibrium

Goods Market: $AE = C + I + G$

$C = C_0 + C_y(Y - T_0)$

$I = I_0 - I_r r$

$G = G_0$

Equilibrium Condition in the Goods Market $\rightarrow AE = Y$

Solving for Y :

$AE = C + I + G = C_0 + C_y(Y - T_0) + I_0 - I_r r + G_0$

- Step 1: Let $A_0 = C_0 + I_0 + G_0$
- Step 2: $Y = A_0 + C_y(Y - T_0) - I_r r$
- Step 3: $Y = A_0 - C_y T_0 - I_r r + C_y Y$
- Step 4: $Y - C_y Y = A_0 - C_y T_0 - I_r r$
- Step 5: $Y(1 - C_y) = A_0 - C_y T_0 - I_r r$
- Step 6: $Y_{IS} = \frac{A_0 - C_y T_0 - I_r r}{1 - C_y}$

Money Market: $M^d = L_0 + L_y Y - L_r r$

$M^s = M_0$

Equilibrium Condition in the Money Market $\rightarrow M^s = M^d$

Solving for r :

$M_0 = L_0 + L_y Y - L_r r$

- Step 1: $M_0 - L_0 - L_y Y = -L_r r$
- Step 2: $r_{LM} = \frac{M_0 - L_0 - L_y Y}{-L_r}$

General IS-LM Equilibrium:

General Equilibrium Condition$\rightarrow IS = LM$

Solve for Y^*:

- Step 1: $Y = \frac{A_0 - C_y T_0}{1 - C_y} - \left[\frac{I_r}{1 - C_y}\right]\left[\frac{M_0 - L_0 - L_y Y}{-L_r}\right]$
- Step 2: $Y = \frac{A_0 - C_y T_0}{1 - C_y} + \frac{I_r M_0 - I_r L_0 - I_r L_y Y}{(1 - C_y) L_r}$

- Step 3: $Y = \frac{L_r A_0 - L_r C_y T_0}{(1-C_y)L_r} + \frac{I_r M_0 - I_r L_0 - I_r L_y Y}{(1-C_y)L_r}$
- Step 4: $Y = \frac{L_r A_0 - L_r C_y T_0 + I_r M_0 - I_r L_0 - I_r L_y Y}{(1-C_y)L_r}$
- Step 5: $Y + \frac{I_r L_y Y}{(1-C_y)L_r} = \frac{L_r A_0 - L_r C_y T_0 + I_r M_0 - I_r L_0}{(1-C_y)L_r}$
- Step 6: $Y \left[1 + \frac{I_r L_y}{(1-C_y)L_r}\right] = \frac{L_r A_0 - L_r C_y T_0 + I_r M_0 - I_r L_0}{(1-C_y)L_r}$
- Step 7: $Y \left[\frac{(1-C_y)L_r}{(1-C_y)L_r} + \frac{I_r L_y}{(1-C_y)L_r}\right] = \frac{L_r A_0 - L_r C_y T_0 + I_r M_0 - I_r L_0}{(1-C_y)L_r}$
- Step 8: $Y \left[\frac{(1-C_y)L_r + I_r L_y}{(1-C_y)L_r}\right] = \frac{L_r A_0 - L_r C_y T_0 + I_r M_0 - I_r L_0}{(1-C_y)L_r}$
- Step 9: $Y^* = \frac{L_r A_0 - L_r C_y T_0 + I_r M_0 - I_r L_0}{(1-C_y)L_r + I_r L_y}$

Solve for r^*:

- Step 1: $r = \frac{M_0 - L_0}{-L_r} + \frac{L_y}{L_r}\left[\frac{A_0 - C_y T_0 - I_r r}{1-C_y}\right]$
- Step 2: $r = \frac{M_0 - L_0}{-L_r} + \frac{L_y A_0 - L_y C_y T_0 - L_y I_r r}{(1-C_y)L_r}$
- Step 3: $r = \frac{-(1-C_y)(M_0 - L_0)}{(1-C_y)L_r} + \frac{L_y A_0 - L_y C_y T_0 - L_y I_r r}{(1-C_y)L_r}$
- Step 4: $r = \frac{(1-C_y)(L_0 - M_0) + L_y A_0 - L_y C_y T_0 - L_y I_r r}{(1-C_y)L_r}$
- Step 5: $r + \frac{L_y I_r r}{(1-C_y)L_r} = \frac{(1-C_y)(L_0 - M_0) + L_y A_0 - L_y C_y T_0}{(1-C_y)L_r}$
- Step 6: $r \left[1 + \frac{L_y I_r}{(1-C_y)L_r}\right] = \frac{(1-C_y)(L_0 - M_0) + L_y A_0 - L_y C_y T_0}{(1-C_y)L_r}$
- Step 7: $r \left[\frac{(1-C_y)L_r}{(1-C_y)L_r} + \frac{L_y I_r}{(1-C_y)L_r}\right] = \frac{(1-C_y)(L_0 - M_0) + L_y A_0 - L_y C_y T_0}{(1-C_y)L_r}$
- Step 8: $r \left[\frac{(1-C_y)L_r + L_y I_r}{(1-C_y)L_r}\right] = \frac{(1-C_y)(L_0 - M_0) + L_y A_0 - L_y C_y T_0}{(1-C_y)L_r}$
- Step 9: $r^* = \frac{(1-C_y)(L_0 - M_0) + L_y A_0 - L_y C_y T_0}{(1-C_y)L_r + L_y I_r}$