Intra-National Evidence of the Fisher Effect

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Abstract

There is a large literature devoted to the size of the response of nominal interest rates to changes in expected inflation, the Fisher effect. The interest in the topic stems from the implications of the Fisher effect for both asset pricing models and monetary neutrality propositions. Unfortunately no consensus has been achieved concerning the Fisher effect. Part of the problem for this has been the low power of the tests employed in Fisher effect studies. In this study we use a data set on local city-wide interest rates and local mortgage interest rates to construct a panel of Fisher relationships within the United States. There are well known power increases in both unit root and cointegration tests using panel techniques. Our panel results, however, are not much different from the single group estimates. The size of the Fisher effect depends more on the particular estimator, i.e. OLS, DOLS, MLE, than on whether used in a single group specification or as a panel.

1 Introduction

There is a large literature devoted to the size of the response of nominal interest rates to changes in expected inflation, the Fisher effect. The long-standing interest in the topic stems from the monetary neutrality implications for different Fisher effect values. Values below unity imply substantial long-run non-neutralities while values equal to one are consistent with long-run superneutrality of money.[1] A related issue is the stationarity of the ex-ante real rate of interest. The class of consumption-based asset pricing models implies that real rates should be proportional to consumption growth, which is clearly a stationary time series. Similarly, the neoclassical growth theory based on dynamic optimization for a representative economic agent implies that the real rate will be proportional to the representative consumer's rate of time preference which will itself be constant in the steady state. In a stochastic environment this steady state condition implies stationarity of the ex-ante real rate.

Unfortunately there is no consensus among economists about the true size of the Fisher effect. There are several problems that plague empirical estimates of the Fisher effect. First is the effect of taxes on the size of the Fisher effect. Since interest income is usually taxed at the same rate as ordinary income, increases in the nominal interest rate that arise from increases in expected inflation will yield less after-tax return to debt holders unless the nominal interest rate increases by more than the increase in expected inflation. This implies a Fisher effect estimate greater than one. A second problem is the generally unobserved nature of the expected inflation rate. When actual realized inflation is used to proxy expected inflation an errors-in-variables bias is induced in the estimate of the Fisher effect. A third problem arising in the estimation of the Fisher effect is the time series properties of the underlying data. If the data are covariance stationary, then standard least squares based estimation techniques are appropriate. But if the data are integrated or non-stationary then the theoretical Fisher relationship suggests cointegration techniques are more appropriate.

Finally, when the series of interest are non-stationary, the power of the cointegration tests and the small sample consistency of the various estimators is an issue.

In this study we use a data set on local city-wide interest rates and local mortgage interest rates to construct a panel of Fisher relationships within the United States. In so doing we hope to overcome most of the problems of previous studies of the Fisher effect. First we examine the time series properties of the individual variables using several commonly applied univariate unit root tests and a relatively new univariate test with good size and power. We then apply two panel unit root tests that have been used extensively in other contexts. There are well known power increases in tests using panel estimators. We use several estimators of the single group Fisher relationship that have been applied elsewhere in the literature. Then we apply three panel cointegration estimators. Our results from the panel techniques provide essentially the same estimates of the size of the Fisher effect as the single group estimators using analogous techniques. The rest of the paper is organized as follows. Section 2 gives some brief background on the recent Fisher effect literature. Section 3 presents a discussion of each of the empirical methodologies employed in the present study. Section four discusses the data and presents the empirical results and section 5 concludes.

2 Recent Evidence on the Fisher Effect

Although the literature examining the Fisher hypothesis extends back before Fisher's original work, the modern debate might be traced back to Fama (1975). In that paper Fama's evidence implied that real interest rates were essentially constant while nominal interest rates adjusted to any changes in expected inflation. Nelson and Schwert (1977), among others, provided evidence contradicting Fama's. The problem has been that almost all of these early estimates of the Fisher effect were significantly less than one. Darby (1975) notes that when interest income is taxed, the nominal rate must respond to changes in inflation by more than one-for-one in order to maintain a constant after-tax real rate of interest. Darby's observation only made the Fisher effect estimates obtained in these early studies appear more anomalous.[2]

There are three explanations for the low Fisher effect estimates. The first is attributed to Mundell (1963) and Tobin (1965) and further clarified by Feldstein (1976), Levi and Makin (1978) and Fried and Howitt (1983). In this explanation, higher inflation encourages economic agents to substitute out of money balances and into other assets. This drives the price of other assets up and their returns down preventing nominal interest rates from rising enough to compensate for the increased inflation. Equation (1) describes the relationship between nominal interest rates and inflation when this Mundell-Tobin effect is operative.

$$\frac{\partial i}{\partial \pi} = \left[(1-\tau)\left[1+\frac{L_i}{I_r(L_y+W_p)}\right]\right]^{-1}, \tag{1}$$

In equation (1), i represents the nominal interest rate, π is the ex-ante inflation rate, τ is the marginal tax rate, L_i is the interest elasticity of money demand, L_y is the income elasticity of money demand, I_r is the interest elasticity of investment demand and W_p is the price level elasticity of the nominal wage in the labor market. The Mundell-Tobin effect arises when $L_i > 0$. Note that this is true even if one were to maintain a full-employment assumption implying $W_p = 1$. Allowing for wage rigidities, i.e. $W_p < 1$, would further erode the response of nominal rates to inflation. [3]

In the long run, however, $L_i = 0$ since output is supply determined and the quantity theory of money holds.[4] Therefore, in the long run, the Fisher effect should reflect fully any changes in inflation as well as the tax consequences of the inflation changes.

The second explanation for low Fisher effect estimates is more recent. Evans and Lewis (1995) suggest that regime changes in the inflation process may lead to a peso problem for nominal debt. The peso problem arises when market participants assign some weight to an extreme realization thus causing observed market prices and/or expectations to appear to be biased. Peso problems are essentially small sample problems. If the data sample spans a long enough time period then the event in question has either occurred or the markets expectations have adjusted to the non-occurrence. Therefore, peso problems should not persist indefinitely and in the long run, again, the Fisher effect should reflect changes in inflation and any tax distortions.

The third cause of low Fisher effect estimates is unrelated to the economic theory underlying the Fisher proposition but is based on the statistical issues involved in estimating the relationship. There are two threads to this line of reasoning. The first is based on the problems of finding a suitable measure for expected inflation. If actual inflation is used to proxy expected inflation, an errors-in-variables bias occurs. This particular problem has given way to a more recent issue surrounding the time series properties of the underlying data. Most recent time series analyses of nominal interest rates have concluded that they are integrated processes.[5] Rose (1988) recognized that if nominal interest rates were non-stationary, then inflation rates must also be non-stationary and cointegrated with nominal rates as a necessary condition for the Fisher effect to hold. Under such circumstances the super-consistency of the cointegration estimators renders the errors-in-variables bias a non-issue. But Rose's empirical analysis yielded an even more troubling result for the Fisher effect. He found that inflation rates were stationary implying that no long-run relationship could exist between nominal interest rates and inflation.

Rose's results set off a flurry of empirical work using unit root consistent techniques to examine the Fisher relationship. Crowder and Hoffman (1996), Crowder (1997) and Crowder and Wohar (1999) all find evidence that inflation is a non-stationary series over the post-WW II period and that nominal interest rates and inflation are cointegrated. The evidence presented in these papers is consistent with a full tax-adjusted Fisher effect. Contradicting these studies are those of Mishkin (1992), Evans and Lewis (1995) (EL), King and Watson (1997) (KW) and Koustas and Serletis (1999) (KS). KW and KS both show that nominal interest rates respond to inflation shocks by much less than is implied by the Fisher effect under a wide range of VAR identifying restrictions. These results are contingent on the presumption that nominal interest rates and inflation rates are not cointegrated.[6] Mishkin and EL both present evidence that nominal interest rates and inflation rates are cointegrated, but that the Fisher effect estimates are well below the theoretically implied values.

2.1 A Theoretical Model

Consider a version of the Lucas asset pricing 'tree' model where an optimizing agent maximizes her present value expected discounted utility with a sequence a composite consumption flow, $\{c_t\}_{t=0}^{\infty}$, with price normalized to 1, and an annual flow of housing services, $\{z_t\}_{t=0}^{\infty}$: $E_t\Sigma_{t=0}^{\infty}\beta^t U(c_t, z_t)$ where E_t is the expectation operator, $\beta < 1$ is the discount factor and $U_{c,t+j} = \frac{\partial U(c_{t+j}, z_{t+j})}{\partial c_{t+j}}$ and $U_{z,t+j} = \frac{\partial U(c_{t+j}, z_{t+j})}{\partial z_{t+j}}$. $U(\cdot)$ is assumed to be continuous and concave in both arguments and satisfy the Inada conditions in both arguments, $U' > 0, U'' < 0, \lim_{x\to 0} U' \to \infty$ and $\lim_{x\to\infty} U' \to 0$.

In each period, t, the household purchases a numeraire consumption basket, c_t, at price 1; next periods housing services z_{t+1} at the *net* price of q_t which is the difference between the selling price and the period mortgage payment; and next periods risk free bonds, b_{t+1} at price p_t. Income is generated each period by wage income, y_t; the price of the bonds in the current period and any dividends/coupon payments, $d_t \geq 0$ received; and the rent the household pays to itself at net price q_t for housing services. We can write the budget constraint as

$$p_t b_{t+1} + q_t z_{t+1} + c_t = y_t + (p_t + d_t)b_t + q_t z_t.$$

The Euler equations take the form of:

$$\frac{U_{c,t}}{U_{c,t+1}} = E_t \beta R_{t+1} \tag{2}$$

and

$$\frac{U_{c,t}}{U_{c,t+1}} = E_t \beta \Big(\theta_{t+1} + \frac{1}{q_t} \cdot \xi_{t+1} \Big) \tag{3}$$

where $R_{t+1} = (p_{t+1} + d_{t+1})/p_t$ is the gross rate of return on bonds; $\theta_{t+1} = q_{t+1}/q_t$ is the net rate of return on the house, after the mortgage payment; and $\xi_{t+1} = (U_{z,t+1}/U_{c,t+1})$ is the marginal rate of substitution between housing services and consumption in time $t+1$. Equation (2) is the standard Euler relationship between the intertemporal marginal rate of substitution and the expected discounted interest rate. Equation (3) shows that the intertemporal marginal rate of substitution equals the house net rate of return plus the marginal rate of substitution between consumption and housing services deflated by the house price.

In the steady state the consumption growth, which is governed by the left hand side of the two Euler equations, is equal. Therefore, equations (2) and (3) can be solved for the relationship between house gross rate of return of the bond and the home gross rate of return:

$$(1 + i_{\ell,t}) = (1 + i_{E,t}) + q_{t-1}^{-1} \cdot \xi_t. \tag{4}$$

where we use the nomenclature for risk free rates from above as $R_t = 1 + i_{\ell,t}$ and $\theta_t = 1 + i_{E,t}$ is the mortgage rate. Notice that $i_{E,t}$ does not represent the actual mortgage mortgage rate, which is an outflow to the household, and has tax implications. Rather is the homeowner's net rate of return accounting for the mortgage rate. Notice that if the house is not in the utility function, that is it does not provide a flow of consumable services, then the risk free rate equals the real estate rate of return. Secondly, this net rate of return is *lower* than the risk free rate because the home, in addition to acting as part of a savings portfolio, also provides a flow of services which standard assets do not. This becomes obvious if we rearrange equation (4) as:

$$i_{\ell,t} - i_{E,t} = q_{t-1}^{-1} \cdot \xi_t, \tag{5}$$

that is the differential between the rate of return of the risk free asset and the home is governed by the deflated contemporaneous marginal rate of substitution. That is, the difference between the risk free rate and net home rate is determined by relative sizes the marginal utility of home services and consumption.

Consider the following: if an agent has a large house, they are willing to give up a lot of house to get more contemporaneous consumption. Equation (2) implies that an increase in consumption today, for any given discount factor, must be countered by a decrease in the risk free rate and the difference between the mortgage and risk free rates declines. Put another way, if ξ is small households are obtaining a lot of housing services relative to consumption, and risk is spread out, as the house contains a lot of intrinsic asset value, therefore the opportunity cost of putting savings in risk free assets is lower, don't need to receive as high a rate of return on assets and the differential shrinks. On the other hand, if houses are small, ξ is large, there is relatively little of the portfolio in the house and higher interest rates are sought elsewhere.

3 Empirical Methodology

3.1 Unit Root Tests

The univariate unit root tests used most commonly in the literature are the augmented Dickey-Fuller (ADF) (Said and Dickey, 1984) and Phillips-Perron (PP) (Phillips and Perron, 1988) tests. It is well known that these univariate unit root tests have notoriously low power against local stationary alternatives and suffer from serious size distortion when the data generating process (DGP) has negative moving average (MA) terms.[7] Elliot, et al. (1996) develop a feasible point optimal test that relies on local GLS detrending. This test has much greater power than standard ADF and PP unit root tests. Recently Ng and Perron (1995, 2000) and Perron and Ng (1996), extending the work done by Elliot, et al. (ERS)(1996), have developed unit root tests that are based upon the local GLS detrending method but also use an autoregressive spectral density estimator of the long-run variance. This class of tests, which they denote the M-tests, has much less size distortion in the presence of MA errors than the standard tests. This is especially true when one chooses lag truncations based upon a modified information criteria developed by Ng and Perron (2000).

The tests proposed by Ng and Perron are motivated by the DGP in (6),

$$y_t = d_t + u_t, \qquad u_t = \rho u_{t-1} + v_t \tag{6}$$

where $v_t = \varphi(L)e_t = \sum_{j=0}^{\infty} \varphi_j e_{t-j}$, $d_t = \zeta' z_t = \sum_{i=0}^{p} \zeta_i t^i$ for $p = 0, 1$. ERS suggest using a GLS detrending method to improve the power of unit root tests. For any series $\{x_t\}_{t=0}^{T}$ define $(x_0^{\overline{\alpha}}, x_t^{\overline{\alpha}}) \equiv (x_0, (1-\overline{\alpha}L)x_t)$ for some chosen $\overline{\alpha} = 1+\overline{c}/T$. The GLS detrended series is defined as,

$$\widetilde{y}_t \equiv y_t - \widehat{\zeta}' z_t \tag{7}$$

where $\widehat{\zeta}$ minimizes $S(\overline{\alpha}, \zeta) = (y^{\overline{\alpha}} - \zeta' z_t^{\overline{\alpha}})'(y^{\overline{\alpha}} - \zeta' z_t^{\overline{\alpha}})$. ERS suggest choosing $\overline{c} = -7.0$ for $p = 0$ and $\overline{c} = -13.5$ for $p = 1$.The test recommended by ERS is the DF^{GLS} statistic given in equation (8).

$$\Delta\widetilde{y}_t = \rho\widetilde{y}_{t-1} + \sum_{j=1}^{k} \gamma_j \widetilde{y}_{t-j} + e_{tk} \tag{8}$$

Ng and Perron recommend two tests that have similar power to the DF^{GLS} but that also have superior size properties in the presence of MA errors. These tests are MZ_ρ, MZ_t, and MSB, collectively referred to as the M tests. These are defined as,

$$MZ_\rho = (T^{-2}\tilde{y}_t^2 - s_{AR}^2)(2T^{-2}\sum_{t=1}^{T}\tilde{y}_{t-1}^2)^{-1} \tag{9}$$

$$MSB = \left[\frac{T^{-2}\sum_{t=1}^{T}\tilde{y}_{t-1}^2}{s_{AR}^2}\right]^{\frac{1}{2}} \tag{10}$$

and $MZ_t = MZ_\rho \times MSB$. All three tests are based on s_{AR}^2, an autoregressive estimate of the spectral density at frequency zero of v_t. This estimate is calculated as,

$$s_{AR}^2 = \frac{\hat{\sigma}_k^2}{[1-\gamma(1)]^2} \tag{11}$$

where $\gamma(1) = \sum_{i=1}^{k}\gamma_i$ and $\hat{\sigma}_k^2 = (T-k)^{-1}\sum_{t=k+1}^{T}\hat{e}_{tk}^2$ and γ_i and $\{\hat{e}_{tk}\}$ are taken from estimation of (8) using OLS. The only piece left is to specify a lag truncation parameter k. Ng and Perron suggest using a modified information criteria (MIC) as in (12),

$$MIC(k) = \ln(\hat{\sigma}_k^2) + \frac{C_T(\tau_T(k)+k)}{T-k_{\max}} \tag{12}$$

where $\tau_T(k) = (\hat{\sigma}_k^2)^{-1}\hat{\rho}\sum_{t=k_{\max}+1}^{T}\tilde{y}_{t-1}^2$ and $k_{\max}$ is the largest lag truncation considered. Ng and Perron show that if $C_T = \ln(T-k_{\max})$ then this reduces to their $MBIC$ or modified Bayesian information criteria.

3.2 Panel Unit Root Tests

But it is well known that the power of such tests is notoriously weak when the autoregressive root is close to,but still less than, unity. Recently several researchers have proposed using a panel unit root testing approach that significantly increases the power of the test against the null.[8] We apply two common panel unit root tests to our data.

The first is a test suggested by Levin and Lin (1992) based on the model in (13),

$$\Delta y_{it} = \rho_i y_{it-1} + z_{it}'\gamma + u_{it},\ i = 1, \ldots, N; t = 1, \ldots, T, \tag{13}$$

where z_{it} is the deterministic component and u_{it} is a stationary process. The Levin and Lin test assumes that $\rho_i = \rho$ for all i. Levin and Lin suggest a t-statistic calculated under the null as

$$t_\rho = \frac{(\hat{\rho} - 1)\sqrt{\sum_{i=1}^{N}\sum_{t=1}^{T} \tilde{y}_{i,t-1}^2}}{s_u} \tag{14}$$

where

$$s_u^2 = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} \tilde{u}_{it}^2 \tag{15}$$

and $\tilde{y}_{i,t}$ and $\tilde{u}_{it}$ are simply y_{it} and u_{it} corrected for the deterministic components z_{it}.

The second panel test is attributed to Im, Pesaran and Shin (1997)(hereafter IPS). It is based on averaging the individual unit root test statistics from standard augmented Dickey-Fuller regressions. Define the t-bar statistic as

$$\bar{t} = \frac{1}{N}\sum_{i=1}^{N} t_{\rho_i}, \tag{16}$$

where t_{ρ_i} is the individual t-statistic for testing the unit root null hypothesis. IPS suggest a statistic normalized by the expected value and variance of the individual t-statistics and call this statistic t_{IPS}. They derive critical values for this statistic via monte carlo simulation.

O'Connell (1998) showed that the Levin and Lin test suffers from extreme size distortion (rejects a true null too often) when the contemporaneous error terms are correlated across groups (referred to as spatial correlation in the literature). O'Connell further demonstrated that once this spatial correlation was controlled for, the power of these tests dropped significantly. Furthermore, Taylor and Sarno (1998) have shown that rejection of the null hypothesis in the panel unit root tests cannot be interpreted as stationarity of all the series in the panel. These tests are uninformative about the number of series that are stationary versus the number that are non-stationary.

3.3 Cointegration Estimators

In this section we describe six cointegration estimators that we employ on each individual group. The first and simplest is a basic OLS regression of nominal interest rate on inflation. This estimator of the cointegrating parameter was first suggested by Engle and Granger (1987). This estimator has the virtue of simplicity and as Engle and Granger demonstrate, the OLS estimator is superconsistent which eliminates the error-in-variables bias asymptotically and allows the econometrician to use the estimated cointegrating parameter as if it were known in subsequent regressions. The problem with this estimator is that it has substantial small sample bias arising from the endogeneity of the regressors and the possible serial correlation of the residuals. Furthermore, the asymptotic distribution of the OLS estimator in a cointegrating model is non-standard making inference on the cointegrating parameter difficult.

Consider the the cointegrated system given in (17) and (18),

$$y_{1t} = \beta y_{2t} + u_{1t} \tag{17}$$

where

$$\Delta y_{2t} = u_{2t} \tag{18}$$

and β is the long-run cointegrating parameter and $u_t \equiv \text{iid}(0,\Sigma)$ with Σ given as,

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{21}' \\ \sigma_{21} & \Sigma_{22} \end{bmatrix}. \tag{19}$$

The maximum likelihood estimator (MLE) of β in (17) is given by,

$$y_{1t} = \beta y_{2t} + \gamma \Delta y_{2t} + u_{1.2t} \tag{20}$$

where $u_{1.2t} = u_{1t} - \sigma_{12}\Sigma_{22}^{-1}u_{2t}$ and $\gamma = \Sigma_{22}^{-1}\sigma_{21}$.[9]

The limiting distribution of this estimator is,

$$T(\beta^* - \beta) \Rightarrow \left(\int_0^1 S_2 S_2'\right)^{-1} \left(\int_0^1 S_2 dS_{1.2}'\right) \tag{21}$$

where S_2 and $S_{1.2}$ are independent Brownian motions. The distribution in (21) is a Gaussian mixture of normals yielding standard asymptotic inference.

Compare the distribution of the MLE in (21) to the limit distribution of the OLS estimator given below,

$$T(\widehat{\beta} - \beta) \Rightarrow \Lambda + \left(\int_0^1 S_2 S_2'\right)^{-1} \left(\int_0^1 S_2 dS_2'\right) \Sigma_{22}^{-1}\sigma_{21} + \left(\int_0^1 S_2 S_2'\right)^{-1} \sigma_{21} \tag{22}$$

where Λ is the distribution of the MLE given in (21). The second term on the right side of (22) is a unit root distribution and the third term on the right side is a bias term arising from the contemporaneous correlation between u_{1t} and the regressor y_{2t}. When $\sigma_{21} = 0$, the OLS and MLE estimators have equivalent limit distributions.

There are several estimators derived in the cointegration literature to eliminate the biases in the OLS estimator. Phillips and Hansen (1990) suggest a fully modified OLS (FM-OLS) estimator. Let $\hat{s}_{21}$ be a consistent estimate of σ_{21}, then a modified estimator of β is given by

$$\beta^{\ddagger} = (y_{2t}' y_{2t})^{-1}(y_{2t}' y_{1t} - T\hat{s}_{21}) \tag{23}$$

which eliminates the bias arising from the third term on the right side of (22). A further modification is necessary in order to remove all the bias from the second term in (22).

$$y_{1t}^{+} = y_{1t} - \hat{s}_{21}' \hat{s}_{22}^{-1} \Delta y2t \tag{24}$$

where $\hat{s}_{22}$ is a consistent estimate of Σ_{22}. The FM-OLS estimator simply replaces y_{1t} in (23) with y_{1t}^{+} and allows standard asymptotic inference.

The dynamic OLS (DOLS) estimator of Saikkonen (1991) and Stock and Watson (1993) uses leads and lags of independent variables to eliminate the correlations of these variables with the errors. Implementation proceeds using the following regression.

$$y_{1t} = \mu_0 + \beta y_{2t} + \delta(L)\Delta y_{2t} + \varepsilon_t \tag{25}$$

where $\delta(L)$ is a two-sided polynomial in the lag operator such that both leads and lags are included in (25). In general the residuals from (25) will be serially correlated requiring use of an autocorrelation-heteroscedasticity consistent covariance estimator in order to make inference in finite samples.

The autoregressive distributed lag (ARDL) estimator suggested by Pesaran and Shin (1998) is also asymptotically equivalent to the MLE. It is based on the OLS regression,

$$y_{1t} = \gamma_0 + \gamma_1 t + \sum_{i=1}^{p} \phi_i y_{1t-i} + \varphi' y_{2t} + \sum_{i=1}^{q-1} \theta_i \Delta y_{2t-i} + \varepsilon_t \tag{26}$$

where the estimate of β is given by $\frac{\varphi}{\phi(1)}$ where $\phi(1) = 1 - \phi_1 - \ldots - \phi_p$.

The last estimator is derived under the assumption that (17) and (18) have a finite order vector autoregressive representation (VAR). The Johansen estimator is the MLE when the process for X_t is given in (27),

$$X_t = \Phi_1 X_{t-1} + ... + \Phi_k X_{t-k} + \mu + \delta t + \varepsilon_t \tag{27}$$

where X_t is $p-$dimensional vector of variables integrated of order one or less, i.e., I(2) and higher orders are ruled out, Φ_j are $(p \times p)$ coefficient matrices, μ is a $(p \times 1)$ vector of constants, δ is a $(p \times 1)$ vector of coefficients on linear trend terms and ε_t is a white noise error vector with non-diagonal covariance matrix Ω.

The Johansen estimator is calculated from the transformed version of (27) into it's error-correction model (VECM) form as in (28).

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + ... + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-1} + \mu + \delta t + \varepsilon_t \tag{28}$$

The long-run multiplier matrix $\Pi = \Phi(1) - I$ can be decomposed into two $(p \times r)$ matrices such that $\alpha\beta' = \Pi$. The $(p \times r)$ matrix β represents the cointegrating vectors or the long-run equilibria of the system of equations. The $(p \times r)$ matrix α is the matrix of error-correction coefficients which measure the rate each variable adjusts to the long-run equilibrium. Maximum likelihood estimation of (28) can be carried out by applying reduced rank regression. Johansen (1988, 1991) suggests first concentrating out the short-run dynamics by regressing ΔX_t and X_{t-1} on ΔX_{t-1}, ΔX_{t-2}, ..., $\Delta X_{t-k+1}, 1$ and t,and saving the residuals as R_{0t} and R_{1t}, respectively. Next calculate the product moment matrices $S_{ij} = T^{-1} \sum_{t=1}^{T} R_{it} R_{jt}^{'}$ and solve the eigenvalue problem $\left| \lambda S_{11} - S_{10} S_{00}^{-1} S_{01} \right| = 0$. Then order the estimated eigenvalues from largest to smallest $(\widehat{\lambda}_1, \widehat{\lambda}_2, ..., \widehat{\lambda}_p)$. The estimate of β, $\widehat{\beta}$, is given by the r-largest eigenvectors associated with the eigenvalues $\widehat{\lambda}$. Hypothesis tests on $\widehat{\beta}$ can be conducted using likelihood ratio (LR) tests with standard χ^2 inference. Let the form of the linear restrictions on β be given by $\beta = H\varphi$ where H is a $p \times s$ matrix of restrictions and φ is a $s \times r$ matrix of unknown parameters. The LR test statistic is given by,

$$T \sum_{i=1}^{r} \ln \left[\frac{(1-\widetilde{\lambda}_i)}{(1-\widehat{\lambda}_i)} \right] \sim \chi^2_{r(p-s)} \tag{29}$$

where $\widetilde{\lambda}_i$ are the eigenvalues from the restricted MLE.

The test for cointegration is a test for the number of non-zero eigenvalues. The likelihood ratio statistic testing the rank of Π, or equivalently the number of non-zero eigenvalues, is given by $-T\sum_{i=r+1}^{p} \ln(1-\widehat{\lambda}_i)$ and is called the trace statistic by Johansen (1988, 1991).[10]

The second technique used to test the individual Fisher relationships is a cointegration test proposed by Horvath and Watson (1995) (henceforward HW) in which the cointegration parameters are pre-specified. This technique is based on estimating (28) while imposing a restricted β. It has the advantage of increased power against the null of no cointegration, but does require knowledge of the cointegrating vector(s), usually from theoretical restrictions derived from economic theory. Specifically, the HW test is a generalization of the Wald test for the significance of α in (28) when β is prespecified.

3.4 Panel Cointegration Estimators

In this section we briefly review the $OLS, FM\!-\!OLSandDOLS$ estimators for panel cointegration.[11] The null hypothesis for the panel cointegration tests, Kao (1999) and Pedroni (1995, 1999) is that the estimated equation is not cointegrated. The LM test in McCoskey and Kao (1998) tests whether the null of the estimated equation is cointegrated.

Consider the following fixed-effect panel regression:

$$y_{i,t} = \alpha_i + x'_{i,t}\beta + u_{i,t}, i = 1, \ldots, N, t = 1, \ldots, T, \tag{30}$$

where $y_{i,t}$ are 1×1, β is an $M \times 1$ vector of slope parameters, α_i are the intercepts, and $u_{i,t}$ are the stationary disturbance terms. It is assumed that $x_{i,t}$ are $M \times 1$ I(1) processes which are themselves not cointegrated such that $x_{i,t} = x_{i,t-1} + \varepsilon_{i,t}$. These assumptions imply that (30) represents a system of cointegrating regressions. Let $w_{i,t} = (u_{i,t}, \varepsilon'_{i,t})'$ and assume that it satisfies the conditions in Kao and Chiang (1998). Then the long-run covariance of $w_{i,t}$ is,

$$\Omega = \sum_{j=-\infty}^{\infty} E(w_{i,j}w'_{i,0})$$

$$= \Sigma + \Gamma + \Gamma' \tag{31}$$

$$= \begin{bmatrix} \Omega_u & \Omega_{u\varepsilon} \\ \Omega_{\varepsilon u} & \Omega_{\varepsilon} \end{bmatrix}$$

where

$$\Gamma = \sum_{j=1}^{\infty} E(w_{i,j}w'_{i,0}) = \begin{bmatrix} \Gamma_u & \Gamma_{u\varepsilon} \\ \Gamma_{\varepsilon u} & \Gamma_{\varepsilon} \end{bmatrix} \tag{32}$$

and

$$\Sigma = \sum_{j=1}^{\infty} E(w_{i,0}w'_{i,0}) = \begin{bmatrix} \Sigma_u & \Sigma_{u\varepsilon} \\ \Sigma_{\varepsilon u} & \Sigma_{\varepsilon} \end{bmatrix} \tag{33}$$

are partitioned conformably with $w_{i,t}$. Then define the one-sided long-run covariance as

$$\Delta = \Sigma + \Gamma$$

$$= \sum_{j=1}^{\infty} E(w_{i,0}w'_{i,0}) \tag{34}$$

$$\Delta = \begin{bmatrix} \Delta_u & \Delta_{u\varepsilon} \\ \Delta_{\varepsilon u} & \Delta_{\varepsilon} \end{bmatrix}.$$

Kao and Chen derive the limiting distributions for the OLS, $FM - OLS$ and $DOLS$ estimators for the regression specification given in (30). They also investigated the finite sample properties of each estimator through Monte Carlo simulation. They found that (i) the OLS estimator has a non-negligible bias, (ii) the $FM-OLS$ estimator does not improve on the OLS estimator in general, and (iii) the $DOLS$ estimator has the best properties of the three.

The OLS estimator of β is,

$$\widehat{\beta}_{OLS} = \left[\sum_{i=1}^{N}\sum_{t=1}^{T}(x_{it}-\overline{x}_i)(x_{it}-\overline{x}_i)'\right]^{-1}\left[\sum_{i=1}^{N}\sum_{t=1}^{T}(x_{it}-\overline{x}_i)(y_{it}-\overline{y}_i)\right] \tag{35}$$

where a bar over a variable denotes its time average. The $FM-OLS$ estimator is constructed by modifying the OLS estimates for endogeneity and serial correlation. Let $\widehat{\Omega}_{\varepsilon u}$ and $\widehat{\Omega}_{\varepsilon}$ be consistent estimates $\Omega_{\varepsilon u}$ and Ω_{ε}, respectively. Define

$$\widehat{y}^{+}_{i,t} = y_{i,t} - \widehat{\Omega}_{\varepsilon u}\widehat{\Omega}_{\varepsilon}^{-1}\varepsilon_{i,t} \tag{36}$$

which modifies the dependent variables for endogeneity. The correction for serial correlation is,

$$\widehat{\Delta}^{+}_{\varepsilon u} = \widehat{\Delta}_{\varepsilon u} - \widehat{\Delta}_{\varepsilon}\widehat{\Omega}_{\varepsilon}^{-1}\widehat{\Omega}_{\varepsilon u} \tag{37}$$

where $\widehat{\Delta}_{\varepsilon u}$ and $\widehat{\Delta}_{\varepsilon}$ are the kernel estimates of $\Delta_{\varepsilon u}$ and Δ_{ε}, respectively. This leads to the $FM-OLS$ estimator of,

$$\widehat{\beta}_{FM} = \left[\sum_{i=1}^{N}\sum_{t=1}^{T}(x_{it}-\overline{x}_i)(x_{it}-\overline{x}_i)'\right]^{-1}\left[\sum_{i=1}^{N}\left(\sum_{t=1}^{T}(x_{it}-\overline{x}_i)\widehat{y}^{+}_{it} - T\widehat{\Delta}^{+}_{\varepsilon u}\right)\right]. \tag{38}$$

Finally, the $DOLS$ estimator is given as,

$$y_{it} = \alpha_i + x'_{it}\beta + \sum_{j=-q}^{q} c_{ij}\Delta x_{it+j} + \upsilon_{it}. \tag{39}$$

Kao and Chen derive the asymptotic distributions of the three estimators as follows:

1. $T\sqrt{N}(\widehat{\beta}_{OLS}-\beta)-\sqrt{N}\delta_{NT} \Longrightarrow N(0, 6\Omega_{\varepsilon}^{-1}\Omega_{u,\ \varepsilon})$,

2. $T\sqrt{N}(\widehat{\beta}_{FM}-\beta) \Longrightarrow N(0, 6\Omega_{\varepsilon}^{-1}\Omega_{u,\ \varepsilon})$,

3. $T\sqrt{N}(\widehat{\beta}_{DOLS}-\beta) \Longrightarrow N(0, 6\Omega_{\varepsilon}^{-1}\Omega_{u,\ \varepsilon})$,

where $\Omega_{u,\ \varepsilon} = \Omega_u - \Omega_{u\varepsilon}\Omega_{\varepsilon}^{-1}\Omega_{\varepsilon u}$ and $\Longrightarrow$ denotes convergence in distribution.

3.5 Panel Cointegration Tests

Kao (1999) describes two types of panel cointegration tests. The Dickey-Fuller (DF) type test and the augmented Dickey-Fuller (ADF) test. These tests can be calculated from the residuals from the panel cointegration estimators as:

$$\widehat{e}_{i,t} = \rho\widehat{e}_{i,t-1} + \nu_{i,t} \tag{40}$$

where the $\widehat{e}_{i,t}$ are the estimated residuals. The null hypothesis of no cointegration can be written as $H_0 : \widehat{\rho} = 1$. Four tests are considered:

1. $DF_\rho = \frac{\sqrt{N}T(\widehat{\rho}-1)+3\sqrt{3}}{\sqrt{10.2}}$

2. $DF_t = \sqrt{1.25}t_\rho + \sqrt{1.875}N$

3. $DF_\rho^* = \frac{\sqrt{N}T(\widehat{\rho}-1)+\frac{3\sqrt{N}\widehat{\sigma}_\upsilon^2}{\widehat{\sigma}_{0\upsilon}^2}}{\sqrt{3+\frac{7.2\widehat{\sigma}_\upsilon^4}{\widehat{\sigma}_{0\upsilon}^4}}}$

4. $DF_t^* = \frac{t_\rho+\frac{\sqrt{6N}\widehat{\sigma}_\upsilon}{2\widehat{\sigma}_{0\upsilon}}}{\sqrt{\frac{\widehat{\sigma}_{0\upsilon}^2}{2\widehat{\sigma}_\upsilon^2}+\frac{3\widehat{\sigma}_\upsilon^2}{10\widehat{\sigma}_{0\upsilon}^2}}}$

where $\widehat{\sigma}_\upsilon^2 = \Sigma_u - \Sigma_{u\varepsilon}\Sigma_{\varepsilon}^{-1}$ and $\widehat{\sigma}_{0\upsilon}^2 = \Omega_u - \Omega_{u\varepsilon}\Omega_{\varepsilon}^{-1}$. While DF_ρ and DF_t are based on the assumption that the regressors are strictly exogenous with respect to the errors, DF_ρ^* and

DF_t^* are appropriate when the regressors are endogenous. For the ADF test the following augmented regression is run,

$$\widehat{e}_{i,t} = \rho \widehat{e}_{i,t-1} + \sum_{j=1}^{\kappa} \zeta_j \Delta \widehat{e}_{i,t-j} + \nu_{i,t} \tag{41}$$

and the test statistic is calculated as,

$$ADF = \frac{t_{ADF} + \frac{\sqrt{6N}\widehat{\sigma}_v}{2\widehat{\sigma}_{0v}}}{\sqrt{\frac{\widehat{\sigma}_{0v}^2}{2\widehat{\sigma}_v^2} + \frac{3\widehat{\sigma}_v^2}{10\widehat{\sigma}_{0v}^2}}}, \tag{42}$$

where t_{ADF} is the t-statistic associated with the hypothesis $H_0 : \widehat{\rho} = 1$ from the estimate given in (41). All of the test statistics have asymptotic standard normal distributions.

The final panel cointegration test considered is one proposed by Larsson et al. (2001) and is based on the Johansen estimator for individual cointegration. Define the Johansen trace test given in 3.3 as $LR_T \{H(r) \mid H(p)\}$ where $\{H(r) \mid H(p)\}$ denotes the null hypothesis of r cointegrating relationships versus the alternative of p cointegrating relationships. Now consider the average LR_T over all i panel groups and define the LR-bar statistic as,

$$\overline{LR}_{NT} \{H(r) \mid H(p)\} = \frac{1}{N} \sum_{i=1}^{N} LR_{i,T} \{H(r) \mid H(p)\}. \tag{43}$$

The proposed test statistic is calculated from (44).

$$\Upsilon_{\overline{LR}} \{H(r) \mid H(p)\} = \frac{\sqrt{N}\left(\overline{LR}_{NT} \{H(r) \mid H(p)\} - E(Z_k)\right)}{\sqrt{Var(Z_k)}} \tag{44}$$

where $E(Z_k)$ is the mean and $Var(Z_k)$ the variance of the asymptotic trace statistic. Larsson et al. (2001) demonstrate that the statistic defined by (44) has a standard normal distribution.

4 Empirical Results

4.1 Data

The effective mortgage rate (EMR) data is available from FNMA annually for a set of 23 cities for the years 1978-2000. Annual city inflation rates were calculated using the city specific annual CPI available from the Bureau of Labor Statistics (BLS).[12]

4.2 Univariate Unit Root Tests

Table 1 presents the univariate unit root tests on the city inflation rates. All test statistics were derived from regressions that included both a constant and a linear trend. In no case can the null hypothesis of a unit root be rejected. Similarly table 2 displays the unit root tests for each of the local nominal interest rates. Again these test statistics were generated from regressions that included both a constant and a linear trend. And again no rejection of the null hypothesis can be found at the 5% level of significance.

4.3 Panel Unit Root Tests

Table 3 presents the panel unit root test results. The panels used to generate these test statistics includes the 23 local city inflation and nominal interest rates. The Levin-Lin test results suggest that the nominal interest rates and the inflation rates are non-stationary. This is consistent with the univariate results in tables 1 and 2. We also present result from the IPS test. Since the results can be sensitive to the underlying ADF regression lag truncation we present the results for three different lag specifications. When one lag is used in the ADF regression the unit root null hypothesis is rejected for both panels. But if the lag is increased to three or five, the null hypothesis is not rejected for the nominal interest rate panel but is rejected for the inflation rate panel. This is problematic for the Fisher relation since real rates should be stationary but a stationary real rate is inconsistent with these results from the IPS test.[13] Several studies have criticized the panel unit root tests on the grounds that the alternative hypothesis is uninformative about possibility of some series in the panel having unit roots when others may be stationary.[14] It is possible that the cointegration analysis may shed more light on the time series properties of the individual series.

4.4 Single Equation Cointegration Estimates

Table 4 displays the estimated cointegrating parameter between nominal interest rates and inflation for the four single equation estimators discussed in section 3.3. In single equation cointegration regressions the appropriate normalization not always theoretically determined. Asymptotically the cointegrating regression will have an R^2 of one implying that the parameter estimate from regressing x on y is the inverse of the estimate from regressing y on x. Ng and Perron (1997) note how the cointegrating parameter estimate can differ substantially in small samples from the implied values in the reverse regression even when the series are actually cointegrated. They suggest choosing the dependent variable as the one with the largest unconditional variance in order to achieve the most consistent cointegrating parameter estimate. In the case of the Fisher equation that would imply that the regression of inflation on nominal interest rate will yield the more accurate estimate of the Fisher effect as the inverse of the estimated cointegrating parameter.

Following this suggestion, each single equation estimator is normalized first on the nominal interest rate, which provides a direct estimate of the Fisher effect, and then on the inflation rate, which provides and indirect estimate of the Fisher effect as the inverse of the estimated cointegrating parameter.

The OLS cointegrating regression results are displayed in columns two and three of table

4. When the nominal interest rate is the dependent variable in the regression there is no evidence of a Fisher effect consistent with a tax-adjusted stationary real rate. These are similar to results obtained in Fisher effect studies in the 1970s and 1980s and led many economists to conclude there was an operative Mundell-Tobin effect. In stark contrast the OLS regressions that specify the inflation rate as the dependent variable all yield estimates consistent with a tax-adjusted Fisher effect. Based on Summers (1983) calculations, a tax-adjusted Fisher effect in the U.S. should be approximately equal to 1.4. A value from the reverse regression, i.e. inflation on nominal rate, should provide an estimate of approximately 0.7 to be consistent with this tax-adjusted Fisher effect.

The results from the ARDL regressions are more generally consistent with the tax-adjusted Fisher effect for both empirical specifications. Out of 46 regressions (23 each way) the hypothesis that the estimated cointegrating parameter is consistent with the Fisher effect is rejected in only 16 of the 46 models.

The FM-OLS results are much closer to the OLS results in that none of the regressions of nominal rates on inflation yield estimates consistent with a tax-adjusted Fisher effect but, except for the Atlanta and San Diego groups, the estimates from the reverse regressions are all consistent with a full Fisher effect.

The results from the DOLS specification are similar to those from the ARDL models. Only eight of the 46 regressions display evidence inconsistent with a tax-adjusted Fisher effect.

4.5 VAR Cointegration Analysis

Table 5 presents the cointegration analysis of each individual Fisher relation using the Horvath-Watson procedure. The empirical model used is the VECM described in section 3.3 with the hypothesized cointegrating parameter of 1.40 imposed. The test for cointegration is a Wald test of the hypothesis that error correction coefficients are jointly insignificant.[15] Horvath and Watson (1995) demonstrate that testing for cointegration with a known cointegration vector yields some power improvements over methods that rely on estimating the coitegrating vector. Column two of table 5 displays the Horvath-Watson Wald test for the null that there are zero cointegrating relationships but a known cointegrating vector under the alternative. Thus rejection of the null hypothesis yields evidence of cointegration between the nominal interest rate and inflation with a known Fisher effect of 1.40. The model generating these results included one lag in the VAR. The null is rejected for only seven out of 24 cases when the 5% level of significance is used. When we use a 10% significance level the number of rejections increases to 12 out 24 cases.[16]

Column 3 of table 5 presents the Wald test for the no cointegration null when the cointegrating relation under the alternative is unknown and must be estimated. Column 4 displays the estimated Fisher effect. These results were also obtained from a VAR model with one lag. There is almost no evidence against the null, even if one uses a 10% level of significance. But, most of the estimated cointegration parameters (i.e. the long-run Fisher effect) are close to the hypothesized value of 1.40.

In contrast, when the VAR model allows four lags and the Fisher effect is assumed known the Wald statistic rejects the null of no cointegration in 18 out of 24 cases, with an additional rejection at the 10% level for the Cleveland model. There is also more evidence in favor of cointegration under the unknown case with four lags than the analogous case with one lag. Column six yields 10 rejections out of 24 cases at the 5% level and 18 out of 24 at the 10% level. In the model with four lags, however, the estimated Fisher effects are much less consistent with the hypothesized value of 1.40.

The results derived from applying the Johansen estimator are presented in table 6. The column labelled $r = 0$ displays the trace test statistics for the null of no cointegration. This hypothesis is rejected at the 5% level for all groups except Honolulu. The next column displays tests statistics that if rejected would imply both nominal interest rates and inflation rates are stationary. None of these statistics are significant consistent with the univariate and Levin-Lin panel unit root tests presented earlier.

The fourth column of table 6 shows the estimated Fisher effects for each city and the fifth column displays the calculated LR test statistics from (29) for the null hypothesis that $\beta = 1.40$. This test has a limiting $\chi^2(1)$ distribution. The restriction on the size of the Fisher effect can only be rejected in one case at the 5% level and four more at the 10% level of significance. This represents fairly strong evidence in favor of a full tax-adjusted Fisher effect.

The next two columns of table 6, sixth and seventh, show the t-statistics for the null hypothesis that the error correction coefficient is statistically insignificant in each equation of the VECM, respectively. If the error correction coefficient can be restricted to zero, then the dependent variable in that VECM equation can be considered weakly exogenous, implying that it is the source of the common trend in the system. Interestingly, the inflation rate is weakly exogenous in all 24 groups. One implication of this result is that nominal interest rates will be poor predictors of future inflation, especially at long horizons.

Another interesting result can be gleaned from the error correction coefficient estimates. The VECM that yielded these particular estimates was estimated using the theoretically implied value of the Fisher effect of 1.40 to specify the equilibrium error. Note that every system yields a dynamically stable implied equilibrium. To see this, let $z_t = i_t - 1.40\pi_{t+1}$ which defines the equilibrium error. The error correction mechanism in the VECM implies that when the equilibrium error is positive, either the nominal interest rate must fall or the inflation rate must rise or both in order to restore the long-run equilibrium. If the equilibrium error were negative, the opposite must be true. Therefore to maintain a dynamically stable equilibrium, the error correction coefficient in the nominal interest rate equation of the VECM, if it is statistically different from zero, must be negatively signed and the opposite must hold for the error correction coefficient in the inflation equation of the VECM. Column 7 of table 6 suggests that inflation rates do not respond at all to the disequilibrium since all of the VECM inflation equations yield insignificant error correction coefficients, ie. the weak exogeneity result discussed earlier. But all of the nominal interest rate equations yield error correction coefficients consistent with the dynamic stability condition. This is in dramatic contrast to the VECM estimates derived under either a pre-specified Fisher effect of 1.00 or using the estimated Fisher effects from column 4. [17] In both of these cases, the systems are characterized by a significant degree of dynamic instability. For example, in the VECM estimated using a Fisher effect value of 1.00, yielded 22 dynamically unstable systems out the 24 estimated. These were all the result of positive and statistically significant nominal interest rate error correction coefficients. We consider this result to be strong evidence of a full tax-adjusted Fisher effect.

4.6 Panel Cointegration Analysis

Tables 7 and 8 display the results from the panel cointegration estimators described in section 3.4 and the tests discussed in section 3.5. Table 7 displays the results from regressing nominal interest rates on inflation and table 8 displays the results from the reverse regression. The panel OLS estimate of the Fisher effect is 0.46 when the regression is specified in teh convential manner. This is well below the theoretically correct value and statistically significantly so. But the FM-OLS estimate is 1.50 and statistically within the range of tax-adjusted Fisher effect estimates suggested by Summers (1983). In between these two estimates is the DOLS estimate which is insignificantly different from one. All panel tests for cointegration reject the null hypothesis of no cointegration at very high levels of marginal significance.[18] It is interesting to note that the panel OLS estimate is not much different from the average of the individual OLS estimates from table 4. The same is true for the DOLS estimator. But the FM-OLS panel estimate differs from the single equation counterpart with the panel estimate larger than any of the single equation estimates and almost double the average of the single equation FM-OLS estimates.

The estimates of the Fisher effect implied from the reverse regressions in table 8 are very similar to each other. All three of these estimates suggest a Fisher effect of approximately two, which is well above the theoretically implied value. Interestingly, all of the panel cointegration tests are much more significant than is the case in table 7. This may suggest a more accurate estimate as suggested by Ng and Perron (1997).

The final panel cointegration test is displayed in the last row of table 6 and is the test suggested by Larsson et al. (2001). The panel test for the null hypothesis of no cointegration is 4.23. This test has a limiting standard normal distribution implying a significant rejection at high marginal levels. The test statistic for the null that both series are stationary is 0.66 which is well below the critical values associated with any reasonable level of significance. Finally, the individual LR tests of the hypothesis that the estimated Fisher effect is equal to 1.40 will yield a joint test that has a limiting $\chi^2(23)$ distribution under the assumption that the individual tests are independent. The calculated value of this statistic is 30.10 which has a p-value of 0.147. The Johansen panel estimator yields results entirely consistent with the full tax-adjusted Fisher effect.

5 Conclusions

In this study we attempt to exploit the cross-sectional information in a panel of Fisher equations from different cities across the U.S. in order to yield better estimates of the Fisher effect and determine whether or not real rates of interest are stationary, as implied by economic theory, or non-stationary.

The results of our analysis are decidedly mixed. On the positive side we find almost no evidence of non-stationarity in the real interest rate. This is true regardless of the empirical methodology employed. But very little consensus was reached with regard to the size of the Fisher effect. It is certainly the case that the estimates from Johansen's MLE, whether applied to individual groups or to the panel as a whole, were the most supportive of a full tax-adjusted Fisher effect. Similarly, the results from reverse regression suggested by Ng and Perron (1997), i.e. inflation rates regressed on nominal interest rates, also yielded estimates consistent with a full tax-adjusted Fisher effect. This was true for all four single equation cointegration estimators, i.e. OLS, ARDL, FM-OLS and DOLS. [19] The Horvath-Watson estimates were also generally consistent with a full Fisher effect.

But the panel cointegration results were so disparate, depending on the particular estimator, that the panel tests shed no new light on this issue. The exceptions were the panel FM-OLS estimate from the regression of nominal interest rate on inflation and the panel MLE. The other panel estimators, while finding cointegration, yielded Fisher effect estimates as low as 0.46 and as large as 2.22.

One problem may be that there is too little cross-sectional variation between the groups. This would imply that there is little information gain by using the panel and we would not expect the panel estimators to perform well in such a situation.

Appendix

Because of the unavailability of city specific interest rates and the availability of individual city mortgage rates, our tests of city Fisher equation relies on a theoretical and statistical relationship between standard interest rates and mortgage rates. In this section we present a simple asset pricing model and a statistical justification for using mortgage rates to proxy for city interest rates.

From the above analysis we first need to establish a link between the effective mortgage rate, $i_{E,t}$ and a set of alternative interest rates which we are given by the one, two, three, five, seven, ten and thirty year t-bill rate, $i_\ell,\ \ell = 1, 2, 3, 5, 7, 10, 30$ years. T-bill data is available monthly from the Board of Governors. Because city specific mortgage data is not available at a monthly frequency, we use the national average flexible mortgage rate from FNMA to proxy for our city specific rates.

Because the national average is constructed from the country as a whole, it is a reasonable instrument for city specific rates. To ensure the nationwide mortgage is a suitable instrument for city specific mortgage rates consider the correlation coefficient between the nationwide effective mortgage rate and the individual city EMR. The correlations are all greater than 0.9762 (New York City), with a maximum of 0.9943 (Washington, DC) and a mean of 0.9872 (between Cleveland and Dallas-Ft. Worth) over the sample period giving confidence in using nationwide mortgage rates as a proxy for city mortgage rates.

From equation (5) we hypothesize there is a cointegrating relationship between the effective mortgage rate and the t-bill rates is given by

$$i_{\ell,t} - i_{E,t} = u_t. \tag{A1}$$

where u_t is stationary and replaces ξ_t, which implies a cointegrating vector of $(1\ \beta)' = (1 - 1)'$.

First, consider some simple descriptive statistics. The top half of Table 1 outlines the mean, standard deviation, σ, and the correlation between the effective mortgage rate and the t-bill rates, $corr(i_E, i_\ell)$. We also tabulate the correlation between changes in the mortgage rate and interest rates $corr(\Delta i_E, \Delta i_\ell)$

As expected the mortgage rates are higher than the t-bill rates but only by 1%-2% points and the volatility of the mortgage rate, given by the standard deviation, is close to that of the 10 and 30 year t-bill rates. Of more interest are the correlation statistics. The closest correlation coefficients are for the three to ten year t-bills. With respect to changes in the various interest rates the correlations are closest to one for the seven and ten year t-bills.

Table A1. Descriptive Statistics

Variable	mean	σ	$corr(i_E, i_\ell)$	$corr(\Delta i_E, \Delta i_\ell)$
Mortgage Rate	9.86	2.46	–	
1 Year t-bill	7.66	3.00	0.938	0.410
2 Year t-bill	7.99	2.85	0.958	0.433
3 Year t-bill	8.12	2.75	0.965	0.433
5 Year t-bill	8.32	2.64	0.968	0.433
7 Year t-bill	8.48	2.56	0.967	0.436
10 Year t-bill	8.54	2.51	0.964	0.438
30 Year t-bill	8.67	2.32	0.952	0.419

Central to our test of city specific Fisher relations is evidence for a strong *statistical* relationship between the effective mortgage rate and the various t-bill rates. To establish this link we conduct a number of cointegration tests to find evidence that there is a strong relationship between the various rates. We consider both the relative interest rates found in equation (5) and a unit root tests on the ratio of bond returns to the mortgage rate. To test the validity of equation (5) we employ the ERS and Ng and Perron *GLS* unit root tests, equations (8)-(10), and the Horvath and Watson VECM test with the restrictions for the cointegrating vector given by: $\Pi = -I_n + \sum_{i=1}^{p} \Pi_i$ is a matrix of coefficients with the restrictions

$$\Pi = \alpha\beta' = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} (1, -1). \tag{A2}$$

We consider both restricted case, given above, and an unrestricted version where we allow the data to estimate the cointegrating vector. For the unrestricted case, theory suggests that $\hat{\beta}$ will be in the neighborhood of -1.[20] Results of these five tests are presented in Table A2. In addition to the relevant test statistics: DF-GLS, MZa, MZt, $W_{0,1}(0,0)$, and $W_{0,1}(0,\alpha_R)$. We also present the first order AR coefficient from the unit root tests, $\hat{\rho}$, and the unrestricted estimated cointegrating vector from the HW VECM, $\hat{\alpha}_U$. 1%, 5% and 10% critical values are tabulated in the last three rows. First, we observe that the majority of risk free rates are cointegrated with mortgage rates, the exceptions being the two shortest term interest rates one and two years. Secondly, there appears to be considerable evidence for cointegration between mortgage rates and 30 treasury bonds rates. However, the strongest cointegration is between the medium term bond rates and the mortgage rates, three to ten years, with the strongest rejection of the null for the relationship between five and ten year bonds.

This observation is important in that most mortgages are held for between five and ten years. From an investment portfolio perspective that implies that medium term bonds and houses are close substitutes. Looking at the estimated AR(1) coefficients for these three relative rates, $\hat{\rho} \in [0.903, 0.922]$, are the smallest values of all the estimates. Using half-lives, this implies that differences between the bond and mortgage rates converge in less than one year: half $\in [0.59, 0.71]$ years.[21] These results are further exhibited by considering the unrestricted cointegrating vectors for these three series from the HW VECM, with $\hat{\alpha}_U \in [0.969, 1.002]$ in the neighborhood of 1. The best estimates are for the seven, $\hat{\alpha}_U = 0.996$, and ten year, $\hat{\alpha}_U = 1.002$ bonds. Further evidence for the portfolio argument.

Table A2. Relationship Between Mortgage Rates and T-bill rates

	$\hat{\rho}$	$DF-GLS^a$	MZa^b	MZt^c	$\hat{\alpha}_U$	$W_{0,1}(0,0)^d$	$W_{0,1}(0,\alpha_R)^e$
(i_E/i_1)	0.970	-2.024**	-8.722**	-2.088**	$(1\ -0.943)'$	18.977**	21.049***
(i_E/i_2)	0.950	-2.479**	-12.630**	-2.506**	$(1\ -0.918)'$	15.629**	22.279***
(i_E/i_3)	0.933	-2.892**	-17.656**	-2.947**	$(1\ -0.948)'$	20.254***	25.128***
(i_E/i_5)	0.907	-3.574**	-26.178**	-3.551**	$(1\ -0.969)'$	22.731***	28.587***
(i_E/i_7)	0.903	-3.553**	-25.958**	-3.526**	$(1\ -0.992)'$	23.574***	28.500***
(i_E/i_{10})	0.922	-3.069**	-20.152**	-3.060**	$(1\ -1.002)'$	21.416***	26.185***
(i_E/i_{30})	0.936	-2.855**	-17.598**	-2.872**	$(1\ -1.094)'$	15.508**	17.861**
1% C.V		-3.42	-23.80	-3.42		19.14	13.73
5% C.V		-2.91	-17.30	-2.91		14.93	10.18
10% C.V		-2.62	-14.20	-2.62		13.01	8.30

Notes: *,**,*** Denote statistical significance at the 10%, 5%, and 1% respectively.

The statistic $W_{0,1}(0,0)$ is calculated using $\alpha_R = (1\ -1)'$,the cointegrating vector $\hat{\alpha}_R$ is normalized as $(1\ \hat{\beta})'$.

Critical Values for the Horvath-Watson test are:19.17, 14.93, 13.01 (Table 1. Horvath and Watson, 1995)

The statistic $W_{0,1}(0,\alpha_R)$ is calculated using $\alpha_R = (1\ -1)'$, the cointegrating vector $\hat{\alpha}_R$ is normalized as $(1\ \hat{\beta})'$. Critical Values for the Horvath-Watson test are: 19.17, 14.93, 13.01 (Table 1. Horvath and Watson, 1995)

Notes

[1]At this point we ignore the implications of taxes for the magnitude of the Fisher effect

[2]Summers (1983) suggests an appropriate Fisher effect would be on the order of 1.3 to 1.5 given the marginal tax rates in the U.S.

[3]See Levi and Makin's (1978) equation (6) for further elaboration on this point. A nice derivation of the Mundell-Tobin effect within a dynamic general equilibrium model is that given by Ahmed and Rogers (1996).

[4]This is simply a recognition that in the steady-state equilibrium the LM curve is vertical.

[5]Cochrane (1991) presents arguments why such a result probably reflects the low power of the tests used. Even so, it is not well understood how one should treat series that appear to be non-stationary in finite samples when their limiting behavior is stationary. The standard treatment is to accept the finite sample properties, but there is little in the time series literature addressing this specific issue.

[6]Both King and Watson (1997) and Koustas and Serletis (1999) test for cointegration between nominal rates and inflation by testing the stationarity of the $r_t = i_t - \pi_{t+1}$. Besides having low power against the null of a unit root, these tests may not be appropriate if the true Fisher effect is not equal to one.

[7]Mishkin (1992) and Crowder and Hoffman (1996) demonstrate the relevance of this issue for U.S. inflation rates over the post-war period

[8]See Papell (1998) and the references therein for a discussion of the panel unit root tests as applied to the PPP hypothesis.

[9]See Phillips and Loretan (1991).

[10]The trace test takes the null hypothesis of at least r cointegrating relationships in X_t versus the alternative of p cointegrating vectors.

[11]See Kao and Chiang (2000), Phillips and Moon (1999) and Pedroni (1995) for a more detailed analysis of the panel cointegration estimators. Much of the discussion in the present paper is taken from Kao et al (1999).

[12]The cities included are: Atlanta, Boston, Chicago, Cleveland, Dallas-Ft. Worth, Denver,Detroit, Honolulu, Houston, Kansas City, Los Angeles, Miami, Milwaukee, Minneapolis-St. Paul, New York City, Philadelphia, Pittsburgh, Portland, OR, St. Louis, San Diego, San Francisco, Seattle, and Washington, DC. Various additional cities mortgage rates were available over the sample period, however the BLS does not collect CPI data for these cities. They are: Columbus, OH, Greensboro, NC, Indianapolis, Louisville, KY, Phoenix, Rochester, NY, Salt Lake City, and Tampa-St. Petersburg, FL. CPI data is available for Tampa-St. Petersburg but only beginning in 1987.

[13]The critical values displayed in table 3 were derived from a small Monte Carlo simulation where the assumed DGP was $x_{it} = 1.5X_{it-1} - 0.5x_{it-2} + \varepsilon_{it}$ and the covariance structure of the errors was chosen to match the empirical structure of the inflation panel as estimated by a VAR(1) model in first differences. Alternative DGP structures yielded similar small sample critical values.

[14]See Taylor and Sarno (1998) and Breuer et al (2001).

[15]Engle and Granger (1987) first demonstrated that under cointegration, at least one of the error correction coefficients must be non-zero.

[16]Extending the significance level to 20% allows rejection in all 24 cases.

[17]We do not present the results from these models to economize on space, but these results are available upon request.

[18]It is easy to see however that these panel cointegration tests suffer from the same uninformative alternative hypothesis as the panel unit root tests.

[19]For example, the average of the reverse regression OLS estimates is 0.85 while the average from the ARDL reverse regressions was 0.72. Both of these are statistically insignificantly different from the inverse of the tax-adjusted Fisher effect of 0.71.

[20]This restriction will be also imposed on the Fisher relation below.

[21]Half-lives are calculated using the standard method: half= $\ln(0.5)/\ln(\hat{\rho})$.

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Table 1: Univariate Unit Root Tests on Inflation

$Inflation$	ADF τ	PP Za	MZa	MZt	DF-GLS
US	-1.96	-7.07	-5.91	-1.65	-1.98
Atlanta	-2.22	-8.44	-6.70	-1.75	-2.21
Boston	-2.96	-11.83	-8.16	-1.95	-2.83
Chicago	-1.94	-6.97	-5.86	-1.65	-1.96
Cleveland	-2.09	-7.49	-5.06	-1.68	-2.07
Dallas	-1.97	-7.26	-6.02	-1.65	-1.98
Denver	-1.63	-5.21	-4.58	-1.45	-1.65
Detroit	-2.13	-7.89	-6.47	-1.75	-2.14
Honolulu	-2.17	-7.69	-6.28	-1.76	-2.15
Houston	-1.67	-5.65	-4.84	-1.45	-1.70
Kansas City	-2.32	-9.44	-7.40	-1.84	-2.34
Los Angeles	-2.54	-10.31	-7.81	-1.92	-2.53
Miami	-2.00	-7.29	-6.03	-1.77	-2.02
Milwaukee	-1.97	-6.90	-5.79	-1.66	-1.97
Minneapolis	-2.05	-7.86	-6.43	-1.70	-2.08
New York	-2.82	-10.43	-7.31	-1.90	-2.67
Philadelphia	-2.84	-11.20	-8.03	-1.98	-2.76
Pittsburgh	-1.87	-6.67	-5.65	-1.61	-1.90
Portland	-1.92	-6.12	-5.05	-1.56	-1.89
St. Louis	-1.87	-6.81	-5.75	-1.60	-1.90
San Diego	-1.84	-7.32	-6.10	-1.58	-1.89
San Francisco	-2.56	-10.65	-8.03	-1.94	-2.57
Seattle	-2.00	-6.94	-5.79	-1.67	-2.00
Washington DC	-2.44	-9.58	-7.40	-1.88	-2.43
5% C.V.	-3.41	-17.30	-17.30	-2.91	-2.91

Note: Entries represent the univariate unit root test statistics discussed in section 3.1. $^*-$ Denotes significance at the 5% level.

Table 2: Univariate Unit Root Tests on Interest Rates

Interest Rate	ADF τ	PP Za	MZa	MZt	DF-GLS
US	-3.26	-8.34	-4.70	-1.51	-2.68
Atlanta	-2.80	-7.17	-4.34	-1.44	-2.37
Boston	-2.96	-7.70	-4.23	-1.43	-2.61
Chicago	-1.94	-7.37	-4.14	-1.42	-2.52
Cleveland	-2.09	-7.31	-4.09	-1.39	-2.49
Dallas	-1.97	-6.79	-3.96	-1.37	-2.35
Denver	-1.63	-7.00	-4.27	-1.42	-2.34
Detroit	-2.13	-8.44	-4.54	-1.48	-2.75
Honolulu	-2.17	-6.95	-4.24	-1.40	-2.30
Houston	-1.67	-7.48	-4.66	-1.50	-2.40
Kansas City	-2.32	-7.27	-4.31	-1.43	-2.42
Los Angeles	-2.54	-7.88	-4.88	-1.54	-2.48
Miami	-2.00	-7.50	-4.43	-1.46	-2.47
Milwaukee	-1.97	-8.50	-5.14	-1.58	-2.61
Minneapolis	-2.05	-7.70	-4.60	-1.49	-2.49
New York	-2.82	-7.48	-4.41	-1.47	-2.49
Philadelphia	-2.84	-7.03	-3.98	-1.39	-2.45
Pittsburgh	-1.87	-7.88	-4.56	-1.48	-2.55
Portland	-1.92	-7.42	-4.63	-1.50	-2.40
St. Louis	-1.87	-7.41	-4.45	-1.46	-2.43
San Diego	-1.84	-7.35	-4.80	-1.52	-2.33
San Francisco	-2.56	-7.72	-4.74	-1.52	-2.48
Seattle	-2.00	-7.23	-4.39	-1.46	-2.40
Washington DC	-2.44	-6.96	-4.09	-1.38	-2.35
5% C.V.	-3.41	-17.30	-17.30	-2.91	-2.91

Note: Entries represent the univariate unit root test statistics discussed in section 3.1. $^*-$ Denotes significance at the 5% level.

Table 3: Panel Unit Root Tests

$Variable \backslash Test$	t_{LL}	$t_{IPS}(1)$	$t_{IPS}(3)$	$t_{IPS}(5)$
Inflation	-1.03	-4.84	-9.07	-5.19
Interest Rate	-6.28	-10.18	-1.18	-3.49
5% C.V.	-6.94	-3.54	-3.52	-3.74

Note: Entries represent the Levin and Lin (1992) and Im *et al.* (1997) panel unit root test statistics discussed in section 3.2. $^*-$ Denotes significance at the 5% level using small sample critical values.

Table 4: Single Equation Cointegration Estimates

Fisher Effect	OLS$_{i\to\pi}$	OLS$_{\pi\to i}$	ARDL$_{i\to\pi}$	ARDL$_{\pi\to i}$	FM-OLS$_{i\to\pi}$	FM-OLS$_{\pi\to i}$	DOLS$_{i\to\pi}$	DOLS$_{\pi\to i}$
US	0.53	0.72⋆	1.01⋆	0.67⋆	0.73	0.71⋆	1.56⋆	0.55⋆
	(0.15)	(0.20)	(0.22)	(0.07)	(0.20)	(0.12)	(0.54)	(0.16)
Atlanta	0.50	0.98⋆	1.88⋆	0.56	0.63	0.98	1.03⋆	0.73⋆
	(0.07)	(0.22)	(0.20)	(0.05)	(0.12)	(0.09)	(0.30)	(0.10)
Boston	0.56	0.70⋆	0.51⋆	0.74⋆	0.75	0.73⋆	1.20⋆	0.56⋆
	(0.15)	(0.19)	(0.49)	(0.09)	(0.21)	(0.11)	(0.34)	(0.23)
Chicago	0.39	0.86⋆	2.53⋆	0.77⋆	0.56	0.86⋆	1.25⋆	0.49⋆
	(0.12)	(0.26)	(1.35	(0.09)	(0.16)	(0.14)	(0.32)	(0.20)
Cleveland	0.41	1.02⋆	0.65	1.31	0.55	0.98⋆	0.75⋆	0.60⋆
	(0.11)	(0.26)	(0.08)	(0.12)	(0.14)	(0.15)	(0.37)	(0.20)
Dallas	0.34	0.97⋆	0.66	−2.08	0.46	0.92⋆	1.13⋆	0.39
	(.011)	(0.30)	(0.30)	(2.63)	(0.15)	(0.16)	(0.81)	(0.14)
Denver	0.31	0.87⋆	0.47⋆	0.05	0.42	0.78⋆	0.24	0.40⋆
	(0.11)	(0.31)	(1.05)	(0.20)	(0.16)	(0.22)	(1.21)	(0.35)
Detroit	0.33	0.81⋆	0.41	0.79⋆	0.49	0.75⋆	1.05⋆	0.26
	(0.12)	(0.29)	(0.34)	(0.14)	(0.18)	(0.17)	(0.99)	(0.25)
Honolulu	0.41	0.81⋆	−0.18	0.84⋆	0.63	0.90⋆	0.92⋆	0.37⋆
	(0.13)	(0.25)	(0.40)	(0.07)	(0.24)	(0.32)	(0.46)	(0.44)
Houston	0.29	0.80⋆	1.07⋆	0.79⋆	0.41	0.77⋆	0.80⋆	0.17
	(0.12)	(0.32)	(0.27)	(0.17)	(0.17)	(0.23)	(1.01)	(0.33)
Kansas City	0.33	0.85⋆	1.09⋆	0.26	0.46	0.85⋆	0.75⋆	0.40
	(0.12)	(0.30)	(0.65)	(0.17)	(0.16)	(0.16)	(0.74)	(0.06)
Los Angeles	0.49	0.95⋆	2.02⋆	0.88⋆	0.65	0.95⋆	1.19⋆	0.55⋆
	(0.12)	(0.22)	(1.52)	(0.08)	(0.16)	(0.15)	(0.25)	(0.25)
Miami	0.49	0.85⋆	1.16⋆	0.27	0.65	0.81⋆	1.15⋆	0.57
	(0.13)	(0.22)	(0.57)	(0.37)	(0.15)	(0.12)	(0.59)	(0.09)
Milwaukee	0.36	0.92⋆	1.19⋆	0.70⋆	0.51	0.91⋆	0.82⋆	0.44⋆
	(0.11)	(0.29)	(0.22)	(0.11)	(0.15)	(0.18)	(0.70)	(0.34)
Minneapolis	0.38	1.03⋆	2.21⋆	0.20	0.49	0.95⋆	0.75⋆	0.88⋆
	(0.10)	(0.28)	(0.68)	(0.16)	(0.12)	(0.14)	(0.77)	(0.15)
New York	0.67	0.67⋆	0.29	0.97⋆	0.87	0.75⋆	1.34⋆	0.59⋆
	(0.16)	(0.16)	(0.57)	(0.15)	(0.21)	(0.13)	(0.24)	(0.33)
Philadelphia	0.48	0.68⋆	−66.12⋆	2.44⋆	0.71	0.76⋆	1.33⋆	0.41⋆
	(0.15)	(0.21)	(1588.2)	(1.92)	(0.23)	(0.16)	(0.42)	(0.24)
Pittsburgh	0.41	0.78⋆	1.36⋆	0.08	0.57	0.77⋆	1.02⋆	0.30⋆
	(0.13)	(0.25)	(0.33)	(0.12)	(0.17)	(0.16)	(0.61)	(0.25)
Portland	0.25	0.53⋆	−2.59⋆	0.61⋆	0.43	0.52⋆	0.70⋆	−0.12
	(0.14)	(0.30)	(5.54)	(0.24)	(0.23)	(0.19)	(1.28)	(0.17)
St. Louis	0.42	0.88⋆	−1.90⋆	1.23⋆	0.56	0.88⋆	1.04⋆	0.49⋆
	(0.12)	(0.25)	(10.35)	(0.60)	(0.14)	(0.12)	(0.45)	(0.11)
San Diego	0.34	1.07⋆	0.89	1.09	0.45	1.10	0.92⋆	0.75⋆
	(0.10)	(0.31)	(0.26)	(0.12)	(0.12)	(0.16)	(0.26)	(0.17)
San Francisco	0.49	0.86⋆	1.20⋆	0.66⋆	0.65	0.75⋆	1.38⋆	0.51⋆
	(0.12)	(0.22)	(0.31)	(0.04)	(0.16)	(0.10)	(0.44)	(0.21)
Seattle	0.33	0.78⋆	0.18	0.55⋆	0.48	0.65⋆	0.63⋆	0.01
	(0.12)	(0.29)	(0.31)	(0.14)	(0.20)	(0.20)	(1.10)	(0.21)
Washington DC	0.52	0.95⋆	13.81⋆	2.77⋆	0.69	1.00⋆	1.06⋆	0.46⋆
	(0.12)	(0.21)	(144.56)	(1.63)	(0.16)	(0.18)	(0.30)	(0.24)

Note: Entries represent slope parameter estimates from the specified Fisher equation regression along with coefficient standard errors in parentheses. OLS$_{i\to\pi}$ is the regression of nominal interest on inflation using ordinary least squares while OLS$_{\pi\to i}$ is the OLS regression of inflation on nominal interest rate. Similar notation applies to the ARDL, FM-OLS and DOLS specifications. ⋆ denotes that estimate is statistically not different from Fisher effect of 1.4.

Table 5: Horvath-Watson Cointegration Tests

	$\ell = 1$			$\ell = 4$		
City	$W_{0,1}(0,\beta_{a_k})$	$W_{0,1}(0,0)$	$\widehat{\beta}_{a_u}$	$W_{0,1}(0,\beta_{a_k})$	$W_{0,1}(0,0)$	$\widehat{\beta}_{a_u}$
US	10.21*	10.88	1.69	13.43*	16.14*	1.59
Atlanta	10.73*	9.67	1.04	15.36*	15.55*	1.80
Boston	8.08	8.03	1.51	13.32*	13.63**	1.54
Chicago	9.52**	9.85	1.19	15.09*	15.11*	2.00
Cleveland	10.87*	11.31	1.31	9.01**	10.43	1.29
Dallas	8.14	8.75	1.14	10.52*	12.87**	1.77
Denver	8.55**	11.75	-3.60	12.13*	12.41**	4.03
Detroit	8.42**	8.66	1.59	12.69*	14.44*	0.54
Honolulu	6.43	6.42	1.19	13.39*	16.38*	1.21
Houston	9.11**	10.21	1.90	7.94	10.61	1.44
Kansas City	11.03*	12.03	1.68	6.22	6.68	2.68
Los Angeles	7.44	7.53	1.05	14.63*	16.54*	1.18
Miami	9.21**	9.15	1.34	7.97	8.60	5.32
Milwaukee	6.95	7.20	1.22	9.21**	10.94	1.39
Minneapolis	10.53*	11.09	1.17	12.14*	12.87**	1.94
New York	5.21	4.89	1.36	12.69*	12.74**	2.07
Philadelphia	7.31	7.12	1.32	16.12*	16.12*	13.32
Pittsburgh	6.93	7.46	1.67	16.47*	16.48*	13.80
Portland	10.24*	10.70	2.31	13.29*	13.29**	-20.97
St. Louis	12.29*	12.97**	1.33	10.52*	12.30**	0.94
San Diego	7.78	9.77	1.08	6.20	13.66**	0.92
San Francisco	5.16	5.40	1.46	15.09*	15.48*	1.61
Seattle	5.71	6.10	2.41	14.85*	16.13*	1.98
Washington DC	7.80	7.08	1.16	14.12*	14.19*	-9.14
5% C.V.	10.18	14.18		10.18	14.18	
10% C.V.	8.30	12.36		8.30	12.36	

Note: ℓ - The lag truncation parameter from equation (28). $W_{0,1}(0,\beta_{a_k})$ - HW test of the null that there is no cointegration with one known cointegration vector with cointegrationg parameter of $\beta = 1.40$ under the alternative. $W_{0,1}(0,0)$ - HW test of the null that there is no cointegration with one unknown cointegration vector under the alternative. $\widehat{\beta}_{a_u}$ - HW estimate of the Fisher effect. $^{*}-$ Denotes significance at the 5% level. $^{**}-$ Denotes significance at the 10% level.

Table 6: Maximum Likelihood Cointegration Estimates

MLE	$r=0^{a}$	$r \leq 0^{b}$	$\widehat{\beta}^{c}$	LR^{d}	$\alpha_i = 0^{e}$	$\alpha_\pi = 0^{f}$	$i_{LM}{}^{g}$	$\pi_{LM}{}^{g}$
US	29.64*	7.44	1.47	0.03	-4.64*	-0.07	0.11	3.35
Atlanta	38.90*	4.82	0.92	6.62*	-5.47*	0.54	1.78	3.34
Boston	40.57*	4.43	1.30	0.26	-6.42*	0.68	1.96	2.84
Chicago	29.97*	5.42	1.08	0.91	-5.23*	0.50	0.19	1.20
Cleveland	27.44*	4.07	0.86	3.55	-4.50*	0.6	0.88	1.64
Dallas	29.06*	4.81	1.00	1.18	-4.59*	0.79	2.24	3.87*
Denver	18.72*	4.50	1.19	0.09	-3.47*	0.76	0.36	0.11
Detroit	33.89*	5.34	1.27	0.13	-5.28*	0.79	0.59	3.66
Honolulu	7.27	1.55	1.37	0.01	-1.54	0.84	1.67	1.99
Houston	21.91*	6.27	1.39	0.00	-3.59*	0.77	0.04	0.29
Kansas City	25.67*	3.14	1.00	1.29	-4.95*	1.21	2.48	0.38
Los Angeles	23.49*	3.34	1.11	0.78	-3.42*	0.99	1.05	1.49
Miami	39.26*	6.41	1.00	0.45	-5.22*	0.53	0.59	1.28
Milwaukee	29.16*	5.75	0.84	2.98	-4.88*	0.54	0.25	1.75
Minneapolis	29.37*	5.38	1.15	0.39	-3.49*	0.96	0.04	0.02
New York	34.92*	5.12	1.20	1.03	-6.91*	-0.28	2.86	3.84*
Philadelphia	29.44*	4.02	1.39	0.00	-5.25*	0.54	0.25	2.88
Pittsburgh	27.05*	4.18	0.94	2.47	-5.23*	0.74	1.55	0.39
Portland	20.81*	4.31	1.86	0.27	-4.21*	0.97	0.57	3.02
St. Louis	31.45*	6.86	1.04	1.13	-4.83*	0.43	0.23	0.86
San Diego	28.66*	5.61	0.78	3.55	-4.37*	0.85	0.64	1.10
San Francisco	29.13*	2.91	1.45	0.02	-3.01*	1.39	0.62	0.22
Seattle	26.80*	4.36	1.61	0.13	-4.04*	0.95	2.90	1.35
Washington DC	24.71*	3.36	0.96	2.83	-4.31*	0.51	3.62	2.26
5% C.V.	17.86	8.07		3.84	±1.96	±1.96	3.84	3.84
Panel MLE								
$\Upsilon_{\overline{LR}}$	4.23^{*h}	0.66^{i}		30.10^{j}				

Note: a - ML test of the null that there is no cointegration. b - ML test of the null that both series are stationary. c - ML estimate of the Fisher effect. d - Likelihood ratio test of the null that $\widehat{\beta} = 1.4$. e - Student t-test of the null that nominal interest rate is weakly exogenous. f - Student t-test of the null that the inflation rate is weakly exogenous. g - LM test for first order residual serial correlation. h - Panel MLE test of the null that there is no cointegration. i - Panel MLE test of the null that both series are stationary. j - Distributed $\chi^2(24)$. $^*-$ Denotes significance at the 5% level.

Table 7: Panel Cointegration Analysis: $i \rightarrow \pi$

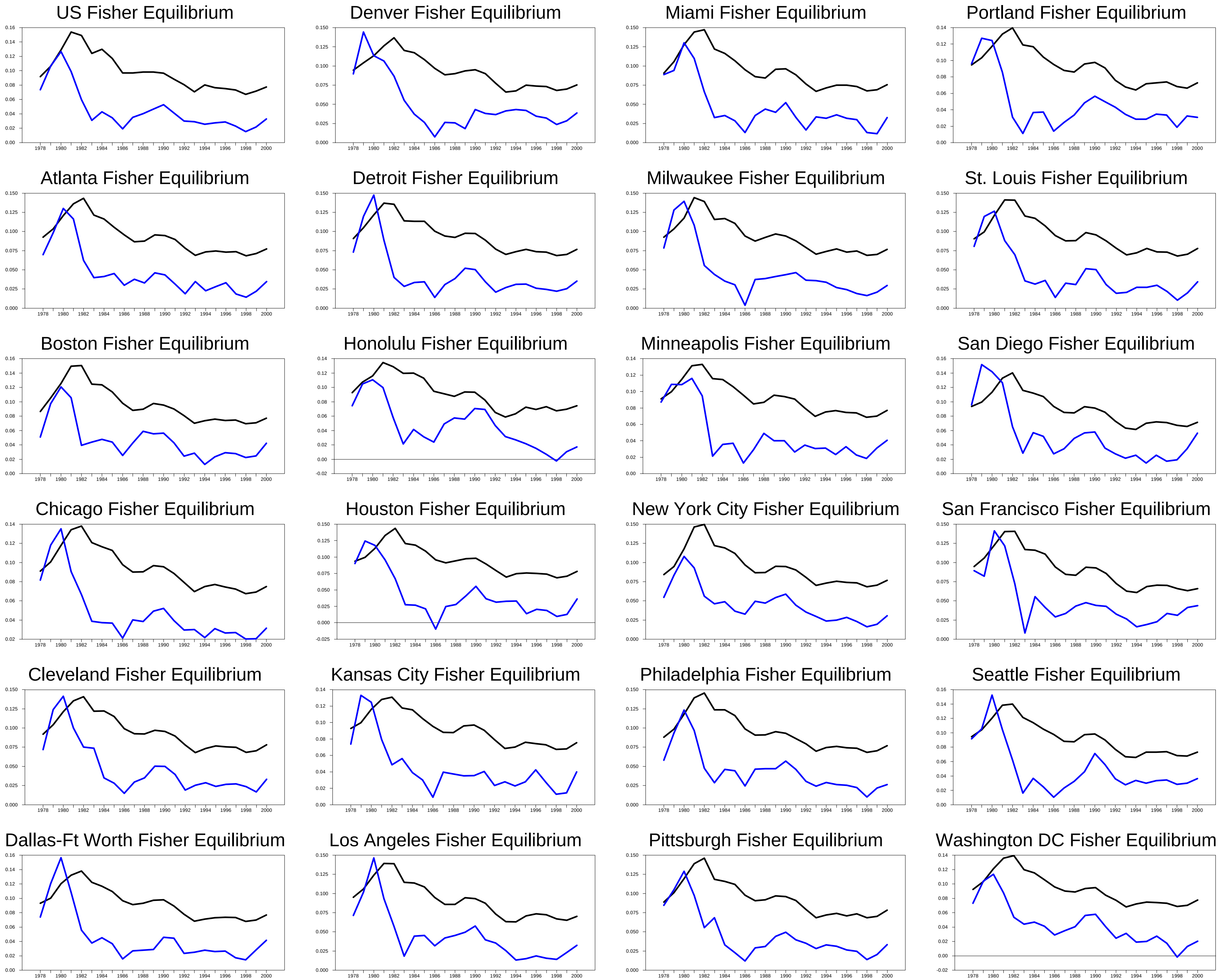
$Estimator$	OLS	$FM-OLS$	$DOLS$
$\widehat{\beta}$ $(\sigma_{\widehat{\beta}})$	0.46 (0.04)	1.50 (0.04)	1.02 (0.05)
$Cointegration\ Test$			
DF_ρ	-2.73	-2.22	-17.41
DF_t	-2.57	-1.93	-15.41
DF_ρ^*	-9.68	-5.34	-23.75
DF_t^*	-3.81	-3.02	-15.33
$ADF(1)$	-6.90	-6.36	-7.90
$ADF(2)$	-4.79	-4.31	-7.44
$ADF(3)$	-4.38	-2.56	-7.53
$ADF(4)$	-7.72	-1.79	-4.22
$ADF(SBC)$	-3.80_6	-4.31_2	-4.22_4
5% C.V.	-1.65	-1.65	-1.65

Note: Entries in row 2 represent the estimates of the Fisher effect and the coefficient standard error in parentheses. Entries in rows 4 through 11 represent the test statistics for the null of no cointegration as discussed in section 3.4. $ADF(SBC)$ denotes the panel cointegration ADF test with lag length chosen by minimizing the Schwartz-Bayesian criterion. The chosen lag is given by the subscript to the test statistic in row 12.

Table 8: Panel Cointegration Tests: $\pi \rightarrow i$

$Estimator$	OLS	$FM-OLS$	$DOLS$
$\widehat{\beta}$ $(\sigma_{\widehat{\beta}})$	0.50 (0.05)	0.50 (0.05)	0.45 (0.06)
$Cointegration\ Test$			
DF_ρ	-17.51	-23.51	-19.34
DF_t	-15.29	-17.37	-15.12
DF_ρ^*	-29.50	-23.01	-24.33
DF_t^*	-14.18	-17.34	-15.17
$ADF(1)$	-5.40	-12.61	-8.40
$ADF(2)$	-3.59	-7.88	-5.15
$ADF(3)$	-2.92	-5.17	-6.15
$ADF(4)$	-3.80	-4.98	-3.93
$ADF(SBC)$	-6.91_6	-4.57_6	-8.40_1
5% C.V.	-1.65	-1.65	-1.65

Note: See notes to table 7.


US Fisher Equilibrium
Denver Fisher Equilibrium
Miami Fisher Equilibrium
Portland Fisher Equilibrium
Atlanta Fisher Equilibrium
Detroit Fisher Equilibrium
Milwaukee Fisher Equilibrium
St. Louis Fisher Equilibrium
Boston Fisher Equilibrium
Honolulu Fisher Equilibrium
Minneapolis Fisher Equilibrium
San Diego Fisher Equilibrium
Chicago Fisher Equilibrium
Houston Fisher Equilibrium
New York City Fisher Equilibrium
San Francisco Fisher Equilibrium
Cleveland Fisher Equilibrium
Kansas City Fisher Equilibrium
Philadelphia Fisher Equilibrium
Seattle Fisher Equilibrium
Dallas-Ft Worth Fisher Equilibrium
Los Angeles Fisher Equilibrium
Pittsburgh Fisher Equilibrium
Washington DC Fisher Equilibrium