Math 1426 **Fall 2007**

Final Exam - Version 1

Print your name legibly as it appears on the class rolls:

Last _____ First _____

ID Number: __ __ __ __ __ __ __ __ __ __

Check the appropriate section:

___	**018 – Mr. Smith, MWF 8am**	**___**	**021 – Dr. Shan**
___	**024 – Mr. Smith, MW 1pm**	**___**	**027 – Dr. Epperson**
___	**030 – Mr. Martines**	**___**	**032 – Dr. Krueger**

Write and bubble on your scantron

Name: **last name (space) first name** ⇐ **NOTE THE ORDER !!**
ID number: **begin in Column A and write all 10 digits**
Test No. **write 1 in Column K**
Section: **write your 3 digit section number in Columns L-N**

Turn cell phones off and put them out of sight. Turn off all beepers and alarms.

Do not write below this line

Part A total (48 points)	**Your score** 3× ____ =_____
17 (10 points)	
18 (11 points)	
19 (10 points)	
20 (10 points)	
21 (11 points)	
Part B total (52 points)	
Final Exam Total (100 points)	

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

The square brackets following an exam question number refer to a section/problem number in the text. Problems numbers preceded by the symbol ~ are modeled on that problem from the text, but not identical to it. Problems numbers without the symbol are identical to or very close to the problem from the text.

INSTRUCTIONS FOR PART A: Write your answers for these questions on the scantron provided and mark only one answer per question. **Your grade will be determined solely by what you mark on your scantron.** Each of the questions in this part counts 3 points, for a total possible score of 48 points. You may use an approved calculator. You may write on this exam or request scratch paper if needed.

1. [2.6/~5] Let $f(x)=\begin{cases} x^2-1 & \text{if } -1\le x<0 \\ 2x & \text{if } 0<x<1 \\ 1 & \text{if } x=1 \\ -2x+4 & \text{if } 1<x<3 \end{cases}$.

   Which of the following statements are true?

   I. f is continuous for all x II. f is differentiable at $x=2$

   III. $\lim_{x\to 0} f(x)$ exists IV. $\lim_{x\to 1} f(x)$ exists

   V. $\lim_{x\to -1} f(x)=-2$

   A. I and III only B. III and V only C. II and IV only
   D. I and IV only E. I, II and IV only

2. [3.9/~13] A ladder 13 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at the rate of 0.5 ft/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 5 feet from the wall?

   A. $-\frac{5}{24}$ ft/sec B. $\frac{5}{24}$ ft/sec C. -30 ft/sec D. 30 ft/sec E. 12 ft/sec

3. [3.3/7] At time t, the position of a body moving along the s-axis is given by $s=t^3-6t^2+9t$ where s is measured in meters. Find the total distance traveled by the body from $t=0$ to $t=2$.

   A. -6 m B. 2 m C. -2 m D. 6 m E. 4 m

4. [3.1] If $f(x)=3x^2+2$, then the difference quotient $\frac{f(3+\Delta x)-f(3)}{\Delta x}$ equals

   A. $3(6+\Delta x)$ B. $\frac{3(\Delta x)^2+2}{\Delta x}$ C. $3\Delta x$ D. $\frac{3(\Delta x)^2-2}{\Delta x}$

   E. $\Delta x+9$

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5. [3.9] Given that x and y are functions of t, find $\left.\frac{dy}{dt}\right|_{x=4}$ if $y = 6\sqrt{x} + x^2$ and $\left.\frac{dx}{dt}\right|_{x=4} = \frac{2}{19}$.

A. $\frac{19}{2}$ B. $\frac{13}{19}$ C. $\frac{22}{19}$ D. 28 E. 1

6. [3.10/~33] Let $y = \ln|\cos x|$. Find the differential dy.

A. $\pm\cot x\,dx$ B. $\cot x\,dx$ C. $-\cot x\,dx$ D. $\tan x\,dx$
E. $-\tan x\,dx$

7. [3.10/~11] Suppose that we know that a function g has derivative $g'(x) = \sqrt{x^2+16}$ for all x, and that $g(3) = -2$. Use a standard linear approximation (tangent line approximation) to estimate the value of $g(3.05)$.

A. -1.75 B. -2.01 C. -1.95 D. -1.9 E. 5.03

8. [4.5/Lab 9] Find the minimum possible value of the sum of a real number and its square.

A. $\frac{1}{2}$ B. $-\frac{1}{2}$ C. $\frac{1}{4}$ D. $-\frac{1}{4}$ E. -1

9. [4.6/~16,~65] Determine the value of c so that $f(x) = \begin{cases} \frac{\sin x - x}{x^3} & \text{if } x \neq 0 \\ c & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$.

A. 1 B. -1 C. $-\frac{1}{3}$ D. $-\frac{1}{6}$ E. $\frac{1}{6}$

10. [4.8/97] Solve the initial value problem:

$$\frac{dr}{d\theta} = -\pi\sin\pi\theta$$
$$r(0) = 0$$

A. $r = -\pi\cos\pi\theta + \pi$ B. $r = -\frac{\pi}{2}\sin^2\pi\theta$ C. $r = \cos\pi\theta - 1$
D. $r = \frac{\cos\pi\theta}{\pi} + C$ E. $r = -\pi\cos\pi\theta + C$

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11. [4.8/40] Find $\int x^{-3}(x+1)\,dx$.

A. $-\frac{1}{2x^2}\left(x^2+x\right)+C$ B. $-\left(\frac{1}{x}+\frac{1}{2x^2}\right)+C$ C. $-3x^{-2}$

D. $-3x^{-2}(x+1)+x^{-3}+C$ E. does not exist

12. [5.1/4] Estimate the area under the graph of $f(x)=4-x^2$ between $x=-2$ and $x=2$ using a lower sum of four rectangles of equal width.

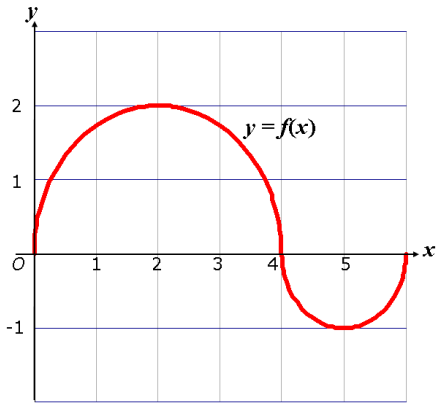
A. 14 B. $\frac{32}{3}$ C. 2 D. $\frac{16}{9}$ E. 6

13. [5.2,5.3] Give an expression in terms of a limit that gives the area between the graph of $y=x^3$, the x-axis and the lines $x=1$ and $x=6$.

A. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{5k}{n}\right)^3\frac{5}{n}$ B. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{6k}{n}\right)^3\frac{6}{n}$ C. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{5(k-1)}{n}\right)^3\frac{6}{n}$

D. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{5k}{n}\right)^3\frac{5}{n}$ E. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{6k}{n}\right)^3\frac{6}{n}$

14. [5.3] The graph of a function f is composed of two semi-circles as shown below. Find $\int_0^6 f(x)\,dx$.



A. 3π
B. 4π
C. $\frac{1}{2}\pi$
D. $\frac{3}{2}\pi$
E. $\frac{5}{2}\pi$

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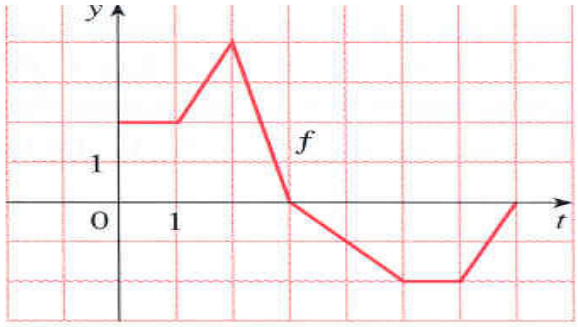
15. [5.4] Let $g(x) = \int_0^x f(t)\,dt$ where the graph of f is given below. Where does g have a maximum?



A. $x = 0$

B. $x = 2$

C. $x = 3$

D. $x = 7$

E. not enough information is given

16. [5.3/ ~9] If $\int_2^0 f(x)\,dx = 1$ and $\int_0^7 f(x)\,dx = 7$, then what is $\int_2^7 f(x)\,dx$?

A. -8 B. 8 C. -6 D. 6 E. 0

INSTRUCTIONS FOR PART B: For these questions, you must write down **all** steps in your solutions. Write legibly and carefully label any graphs or pictures. **Draw a box around your solution.** Partial credit will be given for those parts of your solution that are correct. The total value of the questions in this section is 52 points.

17. **10 pts** [5.5/17] Evaluate $\int \theta \sqrt[4]{1-\theta^2}\,d\theta$.

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18. **11 pts** [5.2, 5.3] Evaluate the definite integral $\int_1^3 x^2 dx$ as the limit of the Riemann sum using the right-hand endpoints.

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19. **10 pts** [4.6/~28] Evaluate $\lim_{t\to 0}\dfrac{\left(\frac{1}{2}\right)^t + t\ln 2 - 1}{t^2}$.

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20. **10 pts** [4.4/68] Sketch the graph of one function that satisfies all the following conditions. Be sure to label all important parts of your graph.

$f(-2)=8, \quad f(0)=4, \quad f(2)=0$

$f'(2)=f'(-2)=0, \quad f'(x)<0 \text{ for } |x|<2, \quad f'(x)>0 \text{ for } |x|>2$

$f''(x)<0 \text{ for } x<0, \quad f''(x)>0 \text{ for } x>0$

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21. **<u>11 pts</u>** [5.6/Example 4] Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

END OF EXAM

If additional sheets of paper are to be graded, ask the proctor about attaching them to the exam. Have you shown all work in Part II? Fill in your scantron form as instructed on the front page. Write name & indicate course section on the front page.