Exam 2, EE5350 and EE4318, Fall 2010

1. Find $z$-transforms of the following in closed form, and their regions of convergence.
   (a) $u(-n)$     (b) $a^n u(n-5)$     (c) $n \cdot u(-n)$     (d) $\cos(w_2 \cdot n) u(n)$

2. An IIR digital filter has the transfer function

   $H(z) = \frac{2 - 7z^{-1}}{1 - 7z^{-1} + 10z^{-2}}$

   (a) Using the partial fraction expansion, or another method, find the causal impulse response $h(n)$, as the sum of two exponentials.
   (b) Give a stable version of the impulse response $h(n)$.
   (c) Letting $H(z) = H(z)/D(z)$, where $D(z)$ is the $z$-transform of $\delta(n)$, give a causal difference equation that calculates $h(n)$ (as in part (a) ) in terms of $\delta(n)$. Give $D(z)$.
   (d) Give the first two terms of $h(n)$ ( $h(0)$ and $h(1)$ ) that result when long division is used to find the causal $h(n)$.
   (e) Give the first two terms of $h(n)$ ( $h(-1)$ and $h(-2)$ ) that result when long division is used to find the anti-causal $h(n)$ from

   $H(z) = \frac{-7z + 2z^2}{10 - 7z + z^2}$

3. Given $H(z)$ in the problem statement of problem 2:
   (a) Give the poles of $H(z)$, and its region of convergence. Are the poles inside the region of convergence ?
   (b) Give the region of convergence of $H(z)$, when $h(n)$ is anticausal. Are the poles inside the region of convergence ?
   (c) How many versions of $h(n)$ can be found from $H(z)$, each corresponding to a different region of convergence ? How many of these versions have a frequency response ?

4. Find $y(n)$ in terms of $x(n)$ and $h(n)$ (which can be complex).

   (a) $Y(k) = H^*(k) \cdot X(k)$,     (b) $Y(k) = \sum_{m=0}^{N-1} H(m - k)N \cdot X(m + k)N$
5. For the infinite length sequence $x(n)$, we want to calculate a frequency domain model in a moving N-sample window as

$$X_n(k) = \sum_{m=0}^{N-1} x(n - N + 1 + m) \cdot W_N^{mk}$$

(a) Express $X_{n+1}(k)$ as $X_n(k)W_N^{-k}$ plus two additional terms.
(b) Replacing $n$ by $n-1$ in part (a), give a final expression for $X_n(k)$ in terms of $X_{n-1}(k)$
(c) Assume $x(n) = 0$ for $n$ negative. What is the smallest value of $n$ for which $X_n(k)$ in part (b) is calculated using no zero-valued samples. (Hint: $X_n(k) = [X_{n-1}(k) - x() + x()]W_N^0$. All 3 terms in the brackets have $x()$.)
(d) Before calculating $X_n(k)$ for $n=0$, how should $X_{n-1}(k)$ be initialized?
(e) For $n$ varying from 0 to $N-2$, modify your answer from part (b), so that no zero-valued quantities are used.

6. A causal FIR digital filter $h(n)$ is to be designed using the inverse DFT. Assume that its desired frequency response $H_d(e^{jw})$ is available for $0 \leq w \leq \pi$.
(a) When generating $H(k)$, $H_d(e^{jw})$ is to sampled as $w = k \cdot \Delta w$, where $\Delta w$ is known. Find the DFT order $N$ in terms of $\Delta w$.
(b) The pseudocode below uses the inverse DFT or FFT to generate $h(n)$. Give the correct value for $W$, in the first line of code.
(c) Give the correct expression for $X$, in terms of $N$, in the second line of code.
(d) Give the correct expression for $Y$, in the third line of code, using $N$ rather than $\Delta w$.
(e) Give the correct expression for $Z$, in the fifth line of code.

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H(0) = H(exp(jW))
For 1 \leq k \leq X
w(k) = Y
H(k) = H(exp(jw(k)))
H(N-k) = Z
End
h(n) = DFT^{-1}\{H(k)\}
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