



# REVIEW MATERIALS FOR REAL ESTATE

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## ANNUAL COMPOUND INTEREST TABLES AT 10%

Year	Compound amount of \$1	Amount of \$1 per period	Sinking fund factor	Present value of \$1	Present value of \$1 per period	Installment to amortize \$1
1	1.1000	1.0000	1.0000	0.9090	0.9090	1.1000
2	1.2100	2.1000	0.4761	0.8264	1.7355	0.5761
3	1.3310	3.3100	0.3021	0.7513	2.4868	0.4021
4	1.4641	4.6410	0.2154	0.6830	3.1698	0.3154
5	1.6105	6.1051	0.1637	0.6209	3.7907	0.2637
6	1.7715	7.7156	0.1296	0.5644	4.3552	0.2296
7	1.9487	9.4871	0.1054	0.5131	4.8684	0.2054
8	2.1435	11.4358	0.0874	0.4665	5.3349	0.1874
9	2.3579	13.5794	0.0736	0.4240	5.7590	0.1736
10	2.5937	15.9374	0.0627	0.3855	6.1445	0.1627
11	2.8531	18.5311	0.0539	0.3504	6.4950	0.1539
12	3.1384	21.3842	0.0467	0.3186	6.8136	0.1467
13	3.4522	24.5227	0.0407	0.2896	7.1033	0.1407
14	3.7974	27.9749	0.0357	0.2633	7.3666	0.1357
15	4.1772	31.7724	0.0314	0.2393	7.6060	0.1314
16	4.5949	35.9497	0.0278	0.2176	7.8237	0.1278
17	5.0544	40.5447	0.0246	0.1978	8.0215	0.1246
18	5.5599	45.5991	0.0219	0.1798	8.2014	0.1219
19	6.1159	51.1590	0.0195	0.1635	8.3649	0.1195
20	6.7275	57.2749	0.0174	0.1486	8.5135	0.1174

# MATERIALS FOR REAL ESTATE INVESTMENT ANALYSIS

## COMPOUND INTEREST

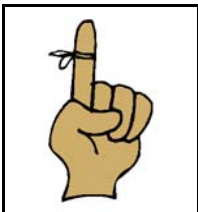
Compound interest is the earning of interest on interest. Compare the accumulation of interest when there is no compounding at 10%.

### With no compounding

Time	Prin.	Int.	Ending Bal
1	1,000	$(1,000 \times .10) = 100$	\$1,100
2	1,100	$(1,000 \times .10) = 100$	\$1,200
3	1,200	$(1,000 \times .10) = 100$	\$1,300
4	1,300	$(1,000 \times .10) = 100$	\$1,400
5	1,400	$(1,000 \times .10) = 100$	\$1,500

### With compounding

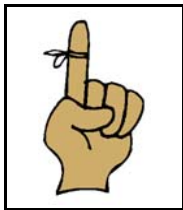
1	1,000	$(1,000 \times .10) = 100$	\$1,100
2	1,100	$(1,100 \times .10) = 110$	1,210
3	1,210	$(1,210 \times .10) = 121$	1,331
4	1,331	$(1,331 \times .10) = 133.10$	1,464.10
5	1,464.10	$(1,464.10 \times .10) = 146.41$	\$1,610.51



The key with compound interest, when interest has been earned it becomes principal for the next compounding period.

There are 5 variables that are used to make compound interest calculations (4 are used in any calculation)

1. Present value (PV) - the value of a sum of money in present dollars.
2. Future value (FV) - the value of a sum of money in the future after interest has been calculated.
3. Interest rate (i) - the rate of interest earned, or the discount rate
4. Number of compounding periods (n) - the number of periods over which interest compounding takes place. This can be expressed in terms of years, months, days, or any other unit of time.
5. Payment (PMT) - The amount of periodic payment



The typical compound interest problem gives you 3 variables and you solve for the 4th.

**EXAMPLE:** You deposit \$500 in a savings account that pays 10% interest compounded annually. How much will you have on deposit at the end of 3 years?

PV = \$500  
 i = 10%  
 N = 3  
 FV = ?

3 variables known, 1 unknown (FV)

YEAR

1	$\$500 \times 10\% = \$50 + \$500 = \$550$
2	$\$550 \times 10\% = \$55 + \$550 = \$605$
3	$\$605 \times 10\% = \$60.50 + \$605 = \$665.50$

We will study 6 ways of making compound interest calculations. We have already studied Future Value of a Lump Sum. Now we will learn a shortcut for making the interest calculations.

## 1. FUTURE VALUE OF A LUMP SUM

We have been making our calculations by figuring the interest and adding it back to the principal.

EX:  $\$500 \times 10\% \text{ interest} = \$50$

$\$500 \text{ principal} + \$50 \text{ interest} = \$550 \text{ total at the end of the year}$

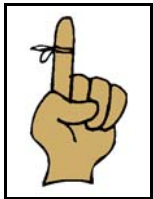
We could recalculate our earlier problem by simply multiplying the principal times 1.1 (1 plus the interest rate, expressed as a decimal) for the number of compounding periods.

$$\begin{array}{ccccccc} 500 & \times & 1.1 & = & 550 & \times & 1.1 & = & 605 & \times & 1.1 & = & \$665.50 \\ 1 & & & & 2 & & & & 3 & & & & \end{array}$$

But there is an even easier way. We have multiplied the principal times 1.1 three times which is the same as  $(1.1 \times 1.1 \times 1.1) \times (\text{the principal})$

Algebraically we can reduce  $1.1 \times 1.1 \times 1.1$  to  $1.1^3$  and then multiply it times the principal.

$$1.1^3 = 1.3310 - \text{This is the factor for this problem}$$



The "factors" that we can derive from the formulas are the basis for the **SIX FUNCTIONS OF ONE DOLLAR TABLE**.

$$1.3310 \times \$500 = \$665.50$$

We can make this factor into a formula as follows:

$$FV_{i,n} = PV (1 + i)^n$$

or in our example =

$$FV_{10\%,3\text{yr}} = \$500 (1 + .1)^3 = \$665.50$$

Calculator:  $PV = -500, n = 3, i = 10$  Solve for:  $FV = \$665.50$

## 2. PRESENT VALUE OF A LUMP SUM

Finding the PV of a future sum is simply the reverse process of finding the FV of a present sum. The process is referred to as discounting.

Suppose we have the opportunity to buy a tract of land that will be worth \$10,000 in 3 years. How much would we be willing to pay for it today?

We know that money received in the future is worth less than money received today, because if we had the money today, we would put it into an investment and earn interest.

If we can find future value by multiplying (1 + int rate), then we can find present value by dividing.

Year

- 1       $10,000 \div 1.1 = \$9,090.91$
- 2       $9,090.91 \div 1.1 = \$8,264.46$
- 3       $8,264.46 \div 1.1 = \$7,513.15$  present value

Once again, we can shorten this process


$$PV_{i,n} = \frac{FV}{(1+i)^n} = FV \frac{1}{(1+i)^n}$$

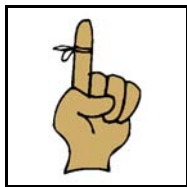
$$PV_{10\%,3\text{yrs}} = \$10,000 \left[ \frac{1}{(1+.1)^3} \right] = 10,000 \times .7513 = \$7,513$$

On the calculator:

FV = \$10,000  
 i = 10  
 n = 3  
 Solve for PV = \$7,513

We can double-check our calculations to show that \$7,513 today is the same as \$10,000 in 3 years if we use a discount rate of 10%.

	<u>Year</u>	
	1	$\$7,513 \times 1.1 = \$8,264.46$
	2	$\$8,264.46 \times 1.1 = \$9,090.91$
	3	$\$9,090.91 \times 1.1 = \$10,000$



Income discounting is a very important concept, since most financial analysis of real estate cash flows deals with the value of cash flows to be received in the future.

### 3. FUTURE VALUE OF AN ANNUITY

An annuity is a series of equal amounts received per period for a specified number of periods. For calculation purposes, there are 2 types of annuities:

- a) Regular Annuity - payments are made at the end of each period.
- b) Annuity due - payments are made at the beginning of each period.

For the time being, we will work with regular annuities.

**EXAMPLE:** If you save \$1,000 per year for 4 years at 10% interest, how much will you have at the end of 4 years?

Year	Beginning balance	Interest	Deposit	Ending balance
1	\$ 0	\$ 0	\$1,000	\$1,000
2	\$1,000	\$100	\$1,000	\$2,100
3	\$2,100	\$210	\$1,000	\$3,310
4	\$3,310	\$331	\$1,000	\$4,641

In equation form:

$$FVA_{i,n} = ANN \left[ \frac{(1+i)^n - 1}{i} \right]$$

Calculation:

$$FVA_{10\%, 4yrs} = \$1,000 \left[ \frac{(1+.1)^4 - 1}{.1} \right] = \$1,000 \left[ \frac{.4641}{.1} \right] = \$1,000 \times 4.6410 = \$4,641$$

On the calculator:

$$N = 4$$

$$i = 10$$

$$PMT = -1,000$$

$$\text{Solve for FV} = \$4,641$$

#### 4. SINKING FUND FACTOR

The sinking fund factor tells you how much you need to save each period in order to accumulate a specific sum.. This is normally expressed as deposits at the end of a period, but can also be expressed as deposits at the beginning of a period. We will work with deposits at the end of a period.

**EXAMPLE:** You need to save money for a down payment on a house. You will need \$10,000 and would like to save it over a period of four years, with deposits at the end of each year. How much do you need to deposit at the end of each year if you can earn 10% per year on your money?

a) Find sinking fund factor

$$SFF_{i,n} = \left[ \frac{i}{(1+i)^n - 1} \right] = \left[ \frac{.1}{(1.1)^4 - 1} \right] = .21547$$

b) multiply the desired sum times the SFF

$$ANN = P(SFF)$$

$$ANN = 10,000 \times .21547 = \$2,154.70$$

c) check:

<u>Year</u>	<u>Beg. bal.</u>	<u>Interest</u>	<u>Deposit</u>	<u>Ending balance</u>
0	0	0	0	0
1	0	0	2,154.70	\$2,154.70
2	2,154.70	215.47	2,154.70	4,524.87
3	4,524.87	452.49	2,154.70	7,132.06
4	7,132.06	713.21	2,154.70	\$9,999.98 \$10,000

## 5. PRESENT VALUE OF AN ANNUITY

The present value of an annuity is the value, in today's dollars, of a stream of equal payment to be received in the future.

**EXAMPLE:** Harold is retiring and would like to receive \$10,000 per year for the next 20 years. If the investment rate is 10%, how much will he have to pay for this annuity?

The value of the annuity is the sum of the present values of each annuity payment.

In equation form:

$$PVA_{in} = ANN \left[ \frac{1 - [1/(1+i)^n]}{i} \right]$$

$$= \$10,000 \frac{1 - [1/(1.1)^{20}]}{.1}$$

$$= \$10,000 \times 8.5136$$

$$= \$85,136$$

ON THE FINANCIAL CALCULATOR:

$$N = 20$$

$$I = 10$$

$$PMT = 10,000$$

$$PV = \$85,135.64$$

## 6. MORTGAGE CONSTANT

Used to figure mortgage payments. Interest cannot be charged until it is earned, so payments are made at the end of each period.

**EXAMPLE:** A borrower wants to borrow \$80,000 to buy a house. The interest rate is 10% per annum, and the loan calls for annual payments over 25 years. What is the amount of the annual payment?

Equation:

$$ANN_{i,n} = PVA \left[ \frac{i}{1 - [1/(1+i)^n]} \right]$$

$$= \$80,000 \left[ \frac{.1}{1 - [1/(1.1)^{25}]} \right]$$

Money talks! Dont forget to clear your calculator memory before you do the next calculation.



$$= \$80,000 \left[ \frac{.1}{.9077} \right]$$

$$= \$80,000 \times .1102 = \$8,813.45$$

On the calculator:

$$PV = 80,000$$

$$i = 10$$

$$n = 25$$

$$PMT = \$ - 8,813.45$$

## NET PRESENT VALUE, INTERNAL RATE OF RETURN, MODIFIED INTERNAL RATE OF RETURN

### a. NET PRESENT VALUE

Net present value is the sum of the values of a series of future cash flows, discounted at a given rate and netted against the original cost of the investment. The calculation can be made easily for a small series of cash flows.

**EXAMPLE:** What is the net present value of the following series of annual cash flows that occur at the end of each year. Use a 10 percent discount rate. Assume that the investor paid \$15,000 for the right to receive these cash flows.

Year	Cash flow	Present value of cash flow
1	\$5,000	\$4,545.46
2	\$7,500	\$6,198.35
3	\$6,000	\$4,507.89
	Sum =	\$15,251.70

The investor paid \$15,000 for the investment (the cash flow at time zero, which is represented as  $Cf_0$ ). We treat this as a negative cash flow, since it represents a cash flow going away from the investor. We then net out the negative cash flow at time zero with the sum of the discounted cash flows to be received in the future ( $\$15,251.70 - \$15,000 = \$251.70$ ). This investment has a net present value of \$251.70. Note that the NPV would change if we used a different discount rate, since the present values of the future cash flows would change. If the discount rate is higher than 10 percent, the present values would shrink and NPV might be negative. If the discount rate is lower than 10 percent, the present values of the future cash flows would be higher, and the NPV would be higher than \$251.70.

This sensitivity to the discount rate means that the investor can use NPV as a simple investment rule. If the investor wants to earn a target rate of interest, say 10 percent, then any investment in which NPV is equal to or greater than zero is a candidate for investment. If NPV is negative, the investor should not invest, since she will not earn her target rate of return. This investment rule is referred to as the ANPV Rule, and while it is useful, it is only a rule of thumb since it ignores other factors such as the risk of the investment.

Your calculator can make this calculation much easier. Your calculator has the ability to store a series of cash flows and then perform operations on the stored series. Learn how to input a series of cash flows on your calculator. To use the NPV function, you must input a discount rate. After inputting the discount rate, the calculator will figure the NPV from the cash flow series.

*b. INTERNAL RATE OF RETURN*

The internal rate of return (IRR) is the actual rate of return earned on a series of cash flows. Sometimes we know what the cash flows are, if the cash flows have already been earned. Often a real estate investor, whether the investor invests in a loan or directly as an equity owner in the property, must estimate cash flows to be received in the future. Of course, in such instance the cash flows are anticipated, not actual. A more technical definition of the IRR is the discount rate that makes NPV equal to zero.

Solving for the IRR is a trial and error process, even on your calculator. The calculation requires trying different discount rates until one is found that makes NPV=0. The table below shows how the internal rate of return can be determined by interpolating between trial discount rates. Assume that the investor paid \$100,000.00 for the investment.

End of year	Cash flow	Present value at 10%	Present value at 15%
1	\$9,000	\$8,182	\$7,826
2	\$9,500	\$7,851	\$7,183
3	\$12,000	\$9,016	\$7,890
4	\$118,000	\$80,596	\$67,467
	Totals	\$105,645	\$90,366

Notice that the present value of the cash flows at a 10% discount rate is \$105,645 making the net present value of the cash flows \$5,645. When NPV is positive, at a given discount rate, that means the IRR must be higher than that discount rate. Conversely, the present value of the cash flows at 15% is \$90,366, making the net present value (\$9,634)> Since NPV is negative at 15%, the IRR must be less than 15%. It is somewhere between 10% and 15%.

Interpolation:

PV at 10%     \$105,645  
 PV at 15%     \$90,366  
 Difference=    \$15,279

PV at smaller discount rate =             \$105,645  
 Less purchase price:                         \$100,000  
    \$5,645

Absolute difference in discount rates=15% - 10% = 5%  
 5% divided by \$15,279 times \$5,645 = .0185 or 1.85%  
 1.85% + (the smaller discount rate) 10% = 11.85% IRR

We can verify this IRR by inputting the series of cash flows into our calculator and solving for the IRR, which is 11.73% (difference due to rounding).

There are several major problems with the IRR as a measure of return:

1. It is insensitive to the scale of the investment. Suppose someone asked you whether you would like to make a 20% return on an investment or a 100% return? You are tempted to take the 100%. But suppose further that the 100% return was based on investing \$1 and getting back \$2 at the end of the year, whereas the 20% investment was based on investing \$1,000 and getting back \$1,200 at the end of the year. By choosing the 20% alternative, you would be \$199 better off at the end of the year. For this reason, NPV is considered to be a better measure of return, because it measures the wealth increase in absolute dollars, not percentages.

2. It does not tell you enough about the risk of the investment. Again, assume that the investor has the choice to two financial instruments, each of which requires a \$100,000 investment.

End of year	Cash flow, investment #1	Cash flow investment #2
1	\$8,000	\$1,000
2	\$11,000	\$1,500
3	\$12,000	\$400
4	\$13,300	\$2,200
5	\$125,000	\$177,460

The IRR from each investment is 13%, but notice that investment #2 has small cash flows at the beginning of the investment period and a very large cash flow at the end. The further in the future that cash flows are received, the more risk exists that the cash flow will not be repaid. The debtor could go bankrupt, die, or have business problems that diminished the probability of payment. Thus, although the IRR's are the same, the investor should choose Investment #1 because of the lower risk.

3. The IRR calculation assumes that all cash flows earned from the investment are immediately reinvested at the IRR rate. Of course, cash earned on an investment is often temporarily placed in a money market account, or reinvested in another type of investment that may have different risk and return characteristics.

4. If the cash flow pattern has negative cash flows (after the initial investment, which will always be negative) the IRR calculation can generate multiple solutions. The multiple solutions are confusing and do not provide the investor with useful information. Since real estate investments (whether debt or equity investments) often have negative cash flows, the investor must resort to another technique to determine the rate of return, One of these solutions is the

Modified Internal Rate of Return, which is discussed below.

*c. MODIFIED INTERNAL RATE OF RETURN*

The Modified Internal Rate of Return (MIRR) calculation eliminates the multiple solution problem of the IRR. THE MIRR calculation requires an assumption about the rate at which the cash flows can be reinvested. It also requires an assumption about the cost of funding the negative cash flows. Once those assumptions are made, the MIRR calculation is made by discounting the negative cash flows back to time zero and compounding the positive cash flows out to the end of the investment period. The determination of the MIRR is then easily made by inputting into the calculator the PV of the negative cash flows as PV, the FV of the positive cash flows as FV, N for the number of periods, and solve for **i**.

**EXAMPLE:**

Assume a 6% (safe rate) for discounting the negative cash flows and a 9% (investment rate) for compounding the positive cash flows. The investment has the following cash flows:

Year	Cash flow	PV of negative cash flows at 6%	FV of positive cash flows at 9%
0	(\$100,000)	(\$100,000)	
1	\$10,000		\$14,116
2	(\$15,000)	(\$13,350)	
3	18,000		\$21,386
4	(\$22,000)	(\$17,426)	
5	\$185,000		\$185,000
Totals		(\$130,776)	\$220,502

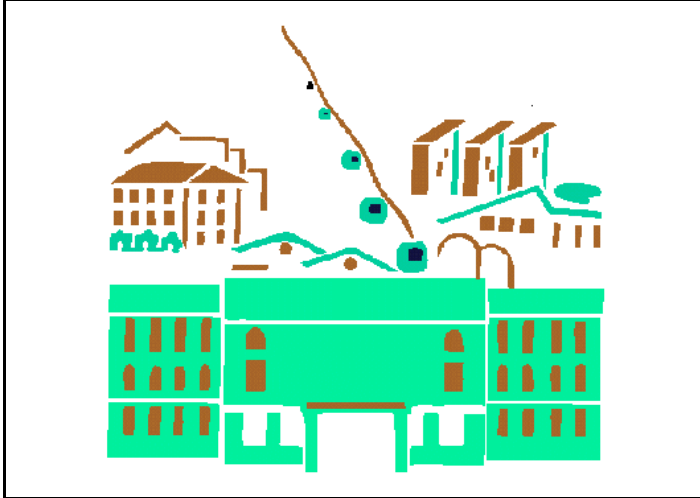
On the calculator:

PV = (130,776

FV = \$220,502

N = 5

solve for  $i = 11.01\% = \text{MIRR}$



## LOAN BALANCES

Loan balances can be calculated by using the AMORT function on your financial calculator. Consult your calculator manual for the procedure applicable for your calculator. However, there is a way of calculating loan balances that works with any calculator. The outstanding balance on any loan at any time is simply the present value of the remaining payments discounted back to present value at the contract loan rate.

### EXAMPLE:

Judy borrows \$120,000 from Capital Mortgage Co. at 9% per annum interest, monthly payments for 30 years. At the end of 6 years, she sells the house and will pay off the loan at the closing of sale (assume that the sale takes place at the end of the sixth year). What will her loan payoff be?

### SOLUTION:

First, calculate the monthly payment, which is \$965.55. You can use the AMORT function to calculate the loan balance, which is \$113,772.56. Or, you could calculate the present value of the remaining payments.

$$\$965.55 = \text{PMT}$$

$$288 = N \text{ (24 years remaining } \times 12)$$

$$9 = I$$

$$\$113,772.90 = \text{PV (slight difference due to rounding)}$$

## LOAN DISCOUNT POINTS

Loan discount points are lump sum interest charges paid by the borrower to the lender to raise the lender's yield above the stated rate on the loan. The lender's yield will be higher if the borrower pays off the loan prior to maturity because the lump sum has a larger effect if it is spread out over a shorter term.

### EXAMPLE #1

Angus borrows \$100,000 at 8.25% for 30 years (monthly payments) from Cashflow Mortgage Co. to buy his house. The lender charges two loan discount points (2% of the loan amount). What will be the lender's yield if the loan is held to maturity?

**SOLUTION:**

The lender receives the 2 discount points at closing from the borrower, so the borrower nets only \$98,000 on the loan, although he will make payments on a \$100,000 loan. First, calculate the payment on the \$100,000 loan. The payment is \$751.27. Then calculate the interest rate for 30 years on a \$98,000 loan (the lender's investment) at a payment rate of \$751.27.

$$360 = N$$

$$(\$98,000) = PV$$

$$\$751.27 = PMT$$

$$I = 8.47\%$$

**EXAMPLE #2**

In the previous problem, suppose Angus paid off the loan at the end of the fourth year. What would be the lender's yield?

**SOLUTION:**

We already know several things: first, the lender will invest \$98,000. Second, Angus will pay \$751.27 for 4 years, plus pay off the remaining loan balance at the end of the fourth year. So, the lender will have an initial negative cash flow of \$98,000 followed by 47 payments of \$751.27 and an 48th payment consisting of \$751.27 plus the loan payoff. We do not know the loan payoff, so we must solve for it. Use a \$100,000 loan amount (remember, the loan payments are based on this amount) and solve for the balance at the end of 4 years. It is \$96,388.28, so the 48th payment to the lender is \$97,139.55 (the monthly payment plus the payoff). Since we have a series of uneven cash flows, we must go into the cash flow menu of our calculator.

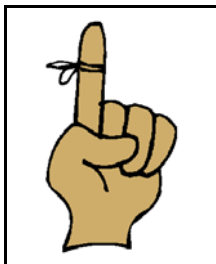
$$(\$98,000) = CF_0$$

$$\$751.27 = CF_1$$

$$47 = N$$

$$\$97,139.55 = CF_2$$

$$I = 0.7379 \times 12 = 8.86\%$$



As you can see, the yield went up substantially due to the early loan payoff and hence the effect of spreading the points over a shorter period. The borrower will not want to pay points if he or she intends to hold the property for a short time. The borrower should calculate the effect of points over the holding period to see if a loan without points would be more desirable.

**AMORTIZATION SCHEDULE**

The following is an amortization schedule for a \$10,000 loan with annual payments at 10% per annum interest.

Year	Payment	Interest	Principal	Balance
0	\$ 0	\$ 0	\$ 0	\$10,000
1	\$1,627.45	\$1,000.00	\$ 627.45	\$ 9,372.55
2	\$1,627.45	\$ 937.25	\$ 690.20	\$ 8,682.35
3	\$1,627.45	\$ 868.23	\$ 759.22	\$ 7,923.13
4	\$1,627.45	\$ 792.31	\$ 835.14	\$ 7,087.99
5	\$1,627.45	\$ 708.80	\$ 918.66	\$ 6,169.33
6	\$1,627.45	\$ 616.93	\$1,010.52	\$ 5,158.81
7	\$1,627.45	\$ 515.88	\$1,111.57	\$ 4,047.24
8	\$1,627.45	\$ 404.72	\$1,222.73	\$ 2,824.51
9	\$1,627.45	\$ 282.45	\$1,345.00	\$ 1,479.50
10	\$1,627.45	\$ 147.95	\$1,479.50	\$ 0

## DURATION

For our purposes in real estate investment analysis, duration is a measure of risk. Duration measures the weighted average until cash flow payment. Given that we know that cash flows are riskier the further out in time, one investment that has a shorter duration than a second investment will be preferable (assuming that both investments have the same rate of return).

To calculate duration, we need to know the cash flows and the discount rate. We find the present value of the cash flows by discounting them back to present value at the given discount rate. The present values are then summed. Calculate the proportion by dividing the PV of each cash flow by the total PV of all cash flows. Then multiply the proportion by the number of the year (or other period, such as months if you are using monthly cash flows). Add the weighted proportions and the result is the duration, expressed in the number of time periods.

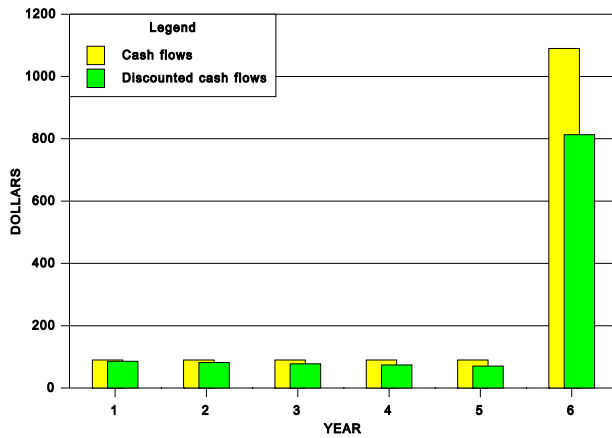
**EXAMPLE:** An investment has the following cash flows. Use a 5% discount rate.

YR	Cash flow	PVCF	Proportion	Prop. X year
1	\$ 90	$\$90 / 1.05 = \$ 85.71$	$\$85.71 / \$1,203.03 = .0712$	$1 \times .0712 = .0712$
2	\$ 90	$\$90 / (1.05)^2 = \$ 81.63$	$\$81.63 / \$1,203.03 = .0679$	$2 \times .0679 = .1352$
3	\$ 90	$\$90 / (1.05)^3 = \$ 77.75$	$\$77.75 / \$1,203.03 = .0646$	$3 \times .0646 = .1938$
4	\$ 90	$\$90 / (1.05)^4 = \$ 74.04$	$74.04 / \$1,203.03 = .0615$	$4 \times .0615 = .2460$
5	\$ 90	$\$90 / (1.05)^5 = \$ 70.52$	$70.52 / \$1,203.03 = .0586$	$5 \times .0586 = .2930$
6	\$1,090	$\$1,090 / (1.05)^6 = \$ 813.37$	$\$813.37 / \$1,203.03 = .6761$	$6 \times .6761 = 4.0566$
		Sum = \$1,203.03	Sum = 1.0000	Duration = 4.9964

The duration of this investment is 4.9964 years, or approximately 5 years.

Duration can be thought of as the "balance point" on a lever and fulcrum diagram. In the diagram at the left, the "balance point" of the discounted cash flows would be approximately at the fifth year, which we have seen mathematically in the table above.

## DURATION OF SIX YEAR BOND



A change in the discount rate will affect the duration of the cash flows. If we increase the discount (yield) rate, the present value of all the cash flows decline, but the cash flows the farthest away would show the largest proportional decrease. Thus, the early cash flows have greater weight and the "balance point" moves to the left (shorter duration). The table below shows the same cash flows as the earlier table, but uses a 7% discount rate.

Year	Cash flow	PVCF	Proportion	Prop. x Year
1	\$90	$\$90 / 1.07 = \$84.11$	$\$84.11 / \$1,095.33 = .0768$	$1 \times .0768 = .0768$
2	\$90	$\$90 / (1.07)^2 = \$78.61$	$\$78.61 / \$1,095.33 = .0718$	$2 \times .0718 = .1436$
3	\$90	$\$90 / (1.07)^3 = \$73.47$	$\$73.47 / \$1,095.33 = .0671$	$3 \times .0671 = .2013$
4	\$90	$\$90 / (1.07)^4 = \$68.66$	$\$68.66 / \$1,095.33 = .0627$	$4 \times .0627 = .2508$
5	\$90	$\$90 / (1.07)^5 = \$64.17$	$\$64.17 / \$1,095.33 = .0586$	$5 \times .0586 = .2930$
6	\$1,090	$\$90 / (1.07)^6 = \$726.31$	$\$726.31 / \$1,095.33 = .6631$	$6 \times .6631 = 3.9786$
		Sum = \$1,095.33	Sum = 1.0	Duration = 4.9441

Conversely, as the discount (yield) rate declines, duration increases. Below is a table with the same cash flows, but a 3% discount rate.

Year	Cash flow	PVCF	Proportion	Prop. x Year
1	\$90	$\$90/1.03 = \$87.38$	$\$84.11 / \$1,325.02 = .0659$	$1 \times .0659 = .0659$
2	\$90	$\$90 / (1.03)^2 = \$84.83$	$\$78.61 / \$1,325.02 = .0640$	$2 \times .0640 = .1280$
3	\$90	$\$90 / (1.03)^3 = \$82.36$	$\$73.47 / \$1,325.02 = .0622$	$3 \times .0622 = .1866$
4	\$90	$\$90 / (1.03)^4 = \$79.96$	$\$68.66 / \$1,325.02 = .0603$	$4 \times .0603 = .2412$
5	\$90	$\$90 / (1.03)^5 = \$77.63$	$\$64.17 / \$1,325.02 = .0586$	$5 \times .0586 = .2930$
6	\$1,090	$1090/(1.03)^6 = \$912.86$	$\$912.86 / \$1,325.02 = .6889$	$6 \times .6889 = 4.1334$
		Sum = \$1,325.02	Sum = 1.0	Duration = 5.0481

## VARIANCE AND REAL ESTATE CASH FLOWS

The **variance** is the average of the squared differences between each observation in a data set and the mean of all observations in the data set. The **standard deviation** is the square root of the variance. Both of these measures are natural indicators of risk that are very useful in analyzing real estate data, although the standard deviation is used more frequently.

One of the characteristics of real estate cash flows is that they vary considerably during the holding period of the investment. This may not be true for a single tenant building with a long lease, but it would usually be true for a multi-tenant building that has short-term leases. The before-tax and after-tax cash flows can vary due to risk factors that affect the components of the cash flow statement. For example, overbuilding of similar types of property in the neighborhood can depress rents and lower the potential gross income. An unexpected increase in utility rates can balloon the operating expenses, thus shrinking the net operating income. The market often prices real estate assets at a multiple of the periodic cash flows, so a downturn in the cash from operations can depress the price of the asset. Since market timing can be crucial in the sale of real property, and real property is illiquid and requires a relatively long time on the market, variance in periodic cash flows creates risk. We can add variance (or more often, standard deviation) to our list of measures of risk for analyzing real property cash flows.

**EXAMPLE:** Two investments have the following periodic cash flows from operations. Calculate the mean and standard deviation of the cash flows from each investment.

<u>Investment #1</u>	<u>Investment #2</u>
\$1,472	\$1,198
\$1,566	\$1,575
\$1,385	\$1,044
\$1,590	\$1,612
\$1,488	\$1,237
\$1,355	\$2,190

The mean cash flow for each investment is \$1,476, but the standard deviation of investment #1 is 93.9979 while the standard deviation of the cash flows from investment #2 is 414.5171. Investment #2 is riskier.

## **REAL ESTATE INVESTMENT PROBLEMS**

### **PROBLEM SET #1**

1. Albert deposits \$4,500 in a savings account paying 8% per annum interest. How much will he have if he leaves this money on deposit for 22 years?
2. Jane has inherited a tract of land. She believes that she can sell the land in 8 years for a price of \$50,000. Assuming that Jane's required rate of return is 9% per year, how much is the property worth in today's dollars?
3. Malcolm wishes to save for a down payment on a house. He will need \$10,000 in 5 years. How much must he save each year if he makes a deposit at the end of every year and he receives 7% per annum on his money?
4. Sarah has an opportunity to buy a lake house from a friend for \$60,000. She will make a 25% down payment and her friend will finance the balance with a mortgage calling for annual payments over 20 years at 8.5% per annum interest. How much are Sarah's payments?
5. Tom will open up an IRA savings account at the end of this year. He will save \$2,000 per year for 40 years in an account paying 9% per annum interest. How much will Tom have in the account when he retires in 40 years?
6. Lucille is considering investing in a parking lot. The tenant is the government, which pays an annual rental of \$2,000. In 15 years the lot will become the property of the government and have no value. How much should Lucille pay for the lot if she requires an 11% per annum return on her investments?

## PROBLEM SET #2

1. Linda has the option of making a \$25,000.00 investment that will earn interest either at the rate of 10% compounded semiannually or 9.5% compounded quarterly. Which would you advise?
2. An investor can buy a tract of farmland today that he believes will sell for \$50,000.00 in eight years. His required rate of return for this type of investment is 12% per annum compounded monthly. How much should he pay for the land today? (assume that there are no taxes or other periodic costs involved)
3. Jack deposits \$1,000 at the end of each month in an account which will earn interest at an annual rate of 10% compounded monthly. How much will he have at the end of six years?
4. Best Properties is considering an investment that will pay \$1,000.00 at the end of each year for the next 15 years. It expects to earn an annual return of 16% on its investment. How much should the company pay today for its investment?
5. Megabucks Development Co. is evaluating an investment that will provide the following returns at the end of the following years: year 1, \$12,000.00; year 2, \$6,500.00; year 3, \$0; year 4, \$8,000.00 and year 5, \$4,500.00. Megabucks believes it should earn an annual rate of return of 14% on its investments. How much should Megabucks pay for this investment?
6. Larry is considering the purchase of an apartment project for \$30,000.00. Larry estimates that he will receive \$7,200.00 at the end of each year for the next four years and \$7,900.00 at the end of each year for the following three years. If he purchases the project, what will be his internal rate of return?
7. Jill is buying a tract of mountain land. The purchase price will be \$80,000.00. The seller has agreed to finance 75% of the purchase price in 25 annual payments at the end of each year at 9.5% interest. How much will Jill's mortgage payments be?

### PROBLEM SET #3

1. Ted and Jane are buying a house for 280,000.00. They will make a 25% down payment. They will be able to obtain a loan at 8.5% per annum interest. What will be their annual payment if their loan is for 15 years? 25 years? What will be the monthly payment if their loan is for 15 years? 30 years?

2. You have \$5,000 to invest and will place it (today) in an account earning 7% per annum interest. If you add \$3,000 to the account at the end of the fifth year, how much will you have on deposit at the end of the tenth year?

3. A real estate investment has the following annual cash flows:

1	\$45,000
2	\$38,000
3	\$52,000
4	\$320,000

If you require a 12% return on your investments, would you be willing to pay \$300,000 for this investment?

4. Suppose that you had paid \$300,000 for the investment in problem 3 above. What would be your internal rate of return?

5. Sam borrows \$1,000,000 by a mortgage with annual payments over 30 years at a rate of 9.75% per annum interest. What are his annual payments? What is the remaining balance on his loan after 5 years? 15 years?

6. Suppose that 10 years after Sam takes out the loan, the mortgage is sold to an investor who

requires a 10.5% rate of return on investments. How much is the investor willing to pay for the loan?

7. Jennifer is planning her retirement. She currently has \$6,783 in an IRA account that will return 8% per annum as long as she keeps the money on deposit. She will save \$2,000 at the end of each year for the next 32 years until she retires in an account that she expects will earn 7% per annum. How much will she have when she retires?

8. Juan has just borrowed \$85,000 at 8.25% per annum over 30 years to buy a house. What are his monthly payments? Suppose Juan wants to retire in 20 years and have his house paid off. How much will he have to add to his monthly payments to pay off the loan in 20 years?

9. Jill invested \$15,000 in a parking lot that she expects to have the following annual rents:

1	\$1,200
2	\$1,300
3	\$1,400
4	\$1,600
5	\$1,800

She expects to sell the lot at the end of the 5th year for \$22,000. What will be her IRR? If Jill requires a 14% rate of return on her investments, what is the NPV of this investment?

10. The ABC Corporation is offering a \$100,000 lottery to its employees. The winner will receive \$10,000 at the end of each year for the next ten years. How much must ABC place in an account today to meet this obligation if the account pays 7.5% per annum?

## PROBLEM SET #4

1. A mortgage has an original principal of \$2,275,000 amortized over 25 years in monthly payments at 9.5% per annum interest.

a) What is the monthly payment?

b) What is the mortgage balance at the end of ten years?

2. Lucretia has \$6,500 on deposit today in an IRA account that will earn 8% per annum annually. She will also deposit \$2,000 per year in an account paying 7.5% per year, starting today. If she retires in 26 years, how much will she have on deposit in her IRA accounts?

3. Pablo has an opportunity to make two investments, but he can only afford to make one of them. Each one costs \$25,000. The first investment can be sold in 15 years for \$98,500 and has no periodic cash flow (use annual compounding). The second investment has a \$200 per month cash flow for 6 years followed by a cash flow of \$400 per month for 8 years. The second investment has no resale value. Which investment is better, from the standpoint of highest IRR?

Investment 1 IRR =

Investment 2 IRR =

4. Tammy Faye requires a 12% before-tax return on her real estate investments. If she invests \$78,000 cash to buy a rental townhouse that has \$120 per month in taxes, insurance and maintenance, and if she can sell the property in 5 years for \$88,000, how much monthly rent must she charge to reach her target rate of return?

5. Tom is buying a small warehouse. The purchase price is \$180,000. The bank is willing to finance 80% of the purchase price for 20 years in monthly installments at 9.75% per annum. What is the amount of the mortgage payment?

6. Suppose that, in problem 5 above, \$1,250 was the maximum monthly payment that Tom could afford. What interest rate (to the nearest tenth of a percent) must Tom obtain to keep his payments at \$1,250 per month? Assume that he wishes to keep the loan amount and term the same.

7. Susan is purchasing an acreage tract of farmland that she is certain will go commercial in the near future. She will pay \$10,000 for the tract today and hopes to sell it for \$25,000 in eight years. What rate of return will she make on the investment?

8. Suppose that, in problem 7, at the end of the fourth year Susan can invest \$3,000 to wage a zoning change that has a 100% probability of success. The zoning change will make the property worth \$30,000 at the end of the eighth year. Assume that Susan requires a 12% rate of return on this investment. What is the NPV?

9. A real estate investment has the following annual cash flows:

<u>Year</u>	<u>Cash Flow</u>
0	(\$250,000)
1	15,000
2	10,000
3	18,000
4	26,500
5	8,750
6	346,200

a) What is the IRR?

b) At a discount rate of 10% per annum, what is the NPV?

10. Mary borrowed \$170,000 to buy a house. The loan had a 30-year term with monthly payments at 9.25% per annum interest. Now she wants to sell the house after 5 years and two months since the loan date. What is the outstanding principal balance on the loan?

11. Tom is buying a small warehouse. The purchase price is \$210,000. The bank is willing to finance 75% of the purchase price for 25 years in monthly installments at 8.75% per annum. What is the amount of the mortgage payment?

12. Suppose that in problem 11 above, Tom wishes to sell the property after exactly eight years. What will his loan balance be at that time?