COMPRESSIVE SENSING OF IMAGE AND COMPARISON WITH JPEG AND JPEG 2000

Name: SANIL FULANI
Student Id: 1000645167
EE 5359 : Multimedia Processing

Note: This report is just a proposal report for the project. It should not be considered as final report. ©Sanil Fulani
TOPICS COVERED

- OVERVIEW OF COMPRESSIVE SAMPLING
- PROCESS OF CONVENTIONAL COMPRESSION
- COMPRESSIVE SENSING CONCEPT FOR IMPLEMENTATION
- COMPARISON WITH JPEG IMAGE COMPRESSION
- CHALLENGE
- APPLICATION
- PROGRESS CHART
Overview

Technological development

• Exploded since 2006
• Pixel growing $\rightarrow$ crunch size $\rightarrow$ better compression algorithm $\rightarrow$ rise of “COMPRESSIVE SENSING (CS)”
• Compressive sensing $\rightarrow$ below Nyquist rate $\rightarrow$ Against Shannon’s Theory
• CS $\rightarrow$ enables ‘Design of Digital Acquisition devices’
• Measurements $\rightarrow$ Inter-products with some random ‘basis’ functions.
• Hardware $\rightarrow$ single- pixel camera
• Signals are sparse.
Schematic of Rice 1-pixel camera
courtesy RICE UNIVERSITY
Physical Implementation of the camera
courtesy RICE UNIVERSITY
Introduction to Data Acquisition

• Shannon/Nyquist Sampling Theorem
  – Must sample more than twice the signal bandwidth,

  – Might end up with a huge number of samples
    ➔ Need to Compress!

  – Doing more work than needed?
Conventional Process of Compression

• After data acquisition → DCT
• Many coefficients → zero → discarded before quantization
• This makes Compressive Sampling applicable where Nyquist rate is high → where compressing sheer volume of samples → problem for transmission and storage
Compressive Sensing

- Split image $\rightarrow$ small non-overlapping blocks of equal size $\rightarrow$ apply DCT on blocks found to be sparse.
- Sparse blocks selection:
  - Let $C$ – a small positive constant.
  - $T$ – an integer threshold i.e. representative of avg no. of non-significant DCT coefficients over all blocks

- No. of DCT coefficients $\rightarrow$ less than $C$
  - $\rightarrow$ larger than $T$

The block selected as reference for Compressing Sampling
Compressive Sensing

\[ K \approx M \ll N \]

Fig. Compressive sensing based data acquisition system
 Concept

• Let $x = \{x[1], \ldots ,x[N]\}$ be a set of $N$ pixels of an image. Let $s$ be the representation of ‘$x$’ in the transform domain, that is:

$$x = \Psi s = \sum_{i=1}^{N} s_i \psi_i$$

• Let $y$ be an $M$-length measurement vector given by: $y = \Phi x$, where $\Phi$ is a $M \times N$ measurement matrix (independent identically distributed (i.i.d.) Gaussian matrix). The above expression can be written in terms of $s$ as:

$$y = \Phi \Psi s$$

• $K < M \ll N \rightarrow$ Reduce Redundancy by selecting $M$-samples of signal.
Signal Recovery

• Orthogonal Matching Pursuit (OMP) algorithm
  – Where, all sampled coefficients $\rightarrow$ less than $C$ $\rightarrow$ set to zero
  – Hence, for $C>0$, sampling process always ‘lossy’
  – i.e. if $N-K$, non-significant samples then $\rightarrow$ atleast $M=K+1$ samples needed for reconstruction
  – It even fails when $M$ is too low, or all DCT coefficients are zero or if division by zero in OMP algorithm appears.
• Other approach

1) L0 norm
   • L0 sparsest coefficients
   • Unfortunately its complex hence fails

2) L2 norm
   • Pros: simple mathematically (involving only a matrix multiplication by the pseudo-inverse of the basis sampled in).
   • Cons: poor results for most practical applications, as the unknown (not sampled) coefficients seldom have zero energy.
• Hence, following Tao, the \textbf{L1 norm}, or the sum of the absolute values, is usually what is minimized.

• Finding the candidate with the smallest L1 norm can be expressed relatively easily as a linear program, for which efficient solution methods already exist. This leads to comparable results as using the L0 norm, often yielding results with many coefficients being zero.

• This optimization also known as \textbf{BASIS PURSUIT}

• excellent approximation via the L1 norm minimization is given by:

\[
\hat{s} = \arg \min_{s'} \|s'\|_1, \text{ such that } \Phi \Psi s' = y.
\]
Block Diagram of JPEG Baseline

Figure 1. DCT-Based Encoder Processing Steps

Figure 2. DCT-Based Decoder Processing Steps
CHALLENGE

• CS replaces conventional sampling and reconstruction → linear measurement scheme
• However, will work ‘ONLY IF SOURCE IS SPARSE’
• Challenge to predict which sources are sparse in a particular domain.
  • Applying CS → whole image → ineffective
  • Hence, split image → small non-overlapping blocks of equal size → apply CS on blocks found to be sparse
APPLICATION

• Analog to Digital Conversion - a fundamental aspect of Wireless Communications.

• Eg. CDMA $\rightarrow$ voice msg $\rightarrow$ 4096 hertz standard freq $\rightarrow$ spreads over radio spectrum $\rightarrow$ span thousands of hertz

• Here $\rightarrow$ signal still sparse $\rightarrow$ so detector recover signal more rapidly then Shannon’s theorem.
Other Applications

- Data Acquisition
- Data Compression
- Image and Video Compression
PROGRESS CHART

- **February 28, 2010** – Research Reading on compressive sensing and information gathering
- **March 20, 2010** – complete research reading and jpeg simulation part
- **April 10, 2010** – complete compressive sensing coding
- **April 20, 2010** – Final touch and documentation report
References

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• Emmanuel Candès and Terence Tao, **The Dantzig Selector: Statistical estimation when p is much larger than n** (To appear in Annals of Statistics)

• Holger Rauhut, Karin Schass, and Pierre Vanderghynst, **Compressed sensing and redundant dictionaries.** (IEEE Trans. on Information Theory, 54(5), pp. 2210 - 2219, May 2008)

• Albert Cohen, Wolfgang Dahmen, and Ronald DeVore, Compressed sensing and best k-term approximation. (Preprint, 2006) [Formerly titled "Remarks on compressed sensing"]
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• Albert Cohen, Wolfgang Dahmen, and Ronald DeVore, Compressed sensing and best k-term approximation. (Preprint, 2006) [Formerly titled "Remarks on compressed sensing"]

• www.jpeg.org

• http://faculty.ksu.edu.sa/hedjar/Documents/MATLAB_Educational_Sites.htm

• http://dsp.rice.edu/cs (Compressive Sensing Resources)
END

THANK YOU

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