1. Conolve $h(n)$ and $x(n)$ to get $y(n)$. Put $y(n)$ in closed form when possible.
   (a) $h(n) = a^n u(n)$ and $x(n) = b^n u(n)$.
   (b) $h(n) = \delta(n-n_o) + \delta(n-n_1)$ and $x(n) = n^3 u(n)$.
   (c) $h(n) = a^n u(-n)$ and $x(n) = b^n u(-n)$.
   (d) $h(n) = u(n+1)-u(n-5)$, $x(n) = u(n+1)-u(n-4)$. Express the result in terms of $r(n)$, where $u(n) * u(n) = r(n+1)$.

2. A linear time invariant) system is described by the recursive difference equation

   \[ y(n) = 5x(n) - \frac{13}{6} x(n-1) + \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) \]

   (a) Find $H(e^{jw})$ in closed form.
   (b) Find the homogeneous solution.
   (c) Find $h(0)$ and $h(1)$
   (d) Find the impulse response $h(n)$.
   (e) In the causal difference equation above, assume that legal values of $n$ range from 0 to $N-1$, and that $x(n)$ is already available. For the pseudocode below give values or expressions for $A$, $B$, $C$, $D$, and $E$.

   \[
   \begin{align*}
   y(0) &= A \\
   y(1) &= B \\
   &\text{For } n = C \text{ to } D \\
   y(n) &= E \\
   \end{align*}
   \]

   End

3. A C/D converter samples $x_c(t)$ with a sampling period $T$ to produce $x(n)$. An ideal lowpass digital filter $h(n)$, with cut-off frequency $w_o$ is applied to $x(n)$ to produce $y(n)$. A D/C converter generates $y_c(t)$ from $y(n)$, again using the sampling period $T$. The cut-off frequency of $x_c(t)$ is 3 radians/sec.
   (a) Find the cut-off frequency of $x(n)$ in radians, as a function of $T$.
   (b) Find the largest sampling period $T$ such that $y(n)$ has no aliasing. (Hint: $x(n)$ may still be aliased)
   (c) Find the smallest sampling period $T$ so that the filter $h(n)$ modifies the spectrum of $x(n)$.
   (d) Give an expression for $h(n)$. 
4. Let \( x(n) \) and \( y(n) \) denote complex sequences with DTFTs \( X(e^{jw}) \) and \( Y(e^{jw}) \).
(a) Find a frequency domain expression for the constant;
\[
C = \sum_{n = -\infty}^{\infty} x(n) \cdot y^*(n)
\]
(b) Find the numerical value of
\[
\sum_{n = -\infty}^{\infty} \frac{\sin(\pi n/2)}{3\pi n} \cdot \frac{\sin(\pi n/8)}{7\pi n}
\]
(c) The pseudocode below should efficiently calculate the sum of part (b), where the limits on \( n \) are replaced by \(-N\) and \(+N\). Give values for \( A, B, C, \) and \( D \).
```
Sum = A
For n = B to C
    Sum = Sum + D
End
```

5. In its passband, a linear time-invariant system has the frequency response
\[
H(e^{jw}) = e^{-jw^3}, \quad |w| < \frac{\pi}{3}, \quad H(e^{jw}) = 0, \quad |w| \geq \frac{\pi}{3}
\]
(a) Find the DTFT of the delayed impulse, \( \delta(n-n_0) \).
(b) Considering your answer to part (a), give the time delay of the filter whose frequency response \( H(e^{jw}) \) is given above.
(c) Give the impulse response, \( h(n) \), of the filter above.
(d) If the input \( x(n) \) is
\[
x(n) = \sum_{k = -\infty}^{\infty} \delta(n+17k)
\]
and \( y(n) \) is \( h(n) \ast x(n) \), express \( y(n) \) in terms of the symbol “\( h() \)” (Hint: do convolution in the time domain).
(e) Using your results from parts (c) and (d), give an exact expression for \( y(n) \).

6. Assume that
\[
X(e^{jw}) = \cosh(5w) \quad \text{for} \quad |w| \leq \pi
\]
where \( \cosh(x) = \frac{1}{2}[e^x + e^{-x}] \).
(a) Find \( x(0) \).
(b) Find \( \lim x(n) \) as \( n \) approaches infinity.
(c) Find an expression for \( x(n) \).