MATH1426-001
Spring 2009

MIDTERM I - PRACTICE QUESTIONS

PART I.

I. 1. [2.2] Find the limit: $\lim_{x \to \pi/2} \frac{1-\sin^2 x}{\cos x}$

A. 1 B. ∞ C. -1 D. 0 E. $-\infty$

I. 2. [2.4] Find the limit: $\lim_{x \to 0} \frac{\cos x - 1}{\sin 2x}$

A. 1 B. $-\infty$ C. -1 D. 0 E. ∞

I. 3. [2.2] Find the limit: $\lim_{x \to 0} \frac{\sqrt{x+3}-\sqrt{3}}{x}$

A. $\frac{1}{\sqrt{3}}$ B. $\frac{\sqrt{3}}{6}$ C. $\frac{\sqrt{3}}{3}$ D. 0 E. does not exist

I. 4. [2.2] Evaluate $\lim_{x \to 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-2}$

A. 0 B. ∞ C. $-1/2$ D. $-1/4$ E. $-\infty$

I. 5. [2.2, 2.4] If $f(x) = \begin{cases} \sqrt{-x} & if\ x < 0 \\ 3-x & if\ 0 \le x < 3 \\ (x-3)^2 & if\ x \ge 3 \end{cases}$, then the $\lim_{x \to 0} f(x)$ is

A. 0 B. 3 C. $\frac{1}{2}$ D. does not exist E. none of these

I. 6. [2.2,2.4] If $f(x) = \begin{cases} \sqrt{-x} & if\ x < 0 \\ 3-x & if\ 0 \le x < 3 \\ (x-3)^2 & if\ x \ge 3 \end{cases}$, then the $\lim_{x \to 2^+} f(x)$ is

A. 1 B. 3 C. $\frac{1}{2}$ D. does not exist E. none of these

I. 7. [2.2] Evaluate the limit $\lim_{x \to 3} \frac{2x^4 - 3x^3 - 81}{x^5 - 10x^3 + 27}$

A. ∞ B. 1 C. 2 D. $\frac{2}{3}$ E. $\frac{1}{2}$

I. 8. [2.4]Evaluate the limit $\lim_{x\to\infty} \frac{\sqrt{2x^4 - 3\pi x^3 - 81}}{\sqrt{x^4 - 10x^3 + 27 - \pi}}$

A. 0 B. 1 C. $\sqrt{2}$ D. ∞ E. 1/2

I. 9. [2.4, 2.5] Find the horizontal asymptotes to the graph of $f(x) = \frac{ax}{x^2 - 1}$ with $a \neq 0$.

A. $y = 0$ B. $y = a$ C. $x = a$ D. $x = 0$ E. none of these

I. 10. [2.6] Find the numbers, if any, at which $f(x) = \begin{cases} \frac{x^2 - 25}{x - 5}, & \text{if } x \neq 5 \\ 10 & \text{if } x = 5 \end{cases}$ is discontinuous.

A. 5 B. 25 C. 5, 25 D. -5 E. None

I. 11. [2.6] Find constants *a* and *b* such that $f(x) = \begin{cases} 2a + b & \text{if } x < 3 \\ 8 & \text{if } x = 3 \\ x^2 + bx - 4 & \text{if } x > 3 \end{cases}$ is continuous for all $x \in \mathcal{R}$.

A. $a = 2, b = 2$ B. $a = 3.5, b = 1$ C. $a = 1.5, b = 1$ D. $a = 2.5, b = 1.5$ E. none of these

I. 12. [2.5] Which one of the following functions has a vertical asymptote at $x = 2$?

A. $y = \frac{x^2 + x - 6}{x - 2}$ B. $y = \frac{x^2 - 4x + 4}{x - 2}$ C. $y = x - 2$ D. $y = \frac{x^2 - x + 6}{x - 2}$ E. $y = \frac{|x - 2|}{x - 2}$

I. 13. [2.6] If $u(t) = \frac{|t - 7|}{t - 7}$, then $u(t)$ is discontinuous at

A. 7 B. -7 C. 0 D. nowhere E. none of these

I. 14. [2.6] If $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \leq x < 3 \\ (x - 3)^2 & \text{if } x \geq 3 \end{cases}$, then $f(x)$ is discontinuous at

A. 0 B. 0 and 3 C. 3 D. nowhere E. none of these

I. 15. [2.1,2.2] If $u(y) = \frac{|y - 9|}{y - 9}$, then $\lim_{y\to 9^-} u(y)$ is

A. 1 B. -1 C. $-\infty$ D. ∞ E. 0

I. 16. [2.6]Evaluate the limit $\lim_{x\to\pi/2} \sin(\cos(x))$

A. π B. $\sqrt{2}/2$ C. 0 D. 1 E. -1

I. 17. [2.7] Find the difference quotient for $f(x) = \frac{1}{x}$ and $x = 2$.

A. $\frac{-1}{2 + \Delta x}$ B. $\frac{\Delta x}{2 + \Delta x}$ C. $\frac{-1}{2(2 + \Delta x)}$ D. $\frac{2}{2 + \Delta x}$ E. $\frac{-1}{2}$

PART II.
For these questions, you should write down all steps in your solutions. Write legibly and carefully label any graphs or pictures. Draw a box around your final answer. Partial credit will be given for those parts of your solution that are correct.

II. 1. [2.2, 2.4] Find the limit (a) $\lim_{x\to 2^+}\sqrt{\dfrac{x^2-4x+4}{x-2}}$; (b) $\lim_{x\to 0^+}\dfrac{\sin^2(\sqrt{x})}{x}$

II. 2. [2.6]Determine a so that the following function is continuous. $f(x)=\begin{cases}2x-1, & \text{if } x<2\\ a+2 & \text{if } x=2\\ ax+1 & \text{if } x>2\end{cases}$

II. 3. [2.6, 3.1]Show that $f(x)=\begin{cases}x^2+1, & \text{if } x\le 1\\ 2x, & \text{if } x>1\end{cases}$ is continuous and differentiable at $x=1$

II. 4. [2.6, 3.1]Show that $f(x)=\begin{cases}x^2+2, & \text{if } x\le 1\\ x+2, & \text{if } x>1\end{cases}$ is continuous but not differentiable at $x=1$

II. 5. [2.7] Use the **definition** of the derivative to find $f'(2)$ given $f(x)=\sqrt{x+1}$

II. 6. [2.2, 2.7(3.1)] Suppose that a function f is differentiable at $x=1$ and $\lim_{h\to 0}\dfrac{f(1+h)}{h}=5$.
Find $f(1)$ and $f'(1)$

II. 7. [2.2,3.1] Suppose that a function f with the property that $f(x+y)=f(x)+f(y)+5xy$ and $\lim_{h\to 0}\dfrac{f(h)}{h}=3$. Find $f(0)$ and $f'(x)$

II. 8. [2.2] Evaluate $\lim_{x\to 5}\dfrac{\sqrt{x-1}-2}{x-5}$ without using a calculator or L'Hôpital's rule.

II. 9. [2.4, 2.5, 2.6] Sketch the graph of a function f that satisfies all of the following conditions.
1) f has domain (-∞, +∞);
2) f is continuous everywhere except at $x = 3$ where it is continuous from the left;
3) $\lim_{x\to 3^+} f(x)=-\infty$;
4) $\lim_{x\to -\infty} f(x)=2$ and $\lim_{x\to +\infty} f(x)=-2$

Solution keys to Part I:
1. D 2. D 3. B 4. D 5. D 6. A 7. B 8. C 9. A 10. E 11. B 12. D 13. A 14. A 15. B 16. C 17. C

Part II:

1. (a) "$\dfrac{0}{0}$" type, $\lim_{x\to 2^+}\sqrt{\dfrac{x^2-4x+4}{x-2}}=\lim_{x\to 2^+}\sqrt{\dfrac{(x-2)^2}{x-2}}=\lim_{x\to 2^+}\sqrt{x-2}=0$.

(b) "$\frac{0}{0}$" type, $\lim_{x\to0^+}\frac{\sin^2(\sqrt{x})}{x}=\lim_{x\to0^+}\frac{\sin^2(\sqrt{x})}{(\sqrt{x})^2}=\lim_{x\to0^+}\left[\frac{\sin(\sqrt{x})}{(\sqrt{x})}\frac{\sin(\sqrt{x})}{(\sqrt{x})}\right]=\lim_{x\to0^+}\frac{\sin(\sqrt{x})}{(\sqrt{x})}\lim_{x\to0^+}\frac{\sin(\sqrt{x})}{(\sqrt{x})}$

Let $t=\sqrt{x}$, thus $t\to0^+$ as $\sqrt{x}\to0^+$ $\Rightarrow \lim_{x\to0^+}\frac{\sin(\sqrt{x})}{(\sqrt{x})}=\lim_{t\to0^+}\frac{\sin t}{t}=1 \Rightarrow \lim_{x\to0^+}\frac{\sin^2(\sqrt{x})}{x}=1$

2. $a=1$.

3. Show continuity:
(1) $f(1)=2$; (2) $\lim_{x\to1^-}f(x)=\lim_{x\to1^-}(x^2+1)=2$ and $\lim_{x\to1^+}f(x)=\lim_{x\to1^-}(2x)=2$ thus $\lim_{x\to1}f(x)=2$; and (3) $\lim_{x\to1}f(x)=f(1)$.

Show differentiability by definition: $f'(1)=\lim_{\Delta x\to0}\frac{f(1+\Delta x)-f(1)}{\Delta x}$

$\lim_{\Delta x\to0^-}\frac{f(1+\Delta x)-f(1)}{\Delta x}=\lim_{\Delta x\to0^-}\frac{[(1+\Delta x)^2+1]-2}{\Delta x}=\lim_{\Delta x\to0^-}\frac{2\Delta x+(\Delta x)^2}{\Delta x}=\lim_{\Delta x\to0^-}(2+\Delta x)=2$ and

$\lim_{\Delta x\to0^+}\frac{f(1+\Delta x)-f(1)}{\Delta x}=\lim_{\Delta x\to0^+}\frac{[2(1+\Delta x)]-2}{\Delta x}=\lim_{\Delta x\to0^+}\frac{2\Delta x}{\Delta x}=2 \Rightarrow f'(1)=\lim_{\Delta x\to0}\frac{f(1+\Delta x)-f(1)}{\Delta x}=2$

4. Show continuity:
(1) $f(1)=3$; (2) $\lim_{x\to1^-}f(x)=\lim_{x\to1^-}(x^2+2)=3$ and $\lim_{x\to1^+}f(x)=\lim_{x\to1^-}(x+2)=3$ thus $\lim_{x\to1}f(x)=3$; and (3) $\lim_{x\to1}f(x)=f(1)$.

By definition: $f'(1)=\lim_{\Delta x\to0}\frac{f(1+\Delta x)-f(1)}{\Delta x}$

$\lim_{\Delta x\to0^-}\frac{f(1+\Delta x)-f(1)}{\Delta x}=\lim_{\Delta x\to0^-}\frac{[(1+\Delta x)^2+2]-3}{\Delta x}=\lim_{\Delta x\to0^-}\frac{2\Delta x+(\Delta x)^2}{\Delta x}=\lim_{\Delta x\to0^-}(2+\Delta x)=2$ and

$\lim_{\Delta x\to0^+}\frac{f(1+\Delta x)-f(1)}{\Delta x}=\lim_{\Delta x\to0^+}\frac{[(1+\Delta x)+2]-3}{\Delta x}=\lim_{\Delta x\to0^+}\frac{\Delta x}{\Delta x}=1$

Hence $\lim_{\Delta x\to0^-}\frac{f(1+\Delta x)-f(1)}{\Delta x}\neq\lim_{\Delta x\to0^+}\frac{f(1+\Delta x)-f(1)}{\Delta x}\Rightarrow\lim_{\Delta x\to0}\frac{f(1+\Delta x)-f(1)}{\Delta x}$ D.N.E., $\Rightarrow f'(1)$ D.N.E. $\Rightarrow f$ is not differentiable at 1.

5. $f'(2)=\lim_{h\to0}\frac{f(2+h)-f(x)}{h}=\lim_{h\to0}\frac{\sqrt{(2+h)+1}-\sqrt{3}}{h}=\lim_{h\to0}\frac{\sqrt{h+3}-\sqrt{3}}{h}$. "$\frac{0}{0}$" type

$\lim_{h\to0}\frac{\sqrt{h+3}-\sqrt{3}}{h}\frac{\sqrt{h+3}+\sqrt{3}}{\sqrt{h+3}+\sqrt{3}}=\lim_{h\to0}\frac{(\sqrt{h+3})^2-(\sqrt{3})^2}{h(\sqrt{h+3}+\sqrt{3})}=\lim_{h\to0}\frac{\not h}{\not h(\sqrt{h+3}+\sqrt{3})}=\lim_{h\to0}\frac{1}{\sqrt{h+3}+\sqrt{3}}=\frac{1}{2\sqrt{3}}=\frac{\sqrt{3}}{6}$

6. $\lim_{h\to0}\frac{f(1+h)}{h}=5 \Rightarrow \lim_{h\to0}\frac{f(1+h)}{h}$ must be "$\frac{0}{0}$" type $\Rightarrow f(1)=0$.

$\Rightarrow f'(1)=\lim_{h\to0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to0}\frac{f(1+h)}{h}=5$

7. $\lim_{h\to0}\frac{f(h)}{h}=3 \Rightarrow \lim_{h\to0}\frac{f(h)}{h}=3$ must be "$\frac{0}{0}$" type $\Rightarrow f(0)=0$.

$$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x)+f(\Delta x)+5x\Delta x-f(x)}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(\Delta x)+5x\Delta x}{\Delta x}=\lim_{\Delta x\to 0}\left(\frac{f(\Delta x)}{\Delta x}+5x\right)$$

$$=\lim_{\Delta x\to 0}\left(\frac{f(\Delta x)}{\Delta x}\right)+5x=3+5x$$

8. $\lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5}$ is of "$\frac{0}{0}$" type.

$$\lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5}\frac{\sqrt{x-1}+2}{\sqrt{x-1}+2}=\lim_{x\to 5}\frac{(\sqrt{x-1})^2-2^2}{(x-5)\sqrt{x-1}+2}\lim_{x\to 5}\frac{\cancel{(x-5)}1}{\cancel{(x-5)}\sqrt{x-1}+2}=\frac{1}{4}.$$

You should also look over your <u>lecture notes</u>, <u>homework assignment</u>, <u>problem-solving labs</u>, and <u>quizzes</u>. The Midterm 1 of Fall 2003, Spring 2004, and Fall 2004 can be found at
http://www.uta.edu/math/pages/main/oldexams/calc1/calc1.htm

Midterm 1: Friday, Feb. 13, 6:00 – 8:00 pm , Room TBD

MAA Review: TBA