MATH1426
Spring 2009

FINAL EXAM PRACTICE QUESTIONS

PART I.

I. 1. [2.6]The intermediate value theorem applies to functions which are

A. continuous at some point *c* only. B. continuous at two points *a* and *b* only.
C. defined on an interval $[a, b]$. D. continuous on an interval $[a, b]$.
E. none of these

I. 2. [2.5, 2.6, 3.1] If $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$, $\lim_{x\to 1^-} f'(x) = +\infty$ and $\lim_{x\to 1^+} f'(x) = +\infty$, which of the following statements are correct?
1) *f* has a vertical tangent line at $x = 1$;
2) *f* has a vertical asymptote at $x = 1$;
3) *f* is continuous at $x = 1$;
4) *f* is differentiable at $x = 1$; 5) *f* has a cusp at $x = 1$.

A. 1) and 3) B. 1) only C. 3) and 5) D. 2) and 3) E. 1) and 4)

I. 3. [2.6] Find the numbers, if any, at which $f(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5} & if\ x \neq 5 \\ 10 & if\ x = 5 \end{cases}$ is discontinuous.

A. 5 B. 25 C. $5, 25$ D. -5 E. None

I. 4. [2.6] Find constants *a* and *b* such that $f(x) = \begin{cases} 2a + b & if\ x < 3 \\ 8 & if\ x = 3 \\ x^2 + bx - 4 & if\ x > 3 \end{cases}$ is continuous for all x.

A. $a = 2, b = 2$ B. $a = 3.5, b = 1$ C. $a = 1.5, b = 1$ D. $a = 2.5, b = 1.5$ E. none of these

I. 5. The solutions to equation $\dfrac{x^2 + x - 6}{x - 2} = 0$ are:

A. 2 B. $2, -3$ C. $-2, 3$ D. -3 E. -2

I. 6. [2.4] If $f(x) = \begin{cases} \sqrt{-x} & if\ x < 0 \\ 3 - x & if\ 0 \le x < 3 \\ (x-3)^2 & if\ x \ge 3 \end{cases}$, then $\lim_{x\to 0} f(x)$ is

A. 0 B. 3 C. $\dfrac{1}{2}$ D. does not exist E. none of these

I. 7. [2.4] If $f(x) = \begin{cases} \sqrt{-x} & if\ x < 0 \\ 3 - x & if\ 0 \le x < 3 \\ (x-3)^2 & if\ x \ge 3 \end{cases}$, then $\lim_{x\to 3} f(x)$ is

A. 0 B. 3 C. $\dfrac{1}{2}$ D. does not exist E. none of these

I. 8. [2.4] If $u(y)=\frac{|y-9|}{y-9}$, then $\lim_{y\to 9^-} u(y)$ is

A. 1 B. -1 C. $-\infty$ D. ∞ E. 0

I. 9. [2.5, 2.6, 3.1, 3.7] Which of the following statements are correct for function $f(x)=\ln|2x|$.

1) $f'(x)=1/|x|$; 2) $f'(x)=1/x$; 3) f has a cusp at $x = 0$;
4) f has a vertical tangent line at $x = 0$; 5) f has a vertical asymptote at $x = 0$;
6) The domain of f is $(-\infty,+\infty)$.

A. 1) and 3) B. 5) only C. 2) and 3) D. 2) and 5) E. 5) and 6)

I. 10. [4.6] Evaluate the limit $\lim_{x\to-\infty} xe^{-x}$

A. 1 B. -1 C. $-\infty$ D. ∞ E. 0

I. 11. [4.6] Evaluate the limit $\lim_{x\to+\infty} xe^{-x}$

A. 1 B. -1 C. $-\infty$ D. ∞ E. 0

I. 12. [4.6] Evaluate the limit $\lim_{x\to 3}\frac{2x^4-3x^3-81}{x^5-10x^3+27}$

A. ∞ B. 1 C. 2 D. $\frac{2}{3}$ E. $\frac{1}{2}$

I. 13. [4.6]Evaluate the limit $\lim_{x\to\infty}\frac{\sqrt{2x^4-3\pi x^3-81}}{\sqrt{x^4-10x^3+27-\pi}}$

A. 0 B. 1 C. $\sqrt{2}$ D. ∞ E. none of these

I. 14. [4.6]Find $\lim_{x\to+\infty}(\sqrt{x^2+1}-x)$

A. e B. $\frac{1}{e}$ C. 1 D. ∞ E. 0

I. 15. [4.6]Evaluate $\lim_{x\to 0}\frac{\tan x}{x+\sin x}$

A. 0 B. 1 C. $\frac{1}{2}$ D. does not exist E. none of these

I. 16. [4.6] Evaluate $\lim_{x\to 0}\frac{\sin x\sin(2x)\sin(4x)}{x^3}$, if it exists.

A. 1 B. 2 C. 4 D. 8 E. does not exist

I. 17. [4.6]Which of the following forms can l'Hopital's rule be applied directly to?

1) $\frac{0}{0}$ 2) $\frac{\infty}{\infty}$ 3) $\frac{-\infty}{\infty}$ 4) $\frac{-\infty}{-\infty}$ 5) $\frac{\infty}{-\infty}$ 6) $0\cdot\infty$ 7) $\frac{0}{\infty}$ 8) $\frac{\infty}{0}$

A. 1,2,3 B. all except 6 C. 1 to 5 D. 1 to 8

I. 18. [2.7, 3.1, 3.2] The slope of the tangent line to the graph of $y=x+\frac{4}{x}$ at point (2, 4) is

A. $1-\frac{4}{x^2}$ B. 0 C. $\frac{1}{2}$ D. 1 E. none of these

I. 19. [2.7, 3.1, 3.2] Find the y-intercept of the line tangent to the graph of $f(x)=\dfrac{x^3+2\sqrt{x}-1}{x}$ at point (1, 2).

A. 0 B. 1 C. $\frac{1}{2}$ D. $\frac{1}{4}$ E. $-\frac{1}{4}$

I. 20. [2.4] Find the horizontal asymptotes to the graph of $f(x)=\dfrac{ax}{x^2-1}$ with $a \neq 0$.

A. $y=0$ B. $y=a$ C. $x=a$ D. $x=0$ E. none of these

I. 21. [4.1,4.3] If $f'(x)=x^2(x-2)(5x-6)$, then *f* has a relative maximum at

A. 0 B. 6/5 C. 2 D. does not exist E. none of these

I. 22. [4.1] All the critical numbers of $g(t)=5t^{2/3}+t^{5/3}$ are

A. 0 and 1 B. -2 and 1 C. -2 and 0 D. -2 E. 1

I. 23. [3.10] Find the differential: $d(2x\cos 3x)$

A. $(-6x\sin 3x+2\cos 3x)\,dx$ B. $(6x\sin 3x+2\cos 3x)dx$ C. $(-2x\sin 3x+2\cos 3x)\,dx$ D. $(2x\sin 3x+2\cos 3x)\,dx$ E. $(-6x\cos 3x+2\sin 3x)dx$

I. 24. [3.6] If $x\cos y+y\cos x=1$, then the value of $\dfrac{dy}{dx}$ at (0, 1) is

A. -1 B. $-\cos 1$ C. $\frac{\sqrt{2}}{2}$ D. does not exist E. not enough information given

I. 25. [3.6] If $f(x)=x^{\sin x}$ for $x>0$, then $f'(\frac{\pi}{2})$ is

A. 0 B. 1 C. 2 D. does not exist E. not enough information given

I. 26. [3.6] Given $y=x^{\ln\sqrt{x}}$, find y'

A. $x^{\ln\sqrt{x}-1}\ln x$ B. $x^{\ln\sqrt{x}+1}\ln x$ C. $x^{\ln\sqrt{x}}\frac{1}{2x}\ln x$ D. $x^{\ln\sqrt{x}}(\frac{1}{\sqrt{x}}+\frac{1}{2x})\ln x$

E. $\dfrac{\ln x}{x}$

I. 27. [4.1/4.5] A piece of wire 30 cm long is cut into two pieces. One piece is bent into a square and the other piece is shaped into a circle. To maximize the total area enclosed, the length of wire for the circle should be

A. 0 cm B. $\frac{30\pi}{8+\pi}$ cm C. 30cm D. does not exist E. not enough information given

I. 28. [5.2] Evaluate $\sum_{k=5}^{20}(3k-k^2+2k^3)$ by using the summation formulas

A. 85760 B. 34750 C. 5230 D. 32430 E. none of these

I. 29. [5.2] Evaluate $\lim_{n\to\infty}\sum_{k=1}^{n}\dfrac{k}{n^2+n}$

A. $+\infty$ B. $-\infty$ C. $\frac{1}{2}$ D. 0 E. none of these

I. 30. [5.2] Give an expression in terms of a limit that gives the area between the graph of $y = x^3$, the x-axis and the lines $x = 2$ and $x = 7$

A. $\lim_{n\to\infty}\sum_{k=1}^{n}(2+\frac{5k}{n})^3\frac{5}{n}$ B. $\lim_{n\to\infty}\sum_{k=1}^{n}(2+\frac{6k}{n})^3\frac{6}{n}$ C. $\lim_{n\to\infty}\sum_{k=1}^{n}(2+\frac{7k}{n})^3\frac{7}{n}$

D. no such limit exists   E. none of these

I. 31. [5.4] Evaluate $\int_0^{2\pi} f(x)\,dx$ where $f(x)=\begin{cases}\cos x & \text{if } 0\le x\le \pi/4\\ \sin x & \text{if } \pi/4 < x \le 2\pi\end{cases}$

A. 0   B. 1   C. -1 D. $\sqrt{2}-1$ E. none of these

I. 32. [5.3] If $\int_{-1}^{1}(2f(x)+g(x))\,dx = 3$ and $\int_{-1}^{1}(f(x)-2g(x))\,dx = 1$, then what is $\int_{-1}^{1}(f(x)+g(x))\,dx$?

A. $\frac{3}{5}$ B. $\frac{8}{5}$ C. 4   D. 2   E. none of these

I. 33. [5.3] If $\int_0^2 f(x)\,dx = 4$ and $\int_0^5 f(x)\,dx = 6$, then what is $\int_2^5 f(x)\,dx$?

A. 0   B. -2 C. 2   D. 10   E. none of these

I. 34. [5.4] Evaluate $\int_0^1 \frac{e^x - e^{2x}}{e^x}\,dx$

A. 2   B. -2 C. $2-e$ D. $2+e$ E. none of these

I. 35. [5.5] Evaluate the indefinite integral $\int \frac{\sin\sqrt{2x}}{\sqrt{2x}}\,dx$

A. $-\cos\sqrt{2x}+C$ B. $x+C$ C. $\cos\sqrt{2x}+C$ D. $-x+C$ E. none of these

I. 36. [5.6] Evaluate the definite integral $\int_1^3 \frac{dt}{(t+1)^2}$

A. 1   B. $\frac{1}{2}$ C. $\frac{1}{3}$ D. $\frac{1}{4}$ E. none of these

I. 37. [5.6] The value of $\int_0^a x\sqrt{a^2-x^2}\,dx$ is

A. $\frac{1}{3}|a|^3$ B. $-\frac{1}{3}|a|^3$ C. $\frac{1}{3}|a|^{3/2}$ D. $-\frac{1}{3}|a|^{3/2}$ E. none of these

I. 38. [5.6] If $a > 0$, then the value of $\int_{-a}^{a} x\sqrt{a^2+x^2}\,dx$ is

A. $-\frac{2}{3}a^{3/2}$ B. 0   C. $\frac{2}{3}a^{3/2}$ D. undefined   E. none of these

I. 39. [5.4] The value of $\frac{d}{dx}\int_{-1}^{x^2}\sqrt{1+t^2}\,dt$ at $x = 2$ is

A. 1   B. $\sqrt{5}$ C. $4\sqrt{17}$ D. $\sqrt{1+t^3}$ E. $\sqrt{1+t^3}-1$

I. 40. [5.4] The value of $\int_0^1 \frac{x^2}{x^2+1}\,dx$ is

A. 2   B. 0   C. 1   D. $1+\frac{\pi}{4}$ E. $1-\frac{\pi}{4}$

I. 41. [5.6] The value of $\int_0^1 \frac{x}{x^2+1}\,dx$ is

A. 1 B. 0 C. $-\infty$ D. $\frac{\ln 2}{2}$ E. none of these

I. 42. [5.1] The graph of function $f(x)$ passes through point (1,2). Given that $f'(x)=6x-2$, find $f(x)$

A. $f(x)=3x^2-2x+2$ B. $f(x)=3x^2-2x+1$ C. $f(x)=3x^2-2x-2$
D. $f(x)=3x^2-2x-1$ E. none of these

PART II.

II. 1. [2.6] Give the value of *M*, in terms of *a*, *b*, and *c*, which makes the function *f*(*x*) continuous if

$$f(x)=\begin{cases} ax^2+bx+c & \text{if } x\le -1 \\ 3c-\dfrac{aM}{2} & \text{if } x>-1 \end{cases}$$

II. 2. [4.6] Evaluate $\lim\limits_{x\to 0}\frac{x2^x}{2^x-1}$.

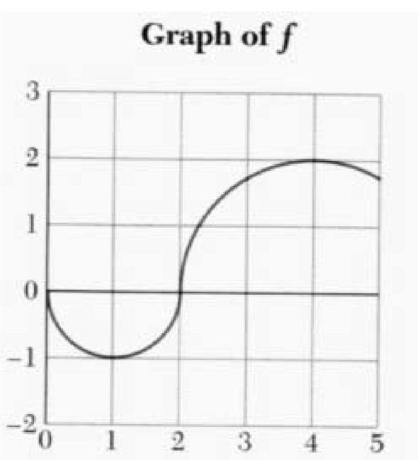
II. 3. [4.1/4.5] A line is drawn through the point (1, 2) so that it forms a right triangle with legs along the x- and y-axes. Find the slope of the line forming the triangle of least area.

II. 4. [5.4] Evaluate $\int_0^{2\pi} f(x)\,dx$ where $f(x)=\begin{cases} \cos x & \text{if } 0\le x\le \pi/4 \\ \sin x & \text{if } \pi/4<x\le 2\pi \end{cases}$

II. 5. [4.3] If $f(x)=x^{2/3}(x-7)$, find (a) all critical numbers; (b) when f is increasing and when decreasing; (c) all critical points and identify as relative maximum, relative minimum, or neither; (d) the second-order critical numbers and tell where the graph is concave up and where concave down; (e) give the coordinate of any inflection point(s).

II. 6. [2.4, 2.5] Find all horizontal and vertical asymptotes of $f(x)=\frac{2x^2-5x+7}{x^2-9}$ without sketching the graph of the function.

II. 7. [5.4/4.3] Let $g(u)=\int_1^u f(x)\,dx$, and the graph of *f* is shown by the graph, where the graph between 0 and 2 is part of a circle with a radius of 1, the graph between 2 and 5 is part of a circle with a radius of 2.



(a) Evaluate *g*(4) and *g*'(4).
(b) Where on the interval [0,5] is *g* increasing? concave up?
(c) Rank these numbers from lowest to highest: -1, 0, 1, g(0), g(2), g(4).

II. 8. [4.8] Given that $g(x)=\sqrt{x^2+7}$, find all functions *f* with the property that $f'(x)=g'(x)$ for all *x* and $f(3)=14$.

II. 9. [4.8, 5.1] Find $\int (\frac{5}{x}+2e^x)\,dx$

II. 10. [5.5] Find $\int \frac{2x+3}{\sqrt{5-x^2}}dx$

II. 11. [5.2] Given $f(x)=x^2+3$, find the exact area of the region under $y=f(x)$ on the interval [1, 3] by using area as the limit of a sum and the summation formulas.

II. 12. [4.6] Find

(a) $\lim_{x\to\infty}\left(1+\frac{r}{x}\right)^x$ (b) $\lim_{x\to 0}(\frac{1}{x}-\csc x)$ (c) $\lim_{x\to 0}\frac{\sqrt{5+2x}-\sqrt{5+x}}{2x}$ (d) $\lim_{x\to\infty}\frac{3^x}{4^x}$

II. 13. [4.1/4.5] Find the minimum possible value of the sum of a real number and its square.

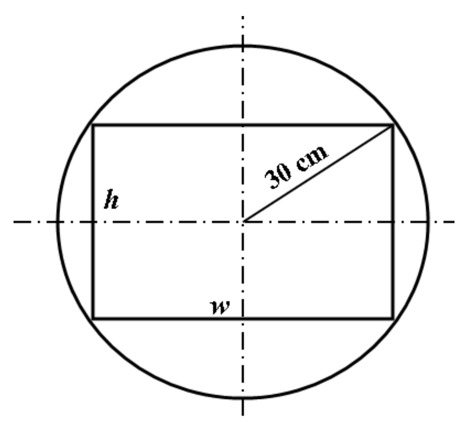
II. 14. [3.9] A balloon has sprung a leak. When the radius is 1 cm, the radius is changing at a rate of –1/4 cm/sec. Find the rate of change of the volume at this time. ($V=\frac{4}{3}\pi r^3$)

II. 15. [4.1] A ball thrown vertically upward at time $t = 0$ (sec) with initial velocity 80 ft/sec and with initial height 96 ft has height function $y(t)=-16t^2+80t+96$. (a) What is the maximum height attained by the ball? (b) When and with what impact speed does the ball hit the ground?

II. 16. [5.2, 5.3] Use the definition of the definite integral to evaluate $\int_1^2 (x^3+x-1)\,dx$.

II. 17. [3.9] An oil tanker slams into the Alaskan coastline. The oil spreads in a circle whose area increases at a constant rate of 6 miles^2/hour. How fast is the radius of the spill increasing when the area is 9 miles^2?

II. 18. [4.1, 4.5] A rectangular beam is cut form a cylindrical log of radius 30 cm. The strength of a beam of width *w* and height *h* is proportional to wh^2. Find the width and height of the beam of maximum strength.



You should also look over your lecture notes, homework assignment, problem-solving labs, and quizzes. The Midterms and Finals of Fall 2003 and Spring 2004 can be found at
http://www.uta.edu/math/pages/main/oldexams/calc1/calc1.htm

Final Exam: Saturday, May 9, 3:00 – 5:30 pm , LS 122

Answers to Part I

I. 1 D I. 2 C I. 3 E I. 4 B I. 5 D I. 6 D I. 7 A I. 8 B I. 9 D I. 10 C I. 11 E I. 12 B I. 13 C I. 14 E I. 15 C I. 16 D I. 17 C I. 18 B I. 19 A I. 20 A I. 21 B I. 22 C I. 23 A I. 24 B I. 25 B I. 26 A I. 27 C

I. 28 A I. 29 C I. 30 A I. 31 D I. 32 B I. 33 C I. 34 C I. 35 A I. 36 D I. 37 A I. 38 B I. 39 C I. 40 E I. 41 D I. 42 B

Part II

II 1. $f(x) = ax^2 + bx + c$ is continuous on $(-\infty, -1)$, $f(x) = 3c - \frac{aM}{2}$ is continuous on $(-1, +\infty,)$.

At $x = -1$,

1) $f(-1) = a - b + c$;

2) $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} ax^2 + bx + c = a - b + c$

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+}\left(3c - \frac{aM}{2}\right) = 3c - \frac{aM}{2}$

$\lim_{x\to -1} f(x)$ exists $\Rightarrow \lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x) \Rightarrow a - b + c = 3c - \frac{aM}{2} \Rightarrow M = \frac{-2(a-b-2c)}{a}$

3) $\lim_{x\to -1} f(x) = f(-1)$ is satisfied.

II 2. $\lim_{x\to 0}\frac{x2^x}{2^x - 1}$ is in the form of " $\frac{0}{0}$ ", using the l'Hopital's rule, we have $\lim_{x\to 0}\frac{x2^x}{2^x - 1} = \frac{1}{\ln 2}$

II 3. m = –2

II 4. $\int_0^{2\pi} f(x)\,dx = \int_0^{\pi/4}\cos x\,dx + \int_{\pi/4}^{2\pi}\sin x\,dx = \sin x\Big|_0^{\pi/4} - \cos x\Big|_{\pi/4}^{2\pi} = (\frac{\sqrt{2}}{2} - 0) - (1 - \frac{\sqrt{2}}{2}) = \sqrt{2} - 1$

II 5.

II 6. To find the horizontal asymptotes, check $\lim_{x\to -\infty} f(x)$ and $\lim_{x\to +\infty} f(x)$

$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty}\frac{2x^2 - 5x + 7}{x^2 - 9} = 2$ and $\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty}\frac{2x^2 - 5x + 7}{x^2 - 9} = 2 \quad \Rightarrow$

$y = 2$ is a horizontal asymptote.

Evaluating $\lim_{x\to 3^-} f(x)$ and $\lim_{x\to 3^+} f(x)$ for possible vertical asymptotes:

Because $\lim_{x\to 3^-}(2x^2 - 5x + 7) = 10$ and $\lim_{x\to 3^-}(x^2 - 9) = 0$ and $x^2 - 9 < 0$ as $x \to 3^-$,

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}\frac{2x^2 - 5x + 7}{x^2 - 9} = -\infty$.

Because $\lim_{x\to 3^+}(2x^2 - 5x + 7) = 10$ and $\lim_{x\to 3^+}(x^2 - 9) = 0$ and $x^2 - 9 > 0$ as $x \to 3^+$,

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}\frac{2x^2 - 5x + 7}{x^2 - 9} = +\infty$.

Therefore, $x = 3$ is a vertical asymptote.

II 7. a) $g(4) = \int_1^4 f(x)\,dx = \int_1^2 f(x)\,dx + \int_2^4 f(x)\,dx = (-A_1) + A_2$ where $A_1 = \frac{1}{2}(\pi\cdot 1^2) = \frac{\pi}{2}$ and

$A_2 = \frac{1}{4}(\pi\cdot 2^2) = \pi$, thus $g(4) = (-A_1) + A_2 = \pi/2$.

Using the Fundamental Theorem of Calculus (Part I), we have

$g'(u) = \frac{d}{du} g(u) = \frac{d}{du}\int_1^u f(x)\,dx = f(u)$. Thus $g'(4) = f(4) = 2$

b) Using the first derivative test for monotonic functions,

$g'(x) = f(x) = 0 \quad \Rightarrow \; x = 0$ or $x = 2$. Inside the domain [0, 5], the critical point is $x = 2$.

x	(0, 2)	2	(2, 5)
$g'(x)$	< 0	$= 0$	> 0
	↘		↗

Using the second derivative test for concavity,

$g''(x) = f'(x) = 0 \quad \Rightarrow \; x = 1$ or $x = 4$. Inside the domain [0, 5], the second-order critical points are $x = 1$ and $x = 4$.

x	(0, 1)	1	(1, 4)	4	(4, 5)
$g''(x)$	< 0	$= 0$	> 0	$= 0$	< 0
Concavity	∩		∪		∩

II 8. $f(x) = \sqrt{x^2 + 7} + 10$

II 9. $\int \left(\frac{5}{x} + 2e^x\right) dx = 5\int \frac{1}{x}\,dx + 2\int e^x\,dx = 5\ln|x| + 2e^x + C$

II 10.

$$\int \frac{2x+3}{\sqrt{5-x^2}}\,dx = \int \frac{2x}{\sqrt{5-x^2}}\,dx + \int \frac{3}{\sqrt{5-x^2}}\,dx$$

$$= \int \frac{2x}{\sqrt{5-x^2}}\,dx + \int \frac{3}{\sqrt{5}\sqrt{1-(x/\sqrt{5})^2}}\,dx \quad \text{Let } u = 5 - x^2, \frac{du}{dx} = -2x, -du = 2x\,dx$$

$$\text{Let } v = x/\sqrt{5}, \frac{dv}{dx} = \frac{1}{\sqrt{5}}, dv = \frac{1}{\sqrt{5}}dx$$

$$= \int \frac{-1}{\sqrt{u}}\,du + 3\int \frac{1}{\sqrt{1-v^2}}\,dv$$

$$= -2\sqrt{u} + 3\sin^{-1} v + C = -2\sqrt{5-x^2} + 3\sin^{-1}(x/\sqrt{5}) + C$$

II 11. $A = 44/3$

II 12. a) Let $y = \left(1 + \frac{r}{x}\right)^x$, $\ln y = x\ln\left(1+\frac{r}{x}\right)$, $\lim_{x\to\infty} \ln y = \lim_{x\to\infty}\left[x\ln\left(1+\frac{r}{x}\right)\right]$, "$\infty \cdot 0$" type

$$\lim_{x\to\infty}\left[x\ln\left(1+\frac{r}{x}\right)\right] = \lim_{x\to\infty}\frac{\ln\left(1+\frac{r}{x}\right)}{\frac{1}{x}}, \text{ "}\frac{0}{0}\text{" type,}$$

$$\lim_{x\to\infty}\frac{\ln\left(1+\frac{r}{x}\right)}{\frac{1}{x}}=\lim_{x\to\infty}\frac{\left[\ln\left(1+\frac{r}{x}\right)\right]'}{\left(\frac{1}{x}\right)'}=\lim_{x\to\infty}\left(r\frac{x}{x+r}\right)=r\lim_{x\to\infty}\frac{x}{x+r}=r\lim_{x\to\infty}\frac{x'}{(x+r)'}=r$$

Thus, $\lim_{x\to\infty}\ln y=-r$, $e^{\lim_{x\to\infty}\ln y}=e^{r}$, $\lim_{x\to\infty}e^{\ln y}=\lim_{x\to\infty}y=e^{r}$, that is $\lim_{x\to\infty}\left(1+\frac{r}{x}\right)^{x}=e^{r}$

II 13. Find the minimum possible value of the sum of a real number and its square.

Let x = the real number, and the sum of a real number and its square as a function of x

$f(x)=x+x^2$ with domain $(-\infty,+\infty)$. Using the first derivative test to find the local min of f,

$$f'(x)=1+2x,$$

Thus $f'(x)=1+2x=0 \quad \Rightarrow x=-1/2$ and $f'(x)$ is defined on the domain. Therefore, the only critical point is $x=-1/2$.

x	$(-\infty,\ -1/2)$	$-1/2$	$(-1/2,\ \infty)$
$g'(x)$	<0	$=0$	>0
	↘	rel. min.	↗

Since $f''(x)=2>0$, the graph of the function is concave up on the domain, the local min is also the absolute min. Conclusion …

II 14. Let r = radius and V = volume of the spherical balloon. $V=\frac{4}{3}\pi r^3$.

Given $\frac{dr}{dt}=-\frac{1}{4}$ when r = 1, find $\frac{dV}{dt}$.

$$\frac{dV}{dt}=\frac{4}{3}\pi 3r^2\frac{dr}{dt}=4\pi r^2\frac{dr}{dt}=4\pi(1^2)\left(-\frac{1}{4}\right)=-\pi\ (\text{cm}^3/\text{s})$$