Math 1426 **Final Exam – Version A** **Spring 2008**

Print your name legibly as it appears on the class rolls:

Last ________________________________ First ________________________________

ID Number: __ __ __ __ __ __ __ __ __ __

Check the appropriate section:

☐ 001 – Dr. Shan

☐ 004 – Dr. Krueger

☐ 007 – Mr. Smith

☐ 010 – Mr. Martines

☐ 013 – Dr. Lin

Fill in your scantron exactly as below

NAME	Last, first (EXACTLY AS YOU WROTE ABOVE)		
SUBJECT	1426- YOUR SECTION #	TEST NO.	FA
DATE		PERIOD	

Turn cell phones off and put them out of sight. Turn off all beepers and alarms.

Do not write below this line

Part I total (48 points)	Your score $3\times$ ______ =______
17 (11 points)	
18 (10 points)	
19 (10 points)	
20 (10 points)	
21 (11 points)	
Part II total (52 points)	
Final Exam Total (100 points)	

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

The square brackets following an exam question number refer to a section/problem number in the text. Problem numbers preceded by the symbol ~ are modeled on that problem from the text, but not identical to it. Problem numbers without the symbol are identical to or very close to the problem from the text.

INSTRUCTIONS FOR PART I: Write your answers for these questions on a scantron (form 882-E or 882-ES) and mark only one answer per question. **Your grade will be determined solely by what you mark on your scantron.** Each of the questions in this part counts 3 points, for a total possible score of 48 points. You may use an approved calculator. You may write on this exam or request scratch paper if needed.

1. [2.7/18] The slope of the tangent to the graph of $f(x)=\sqrt{x^2+x}$ at the point $\left(4,2\sqrt{5}\right)$ is

   A. $\lim\limits_{h\to 0}\dfrac{\sqrt{(4+h)^2-h+4}-2\sqrt{5}}{h}$ B. $\lim\limits_{h\to 0}\dfrac{\sqrt{x^2+x}-\sqrt{h^2+h}}{h}$

   C. $\lim\limits_{h\to 0}\dfrac{\sqrt{(4+h)^2+h+4}-2\sqrt{5}}{h}$ D. $\lim\limits_{h\to 0}\dfrac{\sqrt{h^2+h}-2\sqrt{5}}{h}$

   E. $\lim\limits_{h\to 0}\dfrac{\sqrt{(4+h)^2+h+4}}{h}$

2. [3.5, 3.7/~39] Find $f'(x)$ if $f(x)=\ln\left(\sqrt{\dfrac{x^2+1}{x^2+3}}\right)$

   A. $\dfrac{2x}{(x^2+1)(x^2+3)}$ B. $\sqrt{\dfrac{x^2+3}{x^2+1}}$ C. $2x\sqrt{\dfrac{x^2+3}{x^2+1}}$

   D. $\dfrac{2x}{\sqrt{(x^2+1)(x^2+3)}}$ E. $2x\sqrt{\dfrac{x^2+1}{x^2+3}}$

3. [3.6/36] If r is a function of θ and $\cos r+\tan\theta=e^{2r\theta}$, find $\dfrac{dr}{d\theta}$.

   A. $\dfrac{\sec^2\theta-2re^{2r\theta}}{2\theta e^{2r\theta}}$ B. $\dfrac{\sec^2\theta}{\sin r+2\theta e^{2r\theta}}$ C. $\dfrac{\sec^2\theta-2re^{2r\theta}}{\sin r}$

   D. $\dfrac{\sec^2\theta-2re^{2r\theta}}{\sin r+2\theta e^{2r\theta}}$ E. $\dfrac{-2re^{2r\theta}}{\sin r+2\theta e^{2r\theta}}$

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4. [3.9/11] The length x of a rectangle is **decreasing** at the rate of 2 cm/sec while the width y is **increasing** at the rate of 2 cm/sec. When $x = 12$ cm and $y = 5$ cm, find the rate of change of the area of the rectangle.
   A. -14 cm^2/sec B. 14 cm^2/sec C. 34 cm^2/sec
   D. -34 cm^2/sec E. -4 cm^2/sec

5. [4.2] Let $g(x)$ be a differentiable function with domain $(0, +\infty)$. Let $f(x) = \begin{cases} \sqrt{x^3+1}, & x \geq 0 \\ -x+1, & x < 0 \end{cases}$. If $f'(x) = g'(x)$ on $(0, +\infty)$ and $g(2) = 5$, then
   A. $g(x) = \sqrt{x^3+1} + 2$ B. $g(x) = -x + 7$ C. $g(x) = (x^3+1) - 4$
   D. $g(x) = \sqrt{x^3+8} + 1$ E. $g(x) = \dfrac{3}{\sqrt{x^3+1}} + 4$

6. [4.3/Example 2] Find the intervals where $f(x) = \left(x^2 - 3\right)e^x$ is increasing.
   A. $\left(-\sqrt{3}, 0\right)$ and $\left(1, \sqrt{3}\right)$ B. $(-2, 2)$ and $(7, \infty)$
   C. $(-\infty, -3)$ and $(1, \infty)$ D. $(-3, 0)$ E. $(-\infty, 1)$

7. [4.4/Example 8] For the graph of $f(x) = e^{2/x}$, find the x-coordinate of the inflection point.
   A. there is no inflection point B. e^{-2} C. e^2 D. 1
   E. -1

8. [4.5 Lab] A rectangular box with square base and no top is to have volume of exactly 4000 cm^3. What is the minimum possible surface area of such a box?
   A. 20 cm^2 B. 20 cm^3 C. 1200 cm^2 D. 1200 cm^3
   E. 200 cm^2

9. [4.6/47] Find the limit: $\lim_{x \to 1^+} x^{1/(1-x)}$.
   A. 1 B. $\dfrac{1}{e}$ C. $\dfrac{1}{e^2}$ D. e E. does not exist

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10. [4.8/~33] Evaluate $\int 2t^{-1/3}(t-1)\,dt$.

A. $\frac{6}{5}t^{5/3}-3t^{2/3}+C$ B. $\frac{4(t-1)^2}{3t^{4/3}}+C$ C. $6t^{2/3}+C$

D. $\frac{6}{5}t^{5/3}+3t^{2/3}+C$ E. $\frac{2}{3}t^{-5/3}+\frac{4}{3}t^{-1/3}+C$

11. [4.8/~59] Evaluate $\int \frac{1+\sin 3x}{2}\,dx$.

A. $\frac{x+\cos 3x}{2}+C$ B. $\frac{3}{2}\cos 3x+C$ C. $\frac{1}{2}x+\frac{1}{3\sqrt{1-9x^2}}+C$

D. $\frac{x-\cos 3x}{2}+C$ E. $\frac{3x-\cos 3x}{6}+C$

12. [5.1/~1] Estimate the area of under the graph of $y=x+3$ on the interval $[0,3]$ with a lower sum of 3 rectangles of equal width.

A. 12 B. 15 C. $\frac{27}{2}$ D. $\frac{15}{2}$ E. 27

13. [5.3/~10] If $\int_0^9 f(x)\,dx=37$ and $\int_0^9 g(x)\,dx=16$, find $\int_0^9 [2f(x)+3g(x)]\,dx$.

A. –21 B. 21 C. 53 D. 122 E. 143

14. [5.4/40] Find $\frac{dy}{dx}$ given $y=\int_{2\sqrt{x}}^{2\sqrt{2}} \sin\left(t^4\right)dt$ for $x>2$.

A. $\frac{\sin\left(16x^2\right)}{\sqrt{x}}$ B. $\frac{-\sin\left(16x^2\right)}{\sqrt{x}}$ C. $-\sin\left(16x^4\right)$

D. $-\sin\left(16x^2\right)$ E. $\frac{-\sin\left(16x^4\right)}{\sqrt{x}}$

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15. [5.5/43] Evaluate $\int \frac{dx}{x \ln x}$.

A. $\frac{x}{\ln x} + C$ B. $\ln \frac{1}{x} + C$ C. $\ln(\ln x) + C$

D. $\frac{x}{\ln|x|} + C$ E. $\ln|\ln x| + C$

16. [5.6/4a] Evaluate $\int_0^{\pi} 3\cos^2 x \sin x \, dx$.

A. -2 B. $\frac{\pi}{2}$ C. $\frac{\sqrt{2}}{2}$ D. 3 E. 2

INSTRUCTIONS FOR PART II: For these questions, you must write down **all** steps in your solutions. Write legibly and carefully label any graphs or pictures. **Draw a box around your final answer.** Partial credit will be given for those parts of your solution that are correct. The total value of the questions in this section is 52 points.

17. **11 pts** [4.4/69] Sketch the graph of a **single** function $y = f(x)$ that satisfies **all** the following properties:

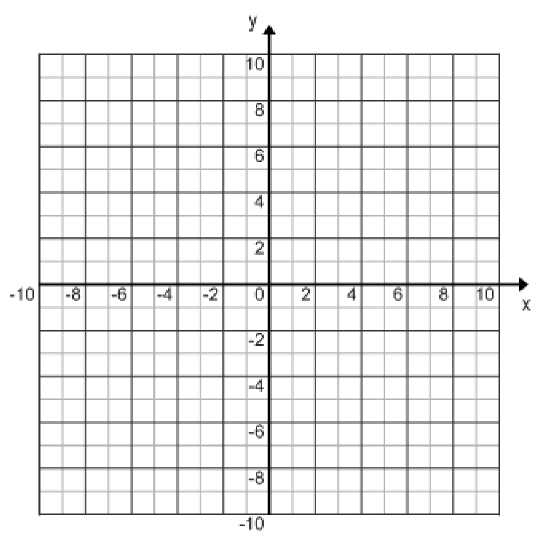
(a) $f''(x)$ exists for all x in $\mathbb{R}$

(b) $f(0) = 2,\ f(2) = 1,\ f(4) = 4,\ f(6) = 7$

(c) $f'(2) = f'(6) = 0$; $f'(x) < 0$ for $x < 2$ and $x > 6$; $f'(x) > 0$ for $2 < x < 6$

(d) $f''(4) = 0$; $f''(x) > 0$ for $x < 4$; $f''(x) < 0$ for $x > 4$

Clearly **label** the local extrema and points of inflection, if there are any.



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18. **10 pts**[4.5/4] A rectangle has its base on the x-axis and its upper 2 vertices on the parabola $y = 54 - 2x^2$. What is the largest area the rectangle can have?

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19. **<u>10 pts</u>** [5.3/Example 4b] Use the **<u>definition</u>** of the integral as a limit of Riemann sums to evaluate $\int_a^b x\,dx$ for $0 < a < b$.

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20. **<u>10 pts</u>**[5.5/~18] Evaluate $\int \frac{x\,dx}{\sqrt{1+2x^2}}$.

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21. **<u>11pts</u>** [5.6/63] Find the area of the region enclosed by $y = x^2 - 2$ and $y = 2$.

END OF EXAM

If additional sheets of paper are to be graded, ask proctor about attaching them to the exam. Have you shown all work in Part II? Fill in your scantron form as instructed on the front page. Write name & indicate course section on the front page.