Signal Processing Project 1: Frequency Domain Interpolation

In the time domain, we can interpolate between samples using bandlimited interpolation. Similarly the DFT $X(k)$, for $0 \leq k \leq N-1$ can be interpolated to get the DTFT $X(e^{jw})$, for arbitrary $w$.

1. Using the expressions,

\[ X(e^{jw}) = \sum_{n=0}^{N-1} x(n) e^{-jwn} \]

\[ x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-nk} \]

\[ W_N = e^{-j2\pi/N} \]

find $X(e^{jw})$ in terms of $X(k)$.

2. Simplify the expression and write a subprogram FRK with parameters $X$, $N$, $W$, $XT$, where $X$ is the DFT array with elements $X(k)$, $W$ is $w$, and $XT$ is the DTFT $X(e^{jw})$ at frequency $w$ or $W$. FRK calculates $XT$ from $X$.

3. Write a subprogram, DIREK, with parameters $X$, $N$, $W$, and $XD$ which calculates the DTFT $XD$ of $x(n)$ $X(e^{jw})$ at frequency $W$.

4. Write a program which;
   (a) Asks the user for $N$ and $W$, and then generates an example signal $x(n)$, such as $x(n) = 1$ for $0 \leq n \leq N/2$ and $x(n) = 0$ for $1+N/2 \leq n \leq N-1$.
   (b) Calculates the DTFT at frequency $W$ using both FRK and DIREK.
   (c) Displays the two DTFT values.

5. Test the program.