I. Introduction

A. Approximating Functions of One Variable, Review

1. Functions of Time

**Goal:** Review approximation techniques for functions of time

a. Example Applications

(1) Approximating message signals in communications
(2) Finding local approximations of analytic functions
(3) Solution of nonlinear differential equations
b. General form of approximation:

\[ y(t) = \sum_{k=1}^{L} a_k \cdot v_k(t) \]

- \( L \) - the number of basis functions
- \( v_k(t) \) - the \( k \)th basis function
- \( a_k \) - the \( k \)th coefficient
- \( t \) - time

c. Example Basis Functions

\[ v_k(t) = (t - t_o)^{(k-1)} \]

\[ v_k(t) = \cos(\omega_k \cdot t + \phi_k) \]

d. Finding the \( a_k \)

1. Integral formulae (Fourier series)
2. Derivative formula (Taylor series)
3. Curve fit to \((y_k, t_k)\) for \(1 \leq k \leq N_v\)
e. Finding the $a_k$ for Orthogonal expansions (like Fourier Series)

Given an error function, which measures the MSE between the original function and its approximation, as

$$E = \frac{1}{T} \int_{0}^{T} \left[ y(t) - \sum_{k=1}^{L} a_k \cdot v_k(t) \right]^2 \, dt$$

or

$$E = \frac{1}{N_v} \sum_{i=1}^{N_v} \left[ y(t_i) - \sum_{k=1}^{L} a_k \cdot v_k(t_i) \right]^2$$

Set $\partial E/\partial a_m = 0$ and solve the resulting equations for $a_m$
f. Basic Ideas:

(1) We pick a set of basis functions $v_k(t)$ which may be
(a) Bounded: $\left| v_k(t) \right| \leq B$
(b) Orthogonal: $\langle v_k(t) \cdot v_m(t) \rangle = K \cdot \delta(k-m)$
(c) Continuous
(2) We find coefficients $a_k$
(3) Such series have approximation theorems that govern how $E$ approaches 0 as $L$ increases, for continuous $y(t)$ etc.
(4) Picking basis functions and $L$ correctly is often critical
2. Designing Nonlinear Approximation \( y(x) \)

a. Example Applications

(1) Approximating nonlinear device characteristics (I versus V)
(2) Finding local approximations of analytic functions
(3) Solution of nonlinear differential equations

b. General form of approximation:

\[
y(x) = \sum_{k=1}^{L} a_k \cdot v_k(x)
\]

- \( L \) - the number of basis functions
- \( v_k(x) \) - the kth basis function
- \( a_k \) - the kth coefficient
- \( x \) - input signal
c. Picking the Basis Functions

\[ v_k(x) = (x - x_o)^{(k-1)} \]

\[ v_k(x) = \cos( w_k \cdot x + \phi_k ) \]

\[ v_k(x) = b_k + c_k x \]

d. Finding the \( a_k \)
(1) Integral formulae (Fourier series)
(2) Derivative formula (Taylor series)
(3) Curve fit to \((x_k, y_k)\) for \(1 \leq k \leq N_v\)

e. Basic Ideas:

(1) We’ve replaced \( t \) by \( x \), and \( y \) is a nonlinear function of \( x \)
(2) We pick basis functions \( v_k(x) \) which may be
   (a) Bounded: \(|v_k(x)| \leq B\)
   (b) Orthogonal: \( <v_k(x) \cdot v_m(x)> = K \cdot \delta(k-m)\)
   (c) Continuous
(3) We find coefficients $a_k$
(4) Such series have approximation theorems that govern how $E$ approaches 0 as $L$ increases, for continuous $y(x)$ etc.
(5) Picking basis functions and $L$ correctly is often critical
B. Usefulness of Multivariate Approximation

**Goal:** Establish the usefulness of approximating $M$ functions of $N$ variables

1. Flight Load Synthesis in Helicopters

**Goal:** Given recorded HUMS (health and usage monitoring system) measurements and loads, find 9 functions of 24 variables which map the measurements into the loads. These 9 functions allow us to determine loads (forces) on helicopter components during flight. Loads and flight time then determine how much critical components degrade, so that they can be replaced long before the helicopter crashes.
Feature and Load Definitions

<table>
<thead>
<tr>
<th>features (inputs)</th>
<th>loads (outputs)</th>
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<tbody>
<tr>
<td>(1) CG F/A load factor</td>
<td>(1) fore/aft cyclic boost tube</td>
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<tr>
<td>(2) CG lateral load factor</td>
<td>oscillatory axial load (OAL)</td>
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<tr>
<td>(3) CG normal load factor</td>
<td>oscillatory axial load (OAL)</td>
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<td>(4) pitch attitude</td>
<td>collective boost tube OAL</td>
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<td>(5) pitch rate</td>
<td>main rotor (MR) pitch link OAL</td>
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<td>(6) roll attitude</td>
<td>MR mast oscillatory perpendicular bending st</td>
</tr>
<tr>
<td>(7) roll rate</td>
<td>MR yoke oscillatory beam bending</td>
</tr>
<tr>
<td>(8) yaw rate</td>
<td>MR blade oscillatory beam bending</td>
</tr>
<tr>
<td>(9) corrected airspeed</td>
<td>MR yoke oscillatory chord bending</td>
</tr>
<tr>
<td>(10) rate of climb</td>
<td>resultant mast bending, sta.</td>
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<tr>
<td>(11) longitudinal cyclic stick position</td>
<td>position</td>
</tr>
<tr>
<td>(12) pedal position</td>
<td>left hand aft pylon link</td>
</tr>
<tr>
<td>(13) collective stick position</td>
<td>right hand aft pylon link</td>
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</table>
2. Flight Condition Recognition in Helicopters

Goal: Given recorded HUMS measurements and flight types, find 50 discriminant functions of 24 variables which map the measurements into 50 types of helicopter flight including level flight, maneuvers, take off etc. The largest output determines the type of flight. Type of flight and flight time then determine how much critical components degrade, so that they can be replaced.
3. Character Recognition in Document Scanners

**Goal:** Given an image of a text character (letter, number, punctuation mark etc.), determine which character is present.

Some Other Applications
(1) Face recognition,
(2) Fingerprint recognition
(3) Vehicle recognition
C. Conventional Multivariate Approximation

Subsection Goal: Describe the approximation of $M$ functions of $N$ variables, and the resulting problems

1. General form of approximation:

$$y_m(x) = \sum_{k=1}^{L} a_{mk} \cdot v_k(x)$$

$$= \sum_{i=1}^{L_1} \sum_{j=1}^{L_2} \cdots \sum_{k=1}^{L_N} a_{m,i,j,\ldots,k} v_i(x_1)v_j(x_2)\cdots v_k(x_N)$$

where $1 \leq m \leq M$.

$v_i(x_k)$ - the $i$th basis function of $x_k$

$v_k(x)$ - the $k$th basis function of $x$, which is a product of $N$ of $v_i(x_k)$ basis functions

$L$ - number of multivariate basis functions

$L_k$ - the number of univariate basis functions of $x_k$ used

$a_k$ - the $k$th coefficient
x - input vector of dimension N
y_m(x) - the mth function of N variables

2. Problems
a. Good basis functions hard to find
b. Mapping unknown, so integral and derivative formulae useless
c. Integral and derivative formulae difficult to evaluate
d. The number of basis functions L can be huge. This is the combinatorial explosion problem.

Combinatorial Explosion in N-Input Polynomials of Degree D

<table>
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<th>N</th>
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D. Basic Neural Net Approach

Idea: Instead of picking a set of basis functions, pick the general form of the basis functions, and let them develop during a learning or training process.

Re-write

\[ y_m(x) = \sum_{k=1}^{L} a_{mk} \cdot v_k(x) = \]

\[ \sum_{i=1}^{L_1} \sum_{j=1}^{L_2} \cdots \sum_{k=1}^{L_N} a_{m,i,j,\ldots,k} v_i(x_1) v_j(x_2) \cdots v_k(x_N) \]

as

\[ y_m(x) = \sum_{k=1}^{L} a_{mk} \cdot v(w_k, x) \]
Comments

(1) The function $v()$ is often bounded and not separable

(2) Basis functions $v(w_k,x)$ aren’t orthogonal

(3) **Basis functions are not picked** but develop during training. $E$ is minimized with respect to $w_k$ and $a_{mk}$, in a gradient algorithm using both $\partial E/\partial w_k(i)$ and $\partial E/\partial a_{mk}$.

(4) The complexity (number of coefficients) of the network is easily incremented by increasing $L$ by 1 (**No combinatorial explosion**).

Example

**Function:** $t = x_1^2x_3^2 + x_4x_6x_8x_{10}$, $N = 10$, $M = 1$. $L$: 1,001

**Training Data:** thousands of pairs $(x_p,t_p)$

**Polynomial Solution:** 1,001 very ill-
conditioned equations in 1,001 unknowns. Has combinatorial explosion as before.

**Neural Net Solution:** The network below has only 81 coefficients and 9 squaring nodes. Yet it can realize the desired function and many others. After eliminating insignificant weights, we have 25 left.
Comments

(1) The conventional approach is impractical.
(2) The neural net has 3 basis functions which develop during training
(3) These basis functions are compositions of simpler functions
(4) The process of finding the best basis functions and the best final network is called training, which is similar to the curve fitting approach in FLNs.

Some Commercial Applications

(1) Forecasting of wind speed and direction, temperature, reservoir inflow, power loads
(2) Parameter estimation in well-logging
(3) Document Processing
(4) Continuous Speech Recognition
(5) Face Recognition
(6) Fingerprint recognition
(7) Waveform and Image Texture Recognition
(8) Vehicle recognition
(9) Endpoint detection in chip fabrication
(10) Flight load synthesis
(11) Flight condition recognition

Some Companies and Organizations Using Neural Nets

(1) FastVDO
(2) Weatherford
(3) Ikonisys
(4) American GNC Corporation
(5) Siemens Electrocom Automation
(6) NASA
(7) Southwest Research Institute
(8) Williams Pyro
(9) Bell Helicopter and Teledyne Controls
(10) Verity Instruments
(11) FAS Technology
(12) Schlumberger Well Services
(13) Exxon-Mobil Research
Recommended Course Sequence

<table>
<thead>
<tr>
<th>Fall</th>
<th>Spring</th>
<th>Summer</th>
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<tbody>
<tr>
<td>EE5302</td>
<td>EE5352</td>
<td>EE5357</td>
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<tr>
<td>EE5350</td>
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<tr>
<td>EE5353</td>
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EE5302: Random Signals and Noise
EE5350: DSP
EE5353: Neural Networks
EE5352: Statistical Signal Processing
EE5356: Digital Image Processing
EE5357: Statistical Pattern Recognition
Employers of Recent IPNNL Graduates

**Internships:** Qualcomm, FastVDO

**MS Jobs:** Qualcomm, FastVDO, Mathworks, States, ServerEngines, Noble Corp., Lockheed, Nokia, Raytheon

**PhD Jobs:** FastVDO, Williams Pyro, Old Dominion University, Ikonisys, SETI (Nasa), American GNC Corporation
Reading Assignment:

Handout entitled “Gradient Techniques for Unconstrained Optimization” located at:

http://www-ee.uta.edu/eeweb/ip/Courses/NN/NN.htm