MATH1426
Spring 2009

MIDTERM II PRACTICE QUESTIONS

PART I.

1.[3.1, 3.2] Find $f'(0)$ for $f(x)=\begin{cases} x+1 & \text{if } x<0 \\ (x-1)^2 & \text{if } x\geq 0 \end{cases}$

A. 0 B. 1 C. -1 D. -2 E. does not exist

2.[2.4] Find the limit: $\lim_{x\to 0}\dfrac{\cos x-1}{\sin 2x}$

A. 1 B. $-\infty$ C. -1 D. 0 E. does not exist

3.[2.2] Find the limit: $\lim_{x\to 0}\dfrac{\sqrt{x+3}-\sqrt{3}}{x}$

A. $\dfrac{1}{\sqrt{3}}$ B. $\dfrac{\sqrt{3}}{6}$ C. $\dfrac{\sqrt{3}}{3}$ D. 0 E. does not exist

4.[2.5] Find the limit: $\lim_{h\to 0^-}\dfrac{h^2+1}{h}$.

A. $-\infty$ B. 0 C. 1 D. $+\infty$ E. -1

5. [3.4,3.5] Suppose *f* is a function with the property that $f'(x)=\cos(x^2)$. Find $g'(x)$, where $g(x)=f(x^2)$.

A. $g'(x)=2x\cos(x^4)$ B. $g'(x)=\cos(x^4)$ C. $g'(x)=\sin(x^4)$
D. undefined. E. none of these.

6. [3.1,3.4] Interpret $\lim_{h\to 0}\left(\dfrac{\cos h-1}{h}\right)$ as a derivative.

A. $\dfrac{d}{dx}(\cos 0)$ B. $\cos'(x)$ C. $\cos'(h)$ D. $\cos'(0)$ E. $\cos'(1)$

7. [3.3] On Earth, you can easily shoot a paper clip straight up into the air with a rubber band. In t sec after firing, the paper clip is $s=64t-16t^2$ ft above your hand. What is the maximum height of the paper clip?

A. 128 ft B. 32 ft C. 96 ft D. 64 ft E. not enough information.

8. [3.6] The equation of the tangent line to the curve of function $y^2=x^3(2-x)$ at (1, 1) is

A. $x+y=0$ B. $y=-x+1$ C. $x-y+2=0$ D. $y=x$
E. $x+y+1=0$

9. [3.2, 3.3]Find the derivative of $f(x)=\dfrac{1-2x}{e^x}$

A. $\dfrac{xe^x - 2e^x}{e^x}$ B. $\dfrac{2xe^x - 3e^x}{e^{2x}}$ C. $\dfrac{xe^x - 2e^x}{e^{2x}}$ D. $\dfrac{2xe^x - 3e^x}{e^x}$

E. None of these.

10.[3.5] Find the derivative $\dfrac{d}{dx}\left[\left(4x^3 + 2\sqrt{x}\right)^2\right]$

A. $2(4x^3 + 2\sqrt{x})(12x^2 + \dfrac{2}{\sqrt{x}})$ B. $(4x^3 + 2\sqrt{x})^2(12x^2 + \dfrac{1}{\sqrt{x}})$

C. $(4x^3 + 2\sqrt{x})(12x^2 + \dfrac{1}{\sqrt{x}})$ D. $2(4x^3 + 2\sqrt{x})(12x^2 + \dfrac{1}{\sqrt{x}})$

E. None of these.

11. [3.2, 3.5] Find the derivative of $f(x) = (x-1)^3(\sqrt[3]{x^{10}} + 1)$ at $x = 0$.

A. 3 B. 1 C. $-\dfrac{1}{3}$ D. -1 E. none of these.

12. [3.4, 3.5] Find d^2y/dx^2 for a parameterized curve $x = (1/2)\tan t,\ y = (1/2)\sec t$ at $t = \pi/3$.

A. $\dfrac{1}{4}$ B. 0 C. $\dfrac{\sqrt{3}}{2}$ D. $\dfrac{\pi}{2}$ E. none of these.

13. [3.8] Find the derivative of $y = (3x-1)^2 \sin^{-1}(x^2)$

A. $6(3x-1)\sin^{-1}(x^2) + 2x(3x-1)^2/(1+x^2)$ B. $(3x-1)^2\cos^{-1}(x^2) + 6(3x-1)\sin^{-1}(x^2)$

C. $6(3x-1)\sin^{-1}(x^2) + 2x(3x-1)^2/(1+x^4)$ D. $2(3x-1)[3\sin^{-1}(x^2) + x(3x-1)/\sqrt{1-x^2}]$

E. $2(3x-1)[3\sin^{-1}(x^2) + x(3x-1)/\sqrt{1-x^4}]$

14. [3.1]The slope of the tangent line to the graph of $y = x + \dfrac{4}{x}$ at $x = 2$ is

A. $1 - 4x^{-2}$ B. 0 C. 1 D. does not exist E. none of these.

15. [3.7] Given that $x^y = y^x$, compute $\dfrac{dy}{dx}$ at the point (2,2).

A. ln2 B. 1 C. 2 D. $\dfrac{1}{2}$ E. none of these.

16. [3.2, 3.5, 3.7] Find the derivative of $y = \dfrac{(1+2x)^{3/2}}{(1+3x)^{4/3}}$

A. $\dfrac{(1+2x)^{3/2}}{(1+3x)^{4/3}}\left(\dfrac{3}{2(1+2x)} - \dfrac{4}{3(1+3x)}\right)$ B. $\dfrac{(1+2x)^{3/2}}{(1+3x)^{4/3}}\left(\dfrac{3}{2(1+2x)} + \dfrac{4}{3(1+3x)}\right)$

C. $\dfrac{(1+2x)^{3/2}}{(1+3x)^{4/3}}\left(\dfrac{3}{1+2x} - \dfrac{4}{1+3x}\right)$ D. $\dfrac{(1+2x)^{3/2}}{(1+3x)^{4/3}}\left(\dfrac{3}{1+2x} + \dfrac{4}{1+3x}\right)$

E. None of these.

17. [3.9] Find $\frac{dx}{dt}$ where $x^2 + xy + 2y^2 = 2$ and $\frac{dy}{dt} = 2$ when $x = 1$ and $y > 0$

A. 3 B. $\frac{19}{6}$ C. $-\frac{12}{5}$ D. 2 E. $\frac{1}{2}$

18. [3.9] Find $\frac{dy}{dt}$ where $y = 6\sqrt{x} + x^2$ and $\frac{dx}{dt} = \frac{2}{19}$ when $x = 4$

A. 1 B. $\frac{19}{2}$ C. $\frac{13}{19}$ D. 28 E. $\frac{22}{19}$

19. [3.3] A stone is falling vertically from the top of a cliff with an zero initial velocity. If the stone hits the ground with a speed of 49 m/s, how high is the cliff? You may assume acceleration due to gravity is 9.8 m/s^2 downwards, that the stone lands at the base of the cliff and that air resistance is negligible.

A. 98 m B. 122.5 m C. 147 m D. 155.5 m E. 196 m

20 [3.6] If $x \cos y + y \cos x = 1$, then the value of dy/dx at (0, 1) is.

A. 0 B. $-\cos 1$ C. $\sqrt{2}/2$ D. -1 E. does not exist

21 [3.2, 3.5] On interval [1, 3], there is a horizontal tangent line to the graph of function $f(x) = \frac{\ln\sqrt{x}}{x}$ at

A. $x = e$ B. $x = 1$ C. $x = 2$ D. $x = 3$ E. $x = e - 1$

22. [3.8] Suppose that we know that a function g has derivative $g'(x) = \sqrt{x^2 + 16}$ for all x, and that $g(3) = -2$. Use a differential approximation (tangent line approximation) to estimate the value of g (3.05).

A. –2.01 B. -1.75 C. -1.95 D. -1.9 E. none of these.

23. [3.8] Find y' where $y = \sec(\tan^{-1} 2x)$.

A. $\frac{4x}{\sqrt{4x^2+1}}$ B. $\frac{8x+1}{\sqrt{4x^2+1}}$ C. $\frac{1}{2x}$ D. $\sqrt{4x^2+1}$ E. $2x$.

24. [3.2, 3.5] Let $f(x) = 4x^3 + 3x^2 - 2x + 1$, find the equation of the normal to the graph of f at $x = -1$

A. $4x - y - 2 = 0$ B. $x + 4y + 7 = 0$ C. $8x + y - 6 = 0$
D. $y = 2x - 1$ E. $x + 4y - 7 = 0$

25. [3.7] Giving $f'(x) = 2(x-1)$ and $f^{-1}(4) = 3$, find $\left(f^{-1}\right)'(4)$.

A. 2 B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. 6 E. $\frac{1}{2}$

26. [3.1] If function f is differentiable at x = c, then which of the following statement is NOT true?

A. $\lim_{x \to c} f(x) \neq f(c)$

B. $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = f'(c)$

C. f is continuous at $x = c$.

D. $\lim_{x \to c} f(x) = f(c)$

E. None of the above.

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27. [3.1, 3.2] Let $f(x) = |x-2|$, find $f'(2)$

A. 0 B. 1 C. –1 D. 2 E. DNE

28. [3.2] If $s(t) = 1 - 2t - t^3$, then the acceleration at $t = 2$ is

A. –12 B. –2 C. –14 D. 2 E. 6

PART II.

1. [2.3] Without using a calculator, find the correct value of k that makes the function $f(x)$ continuous on [0, 11], if f is defined as follows:

$$f(x) = \begin{cases} k \cdot \sin \frac{(x+3)\pi}{6}, & x \le 2 \\ \frac{3 - \sqrt{11-x}}{x-2}, & x > 2 \end{cases}$$

2. [3.2, 3.5, 3.7]Two functions, f and g, are continuous and differentiable for all real numbers. Some values of the functions and their derivatives are shown in this table:

x	0	1	2	3	4
$f(x)$	1/2	1/3	1	4	3
$g(x)$	-2	1	3	2	-1/3
$f'(x)$	3/2	5/3	1/4	0	-4/5
$g'(x)$	-1	2/3	4	-3	-1/3
$f''(x)$	3	2	-1/3	2	1
$g''(x)$	1	0	3	0	1

Based on that luscious table, find the following derivatives:

(a) $\frac{d}{dx} f(2x)$, evaluated at $x = 1$

(b) $\frac{d}{dx}\left(f(x)g(x)\right)$, evaluated at $x = 1$

(e) Let $h(x) = f(g'(x))$, find $h'(2)$

(f) Let $R(x) = f'(g(2x))$, find $R'(1)$

(c) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$, evaluated at $x = 0$

(d) Let $H(x) = f[g(x)]$, find $H'(1)$

(g) Let $r(x) = \ln[f(2x)]$, find $r'(2)$

(h) Let $t(x) = f(x)^{g(x)}$, find $t'(3)$

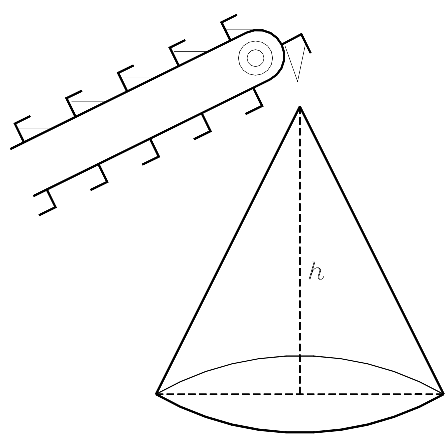
3. [3.9] Two people start from the same point. One walks east at 3 mile/hour and the other walks northeast at 2 mile/hour. How fast is the distance between the people changing after 15 minutes?

4. [3.4]Find what values of constants *a* and *b* does $y = ae^x + bx\sin x$ satisfy $y'' + y = \cos x$?

5. [3.6, 3.7] Find y' from the following implicit functions.
(a) $x\sin(2y) - y\cos x = 2x$; (b) $x^2y^2 = x + y$; (c) $(x-1)y = x^{\ln\sqrt{x}}$

6. [3.3] The position of a particle is given by the equation $s(t) = 10 + 64t - 16t^2$, where *t* is measured in seconds and *s* in ft. (a) Find the velocity *v*(*t*) and acceleration *a*(*t*) at time *t*. (b) What is the velocity and acceleration after 3 seconds? (c) How far does the particle travels after 3 seconds? (d) What is the total distance traveled after 3 seconds?

7. [3.9] Gravel is being dumped from a conveyor belt at a rate of 40 ft^3/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. (a) How fast is the height of the pile increasing when the pile is 10 ft high? (b) At what rate is gravel being dumped if the height of the pile is increasing at a rate of $\frac{2}{\pi}$ ft/min when it is 10 ft high? (The volume V of a cone is $V = \frac{1}{3}\pi r^2 h$)



8. [3.9] An oil tanker slams into the Alaskan coastline. The oil spreads in a circle whose area increases at a constant rate of 6 $miles^2$/hour. How fast is the radius of the spill increasing when the area is 9 $miles^2$?

You should also look over your <u>lecture notes</u>, <u>homework assignment</u>, <u>problem-solving labs</u>, and <u>quizzes</u>. The old Midterm 2 exams can be found at <u>http://www.uta.edu/faculty/hshan/teaching/teaching.shtml</u>

Midterm 2: Friday, Mar. 27, 6:00 – 8:00 pm, Room LS 119

Answers to PART I:

1.E 2. D 3.B 4..A 5.A 6.D 7..D 8.D 9.B 10.D 11.A 12.A 13.E 14.B 15.B 16.C 17.C 18.A 19.B 20.D 21.A 22.B 23.A 24.E 25.C 26.A 27.E 28.A

Answers to PART II:

1. $k = 1/3$; 2. (a) 1/2; (b) 17/9; (c) -5/8; (d) 10/9; (e) -12/5; (f) 16; (g) -8/15; (h) -96ln2.

3. 3.9 mph; 4. $a = 0$, $b = 1/2$;

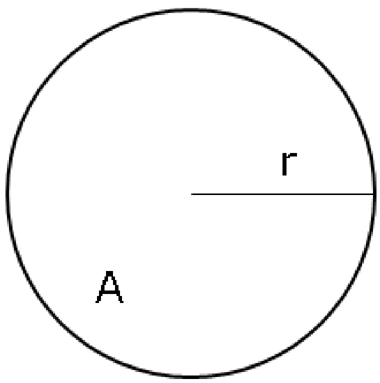
5.(a) $\dfrac{\sin 2y + y\sin x - 2}{\cos x - 2x\cos 2y}$ (b) $-\dfrac{1-2xy^2}{1-2x^2y}$ (c) $\left(\dfrac{\ln x}{x} - \dfrac{1}{x-1}\right)\dfrac{x^{\ln\sqrt{x}}}{x-1}$

6.(a) $v(t) = 64 - 32t$, $a(t) = -32$ (b) $v(3) = -32$ ft/s, $a(3) = -32$ ft/s^2 (c) = 58 ft (d) 80 ft

7. (a) $\dfrac{8}{5\pi}$ (b) 50 ft^3/min

8. Let A = the area of the oil spread, r = the radius.

Giving $\dfrac{dA}{dt}$ = 6 miles^2/hour, find out $\dfrac{dr}{dt}$ when A = 9 miles^2



$$A = \pi r^2$$

$$\frac{d}{dt}A = \frac{d}{dt}(\pi r^2)$$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

When A = 9 miles^2, $r = \sqrt{A/\pi} = \sqrt{9/\pi} = 3\big/\sqrt{\pi}$ mile.

Therefore,

$$\frac{dr}{dt} = \frac{1}{2\pi r}\frac{dA}{dt} = \frac{1}{6\sqrt{\pi}}6 = \frac{1}{\sqrt{\pi}} \approx 0.564 \text{ (mile/hour)}$$