Purchasing Power Parity for Traded and Non-traded Goods: A Structural Error Correction Model Approach

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Abstract

When univariate methods are applied to real exchange rates, point estimates of autoregressive coefficients typically imply very slow rates of mean reversion. However, a recent study by Murray and Papell (2002) calculates confidence intervals for estimates of half-lives for long-horizon and post-1973 data, and concludes that univariate methods provide virtually no information regarding the size of the half-lives. This paper estimates half-lives with a system method based on a structural error correction model for the nominal exchange rate, a domestic price index, a foreign price index, and a monetary variable. The method is applied to estimate half-lives of real exchange rates based on producer price indices (PPI), consumer price indices (CPI), and GDP implicit deflators. The idea is that the traded good component of the PPI is proportionately larger than that of the CPI. If the convergence rate is faster for traded good prices than that for non-traded good prices, half-lives for the real exchange rate based on the PPI should be shorter than those for the real exchange rate based on the CPI and that on the GDP deflator. Our empirical results are consistent with this view.

Keywords: Structural Error Correction Model (SECM); Purchasing Power Parity; Real Exchange Rate; Half-life; Convergence Rate

JEL Classification: C22, F31, F41

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1. Introduction

When univariate methods are applied to real exchange rates, point estimates of autoregressive coefficients typically imply very slow rates of mean reversion. Rogoff (1996) discusses that the remarkable consensus of 3-5 year half-lives of purchasing power parity (PPP) deviations is found among studies using long-horizon data. However, a recent study by Murray and Papell (2002) calculates confidence intervals for estimates of half-lives for long-horizon and post-1973 data, and concludes that univariate methods provide virtually no information regarding the size of the half-lives. This paper estimates half-lives of real exchange rates based on the producer price index (PPI), consumer price index (CPI), and GDP implicit deflators with a system method developed by Kim (2003), who modified Kim, Ogaki, and Yang's estimation method for a Structural Error Correction Model (ECM). This system method employs a two-good version of Mussa's (1982) model with a modification in which the exchange rate exhibits overshooting as in Dornbush's (1976) model. The model includes a gradual adjustment equation, in which the domestic price of the traded good adjusts to the long-run equilibrium level determined by PPP.

In a class of structural ECMs, a single equation instrumental variable (IV) method can be applied to the gradual adjustment equation that describes a gradual adjustment of economic variables toward long-run equilibrium to consistently estimate the structural speed of adjustment coefficient. Kim, Ogaki, and Yang's (2003) system method combines the single equation IV method with Hansen and Sargent's (1982) method, which applies Hansen's (1982) Generalized Method of Moments (GMM) to linear rational expectations models.

In the context of Mussa's (1982) model, the gradual adjustment equation implies the first order autoregression for the real exchange rate defined by the domestic and foreign traded good prices. The autoregressive coefficient is one minus the structural speed of adjustment coefficient in the structural ECM. Thus, the structural speed of the adjustment coefficient can be simply estimated by applying ordinary least squares (OLS) to the real exchange rate autoregression. This coefficient can also be estimated by applying Hansen and Sargent's (1982) method to a system of variables containing the nominal exchange rate, the foreign traded good price, and the money supply. The system method combines these two estimation methods.

In the literature of estimation of half-lives of real exchange rates, the first order autoregressions of real exchange rates have been typically estimated by univariate methods. When a univariate method is combined with Hansen and Sargent's (1982) method as in our system method, then the system method estimator for the autoregressive coefficient is more efficient than the univariate method estimator as long as the linear rational expectations model is correctly specified. When the linear rational expectations model used in this paper is misspecified, the system method estimator is inconsistent. However, if the model is a good approximation, then the estimator's mean may be close to the true value and its variance may be much smaller than the univariate estimators.

In this paper, we are interested in the difference of half-lives of real exchange rates based on traded and non-traded good prices. The half-lives of the real exchange rates based on traded good price indices are expected to be shorter than those based on non-traded good or general price indices. An extreme case of this proposition is that the half-lives of the real exchange rates on traded good price indices are finite because the real exchange rates are stationary, but the half-lives of the real exchange rates based on non-traded and general price indices are infinite because these real exchange rates are nonstationary.

The empirical evidence is mixed for this extreme view. Engel (1999) used a variance decomposition method to find how much variation in the real exchange rate can be explained by the variance in the relative price of the non-traded and traded goods. Under the extreme view, the relative price component will explain 100 percent of the real exchange rate volatility in the long run. Engel (1999) uses several measures of the traded good prices including PPI, and finds no evidence that the relative price component explains most of the real exchange rate volatility at any time horizon he tries. In contrast, Kakkar and Ogaki (1999) use the wholesale price index (WPI), CPI, and GDP deflator as traded, non-traded, and general prices, and find empirical evidence that is consistent with the view that the real exchange rate based on PPI is stationary. Kim (1990) and Ito (1997) also find more favorable evidence for long-run PPP for the WPI based real exchange rate than for the CPI based real exchange rate.

One interpretation for these mixed results is that Engel's (1999) variance decomposition method is not very informative for long-run horizons because his method is designed to be applicable for both short-run and long-run horizons unlike Kakkar and Ogaki's (1999) and Kim's (1990) long-run methods. In this paper, we consider a less extreme view of shorter half-lives for real exchange rates based on traded good price indices compared with those for real exchange rates based on non-traded good and general price indices.

In this paper, we estimate half-lives of real exchange rates based on the PPI, CPI, and GDP deflator. Even though it is ideal to have pure traded and non-traded good price indices that cover all goods for our empirical work, it is impossible to find such price indices. For example, there is a non-traded component in an imported car price because of domestic retailing service. Service consumption is often treated as non-traded, but some types of legal services are traded across borders. The idea behind our use of the CPI, PPI, and GDP deflator is that the traded good component of the PPI is proportionately larger than that of the CPI and the GDP deflator. If the convergence rate is faster for traded good prices than that for non-traded good prices, half-lives for the real exchange rate based on the PPI should be shorter than those for the real exchange rate based on the CPI and that based on the GDP deflator. Because non-traded good components are considered to be important in consumer goods compared with producer goods, the PPI and the WPI have also been used as a traded good price index in the PPP literature (see, e.g., Kakkar and Ogaki, 1999).

Kim (2003) applied the same method as in this paper to a different data set. Kim followed Stockman and Tesar (1995) and used the implicit deflators of non-service consumption and service consumption classified by type and total consumption deflators to construct the real exchange rate for traded, non-traded, and general prices, respectively. The countries used in his study of bilateral exchange rates were Canada, France, Italy, Japan, Sweden, the United Kingdom, and the United States. Kim (2003) used each of the seven currencies alternatively as the base currency in his empirical work.

In the exchange rate model we consider, there are two goods. The only conditions required for the model are that the long-run PPP holds for one of the goods, and that the two goods cover all the goods relevant for the money demand. Therefore, we do not need pure tradable and non-tradable price indices for our empirical work.

The seven countries included in our study are Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States. It is of interest to use different measures of traded and non-traded good prices. Moreover, Kim's (2003) data set does not include Germany because the data are not available. Therefore, it is of interest to compare our results with Kim's. When the system method is applied, our point estimates indicate shorter half-lives for PPI than for CPI and GDP deflators. This result is consistent with Kim's (2003).

2. An Exchange Rate Model with Sticky Prices

2.1. The Gradual Adjustment Equation

Let p_t^T be the log domestic traded goods price level, p_t^{T*} be the log foreign traded goods price level, and e_t be the log nominal exchange rate. We assume that the three variables, p_t^T, p_t^{T*} and e_t are first difference stationary and that PPP holds in the long run, so that the real exchange rate defined by p_t^T - p_t^{T*} - e_t is stationary, or $\boldsymbol{y_t} = (p_t^T, p_t^{T*}, e_t)'$ is cointegrated with a cointegrating vector (1, -1, -1). Let $\mu = E[p_t^T - p_t^{T*} - e_t]$, then μ can be nonzero when different units are used to measure the price levels in the two countries.

To derive the form of a structural ECM, we consider an exchange rate model with sticky prices. We employ Mussa's (1982) model, which may be viewed as a stochastic discrete time version of Dornbush's (1976) model, in which the domestic price of traded goods is assumed to be sticky in the short run and adjust gradually to its long-run equilibrium level determined by PPP with rational expectations.

Employing Mussa's (1982) model, the domestic price of traded goods is assumed to be sticky in the short run and adjust gradually to its equilibrium level in the long run through

$$\Delta p^T_{t+1} = b[\mu + p^{T*}_t + e_t - p^T_t] + E[p^{T*}_{t+1} + e_{t+1} | I_t] - [p^{T*}_t + e_t] \qquad (1)$$

where $\Delta x_{t+1} = x_{t+1} - x_t$ for any variable x_t, $E[\cdot | I_t]$ is the expectation operator conditional on the information, I_t, available to the economic agents at time t, and b is a short-run adjustment coefficient which is a positive constant, $b < 1$. Based on Mussa (1982), the main idea behind equation (1) is that the price level of domestic traded goods adjusts slowly toward its long-run PPP level (i.e. the long-run equilibrium level) of $p^{T*}_t + e_t$. The short-run adjustment speed is slow when b is close to zero, and the adjustment speed is fast when b is close to one. From equation (1), we have

$$\Delta p^T_{t+1} = d + b[p^{T*}_t + e_t - p^T_t] + \Delta p^{T*}_{t+1} + \Delta e_{t+1} + \varepsilon_{t+1} \qquad (2)$$

where $d = b\mu$, and $\varepsilon_{t+1} = E[p^{T*}_{t+1} + e_{t+1}] - [p^{T*}_{t+1} + e_{t+1}]$. Thus, ε_{t+1} is a one period ahead forecasting error, and $E[\varepsilon_{t+1} | I_t]=0$. Equation (2) motivates the form of the structural ECM employed in this paper, and it can be referred as the structural gradual adjustment equation. In the application of this paper, the gradual adjustment equation implies the first order autoregression structure for the real exchange rate defined by traded good prices. To see this, let $s_t = p^{T*}_t + e_t - p^T_t$ be the log real exchange rate. Then equation (2) implies

$$s_{t+1} = -d + (1-b)\, s_t - \varepsilon_{t+1}. \qquad (3)$$

We define the half-life of the log real exchange rate as the number of periods required for a unit shock to dissipate by one half in this first order autoregression.

2.2. The Exchange Rate under Rational Expectations

To obtain a solution for the nominal exchange rate and the domestic traded good in terms of other variables, we now consider the money demand equation and the Uncovered Interest Parity condition. The money demand depends on the general price level rather than the traded good price. The general price level is assumed to be a weighted average of the prices of the traded and non-traded goods. Let

$$P_t = (1-\alpha)\, p_t^T + \alpha\, p_t^N \tag{4}$$

$$P_t^* = (1-\alpha^*)\, p_t^{T*} + \alpha^*\, p_t^{N*} \tag{5}$$

$$m_t = k + P_t - hi_t \tag{6}$$

$$i_t = i_t^* + E[e_{t+1} \mid I_t] - e_t \tag{7}$$

where p_t^N is the log of the price of non-traded goods and p_t^T is the log of the price of traded goods with weights α and $(1-\alpha)$, respectively. The m_t is the log nominal money supply minus the log real national income, i_t is the nominal interest rate in the domestic country, and i_t^* is the nominal interest rate in the foreign country. In (6), we are assuming that the income elasticity of money demand is one. From (4), (5), (6) and (7), we obtain

$$E[e_{t+1} | I_t] - e_t = (1/h)[(1-\alpha)p_t^T - \omega - h(1-\alpha)\{E[p_{t+1}^{T*} - p_t^{T*} | I_t]\}] \tag{8}$$

where $\omega = m_t - k + h r_t^* + \alpha p_t^N$ and $r^* = i_t^* - (1 - \alpha^*)\{E[p_{t+1}^{T*}|I_t] - p_t^{T*}\}$

Following Mussa, solving (1) and (8) as a system of stochastic difference equations for $E[p_{t+j}^T / I_t]$ and $E[e_{t+j} / I_t]$ for a fixed t yields

$$p_t^T = E[F_t \mid I_{t-1}] - \sum_{j=1}^{\infty}(1-b)^j \{E[F_{t-j} \mid I_{t-j}] - E[F_{t-j} \mid I_{t-j-1}]\} \tag{9}$$

$$e_t = \frac{bh + (1-\alpha)}{bh} E[F_t \mid I_t] - p_t^{T*} - \frac{(1-\alpha)}{bh} p_t^T \tag{10}$$

where

$$F_t = (1-\delta)\sum_{j=0}^{\infty}\delta^j \psi_{t+j} \tag{11}$$

$\delta = h / (h + 1 - \alpha)$ and

$$\psi_{t+j} = m_t - k - h r_t^* + \alpha p_t^N + h\alpha^* \{E[p_{t+1}^{T*} \mid I_t] - p_t^{T*}\} \tag{12}$$

$$r_t^* = i_t^* - (1-\alpha^*)\{E[p_{t+1}^{T*} \mid I_t] - p_t^{T*}\}$$

We assume that $\psi(t)$ is first difference stationary. Since δ is a positive constant smaller than one, this implies that F_t is also first difference stationary. From (9) and (10),

$$e_t + p_t^{T*} - p_t^T = \frac{bh + (1-\alpha)}{bh}\sum_{j=1}^{\infty}(1-b)^j\{E[F_{t-j} \mid I_{t-j}] - E[F_{t-j} \mid I_{t-j-1}]\} \tag{13}$$

Since the right hand side of (13) is stationary[1], $e_t + p_t^{T*} - p_t^T$ is stationary. Thus, equation (13) implies that (p_t^T, e_t, p_t^{T*}) is cointegrated, with the cointegrating vector (1, -1, -1).

2.3 Hansen and Sargent's Formula

In this paper, Hansen and Sargent's (1980, 1982) formula for linear rational expectations models is employed to obtain a structural ECM representation from the exchange rate model. From (10), we obtain

$$\Delta e_{t+1} = \frac{bh + (1-\alpha)}{bh}(1-\delta)E[\sum_{j=0}^{\infty}\delta^j \Delta\psi_{t+j+1} \mid I_t] - \frac{(1-\alpha)}{bh}\Delta p_{t+1}^T - \Delta p_{t+1}^{T*} + \varepsilon_{e,t+1} \tag{14}$$

where $\varepsilon_{e,t+1} = \frac{bh + (1-\alpha)}{bh}\{E[F_{t+1} \mid I_{t+1}] - E[F_{t+1} \mid I_t]\}$, so that the law of iterated expectations implies $E[\varepsilon_{e,t+1} \mid I_t] = 0$. Because this equation involves a discounted sum of expected future values of $\Delta\psi_t$, the system method using Hansen and Sargent's (1982) method is applicable.

[1] This assumes that $E_t[F_t]-E_{t-1}[F_t]$ is stationary, which is true for a large class of first difference stationary variable F_t and information sets.

Hansen and Sargent (1982) propose to project the conditional expectation of the discounted sum, $E[\Sigma\delta^j \Delta\psi_{t+j+1} | I_t]$, onto an information set H_t, the econometrician's information set at t, which is a subset of I_t, the economic agents' information set. Let $\hat{E}[\cdot | H_t]$ be the linear projection operator, conditional on the information set H_t.

We take the econometrician's information set at t, H_t, to be the one generated by linear functions of the current and past values of Δp_t^{T*}. Then, replacing the best forecast of the economic agents, $E[\Sigma\delta^j \Delta\psi_{t+j+1} | I_t]$ by the econometrician's linear forecast based on H_t in equation (14), we obtain

$$\Delta e_{t+1} = \frac{bh + (1-\alpha)}{bh}(1-\delta)\hat{E}[\sum_{j=0}^{\infty}\delta^j \Delta\psi_{t+j+1} | H_t] - \frac{(1-\alpha)}{bh}\Delta p_{t+1}^{T} - \Delta p_{t+1}^{T*} + u_{2,t+1} \quad (15)$$

where

$$u_{2,t+1} = \varepsilon_{e,t+1} + \frac{bh + (1-\alpha)}{bh}(1-\delta)[\{E[\sum \delta^j \Delta\psi_{t+j+1} | I_t] - \hat{E}[\sum \delta^j \Delta\psi_{t+j+1} | H_t]\}]$$

Because H_t is a subset of I_t, we obtain $\hat{E}[u_{2,t+1} | H_t] = 0$.

Since $\hat{E}[\cdot | H_t]$ is the linear projection operator onto H_t, there exist possibly infinite order lag polynomials $\beta(L)$, $\gamma(L)$, and $\xi(L)$, such that

$$\hat{E}[\Delta p_{t+1}^{T*} | H_t] = \beta(L)\Delta p_t^{T*} \quad (16)$$

$$\hat{E}[\Delta\psi_{t+1} | H_t] = \gamma(L)\Delta p_t^{T*} \quad (17)$$

$$\hat{E}[\sum_{j=0}^{\infty}\delta^j \Delta\psi_{t+j+1} | H_t] = \xi(L)\Delta p_t^{T*} \quad (18)$$

Then, following Hansen and Sargent (1980, appendix A), we obtain the restrictions imposed by (15) on $\xi(L)$:

$$\xi(L) = \frac{\gamma(L) - \delta L^{-1}\gamma(\delta)\{1-\delta\beta(\delta)\}^{-1}\{1-L\beta(L)\}}{1-\delta L^{-1}} \tag{19}$$

Assume that linear projections of Δp_{t+1}^{T*} and $\Delta\psi_{t+1}$ onto H_t have only a finite number of Δp_t^{T*} terms:

$$\hat{E}[\Delta p_{t+1}^{T*} \mid H_t] = \beta_1 \Delta p_t^{T*} + \beta_2 \Delta p_{t-1}^{T*} + \ldots + \beta_p \Delta p_{t-p+1}^{T*} \tag{20}$$

$$\hat{E}[\Delta\psi_{t+1} \mid H_t] = \gamma_1 \Delta p_t^{T*} + \gamma_2 \Delta p_{t-1}^{T*} + \ldots + \gamma_{p-1} \Delta p_{t-p+2}^{T*} \tag{21}$$

Here, we assume $\beta(L)$ is of order p and $\gamma(L)$ is of order $p-1$ to simplify the exposition, but we do not lose generality because any β_i and γ_i can be zero. Then, as in Hansen and Sargent (1982), (19) implies that $\xi(L) = \xi_0 + \xi_1 L + \ldots + \xi_p L^p$, where

$$\begin{aligned} \xi_0 &= \gamma(\delta)\{1-\delta\beta(\delta)\}^{-1} \\ \xi_j &= \delta\gamma(\delta)\{1-\delta\beta(\delta)\}^{-1}(\beta_{j+1} + \delta\beta_{j+2} + \ldots + \delta^{p-j}\beta_p) \\ &\quad + (\gamma_j + \delta\gamma_j + \ldots + \delta^{p-j}\gamma_p) \end{aligned} \tag{22}$$

for $j = 1, \ldots, p$.

Then,

$$\hat{E}[\sum_{j=0}^{\infty} \delta^j \Delta\psi_{t+j+1} \mid H_t] = \xi_1 \Delta p_t^{T*} + \xi_2 \Delta p_{t-1}^{T*} + \ldots + \xi_p \Delta p_{t-p+1}^{T*} \tag{23}$$

Combining (2), (15), (20), and (21) with (23), we obtain a system of four equations:

$$\Delta p_{t+1}^{T} = d + \Delta p_{t+1}^{T*} + \Delta e_{t+1} - b[p_t^T - p_t^{T*} - e_t] + u_{1,t+1} \tag{24}$$

$$\begin{aligned} \Delta e_{t+1} &= -\frac{(1-\alpha)}{bh}\Delta p_t^T - \Delta p_t^{T*} + \\ &\quad \mu\xi_1 \Delta p_t^{T*} + \mu\xi_2 \Delta p_{t-1}^{T*} + \ldots + \mu\xi_p \Delta p_{t-p+1}^{T*} + u_{2,t+1} \end{aligned} \tag{25}$$

$$\Delta p_{t+1}^{T*} = \beta_1 \Delta p_t^{T*} + \beta_2 \Delta p_{t-1}^{T*} + \ldots + \beta_p \Delta p_{t-p+1}^{T*} + u_{3,t+1} \tag{26}$$

$$\Delta\psi_{t+1} = \gamma_1 \Delta p_t^{T*} + \gamma_2 \Delta p_{t-1}^{T*} + \ldots + \gamma_{p-1} \Delta p_{t-p+2}^{T*} + u_{4,t+1} \tag{27}$$

where $\mu = \frac{bh + (1-\alpha)}{bh}(1-\delta)$, and $u_{1,\,t+1} = \varepsilon_{t+1}$.

Given the data for $[\,\Delta p^{T}_{t+1},\ \Delta e_{t+1},\ \Delta p^{T*}_{t+1},\ \Delta\psi_{t+1}]'$, the system method can be applied to these four equations. There exist additional complications for obtaining data for $\Delta\psi_{t+1}$, which will be discussed later in this paper.

3. Structural Models and Error Correction Models

Let $\mathbf{Y_t}$ be an n-dimensional vector of first difference stationary random variables, and assume that there exists ρ linearly independent cointegrating vectors, so that $\mathbf{A'Y_t}$ is stationary, where $\mathbf{A'}$ is a $(\rho \times n)$ matrix of real numbers whose rows are linearly independent cointegrating vectors.

Consider a standard ECM.

$$\mathbf{\Delta Y_{t+1} = k + QA'Y_t + F_1\Delta Y_t + F_2\Delta Y_{t-1} + \ldots + F_p\Delta Y_{t-p+1} + v_{t+1}} \tag{28}$$

where $\mathbf{k}$ is an $(n \times 1)$ vector, $\mathbf{Q}$ is an $(n \times \rho)$ matrix of real numbers, and $\mathbf{v_t}$ is a stationary n-dimensional vector of random variables with $\hat{E}\,[\mathbf{v_{t+1}}|H_t]=0$.

A class of structural models can be written in the following form of a structural ECM:

$$\mathbf{C_0\Delta Y_{t+1} = d + BA'Y_t + C_1\Delta Y_t + C_2\Delta Y_{t-1} + \ldots + C_p\Delta Y_{t-p+1} + u_{t+1}} \tag{29}$$

where $\mathbf{C_i}$ is an $(n \times n)$ matrix, $\mathbf{d}$ is an $(n \times 1)$ vector, and $\mathbf{B}$ is an $(n \times \rho)$ matrix of real numbers. Here, $\mathbf{C_0}$ is a nonsingular matrix of real numbers with ones along its principal diagonal, and $\mathbf{u_t}$ is a stationary n-dimensional vector of random variables with $\hat{E}\,[\mathbf{u_{t+1}}|H_t] = 0$. Even though the cointegrating vectors are not unique, we assume that there is a normalization that uniquely determines $\mathbf{A}$, so that parameters in $\mathbf{B}$ have structural meanings.

The exchange rate model with sticky price can be written in the structural ECM form (29) as in the system of four equations (24)-(27): we have $\mathbf{y_t}$ = [Δp^T_{t+1}, Δe_{t+1}, Δp^{T*}_{t+1} , $\Delta \psi_{t+1}$]′, $\mathbf{B}$ = [-b, 0, 0, 0]′, $\mathbf{A}$ = [1, -1, -1, 0]′,

$$\mathbf{C_0} = \begin{bmatrix} 1 & -1 & -1 & 0 \\ (1-\alpha)/bh & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{30}$$

and

$$\mathbf{C_j} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \mu\xi_j & 0 \\ 0 & 0 & \beta_j & 0 \\ 0 & 0 & \gamma_j & 0 \end{bmatrix} \tag{31}$$

for $j = 1,\ldots,p$

Comparing equation (28) with equation (29), in many applications of standard ECMs given in equation (28), elements in $\mathbf{Q}$ are given structural interpretations as parameters of the speed of adjustment toward the long-run equilibrium represented by $\mathbf{A'Y_t}$. However, if we assume that in equation (29) $\mathbf{C_0}$ is nonsingular, and pre-multiply both sides of (29) by $\mathbf{C}_0^{-1}$, we obtain the standard ECM given in equation (28), where $\mathbf{k}$ = $\mathbf{C}_0^{-1}\mathbf{d}$, $\mathbf{Q} = \mathbf{C}_0^{-1}\mathbf{B}$, $\mathbf{F_i} = \mathbf{C}_0^{-1}\mathbf{C_i}$, and $\mathbf{v_t} = \mathbf{C}_0^{-1}\mathbf{u_t}$. Thus, the standard ECM, estimated by Engle and Granger's (1987) two step method or Johansen's (1988) Maximum Likelihood method, is a reduced form model. Hence, it cannot be used to recover structural parameters in $\mathbf{B}$, nor can the impulse response functions based on $\mathbf{v_t}$ be interpreted in a structural way, unless some restrictions are imposed on $\mathbf{C_0}$.

As in a VAR, various restrictions are possible for $\mathbf{C_0}$. One example is to assume that $\mathbf{C_0}$ is lower triangular. If $\mathbf{C_0}$ is lower triangular, then the first row of **Q** in equation (28) is equal to the first row of **B** in equation (29), and structural parameters in the first row of **B** are estimated by the standard methods to estimate an ECM.

However, in the exchange rate model we present in this paper, we are interested in b that represents a structural parameter. In estimating b in the model, the restriction that $\mathbf{C_0}$ in equation (29) is lower triangular is not attractive. As we can see from equation (30), the structural ECM from the two-good version of the exchange rate model does not satisfy the restriction that $\mathbf{C_0}$ is lower triangular for any ordering of the variables.

Based on equation (30), we can see the relationship between the structural ECM and the reduced form ECM in the exchange rate model. Because

$$\mathbf{C_0^{-1}} = \begin{bmatrix} \frac{bh}{bh+(1-\alpha)} & \frac{bh}{bh+(1-\alpha)} & 0 & 0 \\ -\frac{(1-\alpha)}{bh+(1-\alpha)} & \frac{bh}{bh+(1-\alpha)} & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{32}$$

$\mathbf{Q} = \mathbf{C}_0^{-1}\mathbf{B}$ = $[-b^2h/(bh+(1-\alpha)], b(1-\alpha)/(bh+(1-\alpha)), 0, 0]'$ in the reduced form model, and $\mathbf{B}$ = $[-b, 0, 0, 0]'$ in equation (29). The speed of adjustment coefficient for the domestic price is b in the structural model, while it is $b^2h/[bh+(1-\alpha)]$ in the reduced form model. The error correction term does not appear in the second equation for the exchange rate in the structural ECM, while it appears with the speed of adjustment coefficient of $b(1-\alpha)/[bh+(1-\alpha)]$ in the reduced form model.

4. The System Method

To implement the system method, we need data for $\Delta\psi_t$, which requires knowledge of α and h. To compute α, weights on the non-traded goods, we followed Kakkar and Ogaki (1999). We applied a cointegrating regression of log real exchange rate defined by the GDP deflator onto log relative price in Japan and log relative price in a foreign country to estimate α and α^*.[2] For h, even though h is unknown, a cointegrating regression can be applied to money demand if money demand is stable in the long run, as in Stock and Watson (1993). For this purpose, we augment the model as follows:

$$m_t = k + P_t - hi_t + \eta_{m,t} \tag{33}$$

where $\eta_{m,t}$ is the money demand shock, which is assumed to be stationary, so that money demand is stable.

By redefining m_t as $m_t - \eta_{m,t}$, the same equations as those in section two are obtained. For the measurement of $\Delta\psi_t$, note that the *ex ante* foreign real interest rate can be replaced by the *ex post* foreign real exchange rate because of the law of iterated expectations. Using the money market clearing condition (33) and (12), we obtain

$$\Delta\psi_{t+1} = \Delta P_{t+1} - h\Delta i_{t+1} + h\,\Delta i^*_{t+1} + \alpha\Delta p^N_{t+1} - h(1-2\alpha^*)[\Delta p^{T*}_{t+2} - \Delta p^{T*}_{t+1}] \tag{34}$$

Hence, when h, α, and α^* are obtained, $\Delta\psi_t$ can be obtained from the prices of traded and non-traded goods and interest rate data without data for monetary aggregate and national income.

We have now obtained a system of four equations (24), (25), (26), and (27). Because $E[u_{1,t}/I_t] = 0$, we can choose instrument variables, $z_{1,t}$, for $u_{1,t}$ from I_t and, since

[2] Here, the relative price is CPI/PPI. See Kakkar and Ogaki (1999) for details.

$\hat{E}\,[u_{i,t}/H_t] = 0$, instrumental variables, $z_{i,t}$, for $u_{i,t}$ can be selected from H_t for $i = 2, 3, 4$.

Because the speed of adjustment, b, for p_t^T affects the dynamics of the other variables[3], there are cross-equation restrictions involving b in many applications to the restrictions in (22). Using the moment conditions $E[z_{i,t}\,u_{i,t}] = 0$ for $i = 1,\ldots,4$, we form a GMM estimator, imposing the restrictions from (22) and the other cross-equation restrictions implied by the model.

Given the cointegrating vector, this system method provides more efficient estimators than the single equation method, as long as the restrictions implied by the model are true. On the other hand, the single equation method estimators are more robust because misspecification in the other equations does not affect their consistency. The cross-equation restrictions can be tested by Wald, Likelihood Ratio type, and Lagrange Multiplier tests in a GMM framework (e.g., see Ogaki (1993a)). When the restrictions are nonlinear, Likelihood Ratio type and Lagrange Multiplier tests are known to be more reliable than Wald tests.

5. Empirical Results

In this paper, we use each of the seven currencies alternatively as the base currency. We use PPI, CPI, and the GDP deflators from 1973 Q1 to 2001 Q1 to construct the real exchange rates. The countries are Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States. Both the CPI and PPI are from the OECD Main Economic Indicators. For the PPI, we use the manufacturing industry products in

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[3] Note that only p_t^T adjusts slowly, but b affects the dynamics of other variables because of interactions of p_t^T with those variables.

domestic WPI for Japan, the manufacturing output price for Germany, and the home market (excluding VAT) for Italy, and the domestic market (excluding food, beverages, and tobacco and petroleum) for the United Kingdom. For the GDP deflator, we use data from the OECD Main Economic Indicators except for Japan and Germany. For Japan, the data are obtained from the National Accounts published by the Cabinet Office of the Government of Japan. For Germany, the data published by the Bundesbank are obtained from Data Stream. Monthly average foreign exchange rates with the U.S. dollar as the base currency are from the OECD Main Economic Indicators. To estimate the interest elasticity of money demand, we use the sum of M1 and quasi money as the measure of M2 as the IFS suggests. The three-month T-bill rates are used for the interest rate data, but for Japan three-month deposit rates are employed because Japanese T-bill rates are not available for an early part of the sample.

In the present study, the estimation procedure has two steps. First, we estimate the monetary equilibrium equation using Park's (1992) Canonical Cointegrating Regression (CCR) to obtain the interest elasticity of money demand. Second, the speed of price adjustment is estimated by applying GMM to the structural ECM.

Tables 1 and 2 present the results of cointegrating regression for the money demand equations of the GDP deflator and the weights on the non-traded goods, α and α^*, for each country. We report the third stage estimates of CCR for the coefficients and the fourth stage test results. In Table 1, the deterministic cointegrating restrictions are not rejected for most countries except the U.K. and Japan, and the null of stochastic cointegration is not rejected for most countries with the exception of Canada, Germany, and Japan at the 5 % level of significance. To compute α and α^* in Table 2, weights on the CPI, we followed

Kakkar and Ogaki (1999). The results in Table 2 show that we have not only theoretically correct signs but also the theoretically correct magnitudes for most countries except Italy, whose weight on the CPI has the theoretically incorrect sign. Furthermore, the deterministic cointegration restriction and the stochastic cointegration are not rejected at the 5 percent level for each country.

Tables 3, 4, and 5 report the results of GMM estimation for the PPI, CPI, and GDP deflators using the system method, equations (24)~(27).[4] We also report the estimation results with additional sample period, namely 1973:Q1~ 1990:Q2, to see if German Economic and Monetary Union affects our results. The instrumental variables are Δp^{T*}_{t-3} and Δp^{T*}_{t-4}, which are foreign traded goods prices in all cases.[5] For each country, the estimation results are reported under the assumption that PPP holds in the long run. In the system method, the structural speed of the adjustment coefficient, *b*, appears in two equations: the gradual adjustment equation, (24), and the Hansen-Sargent equation, (27). The model imposes the restriction that the coefficient b in the gradual adjustment equation is the same as the coefficient *b* in the Hansen-Sargent equation. We report results with and without this restriction imposed for the system method of estimation. In the case of unrestricted estimation, $b_{u,hs}$ is the estimate of *b* from the Hansen-Sargent equation, and $b_{u,ga}$ is the estimate of *b* from the gradual adjustment equation. The restricted estimate is denoted by b_r. The likelihood ratio type test statistic denoted by LR is used to test the restriction. In most cases, this restriction is not rejected at the 5 % level. Furthermore, for the test of the Hansen-Sargent restrictions we also report the likelihood ratio type test

[4] For the results of GDP deflators(general prices), we used the system method of Kim, Ogaki, and Yang (2003) for a single-good model.
[5] The selection of the instrumental variables is based on Akaike Information Criteria (AIC).

statistic, denoted by LR1.[6] For all cases the null hypothesis is not rejected at the ten percent level, which is evidence in favor of the Hansen-Sargent restrictions.

To obtain the half-life estimate, we use the restricted estimate of the structural speed of the adjustment coefficient, *b*, in each case. Because 1- *b* is the AR coefficient for the first order AR representation as in equation (3), and because our data are quarterly, the half life is calculated as 0.25 ln*(*0.5*)/*ln*(1-b).* All restricted estimates for the structural speed of the adjustment coefficient have the theoretically correct positive sign. Furthermore, most of them are significant at the 5 percent level.

The results in Table 3 show that the estimated half-lives of the PPI-based real exchange rates range from 0.08 to 0.99 year. All half-life estimates are shorter than one year and much shorter than the consensus of 3-5 years explained by Rogoff (1996) and others.[7] For the GDP deflator-based real exchange rates in Table 5, the estimated half-lives range from 0.16 to 1.48 years. For the CPI-based real exchange rates in Table 4, the half-life estimates fall in the 0.20- to 2.95-year range. When comparing to the adjustment speeds over the full samples to those for subsamples, the results are not very different for the full sample and the subsample.

In most cases, the point estimate for the half-life of the GDP deflator-based real exchange rate is larger than that of the PPI-based real exchange rate and is smaller than that of the CPI-based real exchange rate for each pair of countries. Similarly, in most cases,

[6] This test is done by conducting the likelihood ratio type test comparing the J statistics with the Hansen-Sargent restriction from the linear rational expectations model and unrestricted one with free parameters.
[7] Frankel (1986) uses 116 year long data for the WPI based dollar/pound real exchange rate and reports a half-life of 4.6 years. Abuaf and Jorion (1990) use Lee (1976) data for WPI based real exchange rates for the US and eight countries report 3.3 years of half-lives; Glen (1992) and Cheung and Lai (1993) find similar results with the data. Lothian and Taylor (1996) use two centuries of data for the dollar-pound rate and the franc-pound rate, and find half-lives of 4.7 and 2.5 years, respectively. Diebold, Husted, and Rush (1991), using data for the gold standard period, find an average half-life of 2.8 years.

the standard error for the half-life of the GDP deflator-based real exchange rate is larger than that of the PPI-based real exchange rate, and is smaller than that of the CPI-based real exchange rate.

6. Conclusions

In this paper, we used a system method based on a structural ECM to estimate half-lives of PPI-, CPI-, and GDP deflator-based real exchange rates for the G-7 countries. The empirical results in this paper can be summarized in three ways. First, our results indicate that the system method based on a structural ECM provide uniformly shorter half-lives than the consensus of 3-5 years explained by Rogoff (1996). They also show that all of our half-life estimates for the PPI-based real exchange rates are less than one year. For each country, the point estimate for the half-life of the GDP deflator-based real exchange rate is larger than that of the PPI-based real exchange rate and is smaller than that of the CPI-based real exchange rate. Even for the CPI-based real exchange rate, our estimates of the half-lives range from 0.20 to 2.95 years.

Some recent studies, using producer price indices and tradable sector deflators, which apply panel unit root tests to real exchange rates, report strong evidence against the unit root null and estimate the half-life of PPP deviation to be 3-5 years.[8] Note that, even for the rates of traded goods, this remarkable consensus of 3-5 year half-life is the same as that found for real exchange rates for general prices in many studies. These studies that

[8] Wu (1996) uses quarterly CPI and WPI, and reports a half-life of around 2.5 years. Wei and Parsley (1995) use the tradable sector deflator and find that the half-life of PPP deviation is still 4-5 years. Chinn and Johnston (1996) employ CPI and estimate a cointegrating relationship, and the half-life of deviations from the equilibrium defined by the cointegrating vector is 4-5 years. Papell (1997, 2001), Fleissig and Strauss (2000), and Papell and Theodoridis (2001) find shorter half-lives of 2 to 2.5 years. Murray and Papell (2001) confirm Rogoff's original claim of 3-5 years.

attempt to solve the PPP puzzle of the 3-5 year half-life typically conduct Dickey-Fuller or Augmented Dickey-Fuller regression, and the half-life is calculated from the coefficient of the lagged real exchange rate. However, this suggests that the point estimates and the empirical results from the univariate methods may not provide the structural interpretation of the adjustment speed and the half-life of PPP deviation.

Second, our results indicate that a sharper estimation of the half-life is possible when we use price indices with large traded good price components together with a system method for each country. This is because the standard error for the half-life of the GDP deflator-based real exchange rate is larger than that of the PPI-based real exchange rate, and is smaller than that of the CPI-based real exchange rate.

Third, our estimates suggest that theories of international price determination should treat traded and non-traded goods differently to match their differential convergence rates. All of the European and other real exchange rates for PPI show that their half-lives tend to be shorter than those for the GDP deflator and CPI. These real exchange rates for PPI are among the most likely to exhibit evidence of short-run and long-run PPP, because trade between European countries as well as major trading partners has relatively low transaction costs and relatively stable non-tariff barriers to trade. This result is interesting, because it confirms that traded goods prices tend to adjust faster than general prices and non-traded goods prices, implying shorter half-lives for PPI-based rates than for general prices and CPI-based rates. Moreover, it may be that traded good convergence rates are more plausible estimates of the impact of nominal rigidities while considerations such as international factor immobility and non-traded components of goods prices are important for the dynamic behavior of the overall price index.

Among the three indices used in this paper, the non-traded good price component in the CPI is considered to be the largest and the traded good price component in the PPI is considered to be the largest. This observation readily explain our result that the half-life of the real exchange rate is the longest when the CPI is used, and the shortest when the PPI is used. Our result is consistent with the results regarding the long-run PPP in Kim (1990), Ito (1997), Kakkar and Ogaki (1999), and Kim (2003). Our result is in contrast with Engel's (1999) results that find no evidence for faster convergence to the PPP level for the PPI-based real exchange rates compared with the CPI-based real exchange rates.

In future work, we plan to relax the UIP assumption. For example, in Lim and Ogaki's (2003) model, the UIP essentially holds for the long-term interest rate differential, but the forward premium anomaly exists for the short-term interest differential. It may be possible to develop a system method based on the UIP for the long-term interest rate differential.

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Table 1. Money Demand Equation of GDP Deflator

Country	h[a]	H(0, 1)[b]	H(1, 2)[c]	H(1, 3)[d]
Canada	25.0011	0.0217	9.5442	14.2803
	(9.2315)	(0.8828)	(0.0020)	(0.0007)
France	14.5611	2.5667	1.9930	3.0491
	(3.6431)	(0.1091)	(0.1580)	(0.2177)
Italy	25.6649	3.1159	1.7147	1.9280
	(9.7020)	(0.7752)	(0.1903)	(0.3813)
Japan	6.125	5.071	4.160	10.787
	(2.198)	(0.024)	(0.041)	(0.004)
Germany	18.7127	3.6041	5.5793	8.9326
	(4.7219)	(0.0576)	(0.0181)	(0.0114)
U.K.	9.6147	14.8621	0.0923	0.8609
	(5.6209)	(0.0001)	(0.7612)	(0.6501)
U.S.	49.9178	0.0709	2.2288	2.4315
	(15.8720)	(0.7899)	(0.1354)	(0.2964)

Note: Results for $m_t = k + P_t - hi_t + \eta_{m,t}$
For column (a): Standard errors are in parentheses
For column (b)-(d): P-values are in parentheses.

Table 2. Weight on CPI, α and α^*

Country	α[a]	α^*[a]	H(0, 1)[b]	H(1, 2)[c]	H(1, 3)[d]
Canada	0.3794	0.1558	3.5560	1.2379	2.6892
	(0.0825)	(0.0518)	(0.0593)	(0.2658)	(0.2606)
France	0.3222	0.3545	0.0051	0.6855	4.2166
	(0.0748)	(0.0489)	(0.9425)	(0.4076)	(0.1214)
Italy	0.2177	-0.2921	0.9701	0.9740	1.9812
	(0.1329)	(0.1365)	(0.3246)	(0.3236)	(0.3713)
Germany	0.6383	0.0534	3.0071	3.8143	5.2977
	(0.0971)	(0.1159)	(0.0829)	(0.0508)	(0.0707)
U.K.	0.4492	0.8129	3.3913	0.8517	6.5428
	(0.0470)	(0.1069)	(0.0655)	(0.3560)	(0.0379)
U.S.	0.1250	0.4225	1.8838	0.4124	0.7249
	(0.0357)	(0.0800)	(0.1698)	(0.5207)	(0.6959)

Note: Results for $s_t = \theta + \alpha q_t - \alpha^* q_t^* + \xi_t$ where s_t is the log real exchange rate defined GDP deflator, q_t is CPI/PPI in Japan, and q_t^* is CPI/PPI in a foreign country.
For column (a): Standard errors are in parentheses
For column (b)-(d): P-values are in parentheses.

Table 3. System Method Results for PPI-based Real Exchange Rates

Currencies	Half-life (a)	b_r (b)	J_r (c)	$b_{u, hs}$ (d)	$b_{u,ga}$ (e)	J_u (f)	LR (g)	LR1 (h)
US/JP	0.37	0.3702	7.2969	1296.3	0.2363	7.0414	0.2555	1.4913
	(0.0741)	(0.0423)	(0.1210)	(94200)	(0.1900)	(0.0516)	(0.6132)	(0.4744)
UK/JP	0.28	0.4655	5.1995	-0.0754	1.1210	2.7345	2.4650	4.2922
	(0.1281)	(0.1818)	(0.2674)	(0.6702)	(0.5284)	(0.4344)	(0.1164)	(0.1169)
CA/JP	0.41	0.3427	2.5427	-1.2754	0.2012	1.8432	0.6995	0.1539
	(0.2186)	(0.0932)	(0.6370)	(0.7582)	(0.2313)	(0.6055)	(0.4029)	(0.9259)
FR/JP	0.90	0.1749	2.7932	0.1295	0.1933	1.9487	0.8445	1.4011
	(0.1438)	(0.0059)	(0.5930)	(0.1594)	(0.2314)	(0.5832)	(0.3581)	(0.4963)
GE/JP	0.30	0.4428	2.0862	0.1237	0.4327	1.8297	0.2565	0.0966
	(0.1084)	(0.1252)	(0.7199)	(0.1252)	(0.4404)	(0.6084)	(0.6125)	(0.9528)
(73:I~90:II)	0.26	0.4882	1.7985	0.3890	0.9866	0.4016	1.3969	0.0585
	(0.0670)	(0.1163)	(0.7727)	(0.2097)	(0.4895)	(0.9398)	(0.2372)	(0.9711)
IT/JP	0.44	0.3247	1.2565	1245.3	0.1638	1.1665	0.0900	0.0411
	(0.4163)	(0.1454)	(0.8687)	(32196)	(0.1298)	(0.7610)	(0.7641)	(0.9796)
UK/US	0.25	0.4969	3.1303	0.0109	0.3691	2.2998	0.8305	1.3382
	(0.0621)	(0.1161)	(0.5362)	(0.0001)	(0.3033)	(0.5125)	(0.3622)	(0.5121)
JP/US	0.65	0.2332	2.7011	622.51	0.0385	1.2270	1.4741	0.3288
	(0.9385)	(0.1014)	(0.6090)	(17225)	(0.0985)	(0.7465)	(0.2247)	(0.8484)
CA/US	0.99	0.1599	1.3904	-1280.2	1.4175	1.0897	0.3007	0.0254
	(1.4152)	(0.0432)	(0.8458)	(24475)	(0.7394)	(0.7671)	(0.5834)	(0.9873)
FR/US	0.10	0.8282	3.2311	0.5167	0.7541	2.3257	0.9054	0.1544
	(0.0061)	(0.1951)	(0.5199)	(0.0455)	(0.2174)	(0.5076)	(0.3413)	(0.9257)
GE/US	0.53	0.2767	4.7774	441.51	1.1513	1.8802	2.8972	0.6221
	(0.6510)	(0.1277)	(0.3109)	(69490)	(0.2438)	(0.5976)	(0.0887)	(0.7326)
(73:I~90:II)	0.46	0.3126	3.5341	0.3308	3.8195	1.9107	1.6234	1.8490
	(0.3053)	(0.0929)	(0.4726)	(0.1172)	(2.5034)	(0.5911)	(0.2026)	(0.3967)
IT/US	0.12	0.7613	0.3345	0.0139	2.2643	0.2137	0.1208	0.1641
	(0.0130)	(0.2207)	(0.9874)	(0.0109)	(1.6649)	(0.9753)	(0.7281)	(0.9212)
US/UK	0.48	0.3027	4.0184	0.4131	1.1457	3.2831	0.7353	0.1973
	(0.3801)	(0.1028)	(0.4035)	(0.0225)	(0.2426)	(0.3500)	(0.3911)	(0.9060)
JP/UK	0.20	0.5754	2.0641	1129.6	0.2187	1.9893	0.0748	0.6517
	(0.0472)	(0.1715)	(0.7239)	(66268)	(1.3084)	(0.5746)	(0.7844)	(0.7219)
CA/UK	0.38	0.3646	1.7441	-1.0422	0.3144	1.7209	0.0232	0.2418
	(0.2335)	(0.1257)	(0.7827)	(1.2608)	(0.3579)	(0.6322)	(0.8789)	(0.8861)
FR/UK	0.12	0.7684	2.8361	0.4933	0.9099	2.7174	0.1187	0.6843
	(0.0136)	(0.2465)	(0.5856)	(0.1041)	(0.5688)	(0.4372)	(0.7304)	(0.7102)
GE/UK	0.61	0.2457	4.7501	1.3017	2.5730	3.6556	1.0945	0.5691
	(0.3161)	(0.0490)	(0.3139)	(7.9849)	(0.4883)	(0.3011)	(0.2954)	(0.7523)
(73:I~90:II)	0.59	0.2535	1.7401	-0.0218	-0.0191	1.4433	0.2968	0.8239
	(2.2807)	(0.0604)	(0.7834)	(0.0083)	(0.0267)	(0.6954)	(0.5858)	(0.6623)
IT/UK	0.43	0.3289	5.5458	0.3795	1.0832	3.9265	1.6193	3.8965
	(0.3610)	(0.1322)	(0.2357)	(0.2001)	(0.4262)	(0.2695)	(0.2031)	(0.1425)

Table 3 (continued)

Currencies	Half-life (a)	b_r (b)	J_r (c)	$b_{u, hs}$ (d)	$b_{u,ga}$ (e)	J_u (f)	LR (g)	LR1 (h)
US/CA	0.10	0.8283	3.1364	0.5114	0.8212	2.9511	0.1853	0.6545
	(0.0045)	(0.1449)	(0.5352)	(0.8777)	(0.1642)	(0.3992)	(0.6668)	(0.7209)
UK/CA	0.23	0.5339	2.5849	0.3384	2.7806	1.5497	1.0352	0.1972
	(0.0420)	(0.1079)	(0.6294)	(0.2081)	(1.3857)	(0.6708)	(0.3089)	(0.9061)
JP/CA	0.13	0.7454	1.1938	0.1412	0.1783	0.3757	0.8181	0.3393
	(0.0169)	(0.2511)	(0.8791)	(0.4599)	(1.4478)	(0.9451)	(0.3657)	(0.8439)
FR/CA	0.19	0.6004	2.4459	0.3637	0.1796	2.4453	0.0006	0.5578
	(0.0189)	(0.0846)	(0.6543)	(0.0021)	(0.0434)	(0.4952)	(0.9804)	(0.7566)
GE/CA	0.19	0.6071	3.9237	0.3658	0.5451	3.3889	0.5348	0.4599
	(0.0499)	(0.2352)	(0.4164)	(0.1039)	(0.2661)	(0.3354)	(0.4645)	(0.7945)
(73:I~90:II)	0.17	0.6349	0.1021	351.37	2.1037	0.0972	0.0049	0.0061
	(0.0433)	(0.2560)	(0.9987)	(27572)	(2.0078)	(0.9921)	(0.9441)	(0.9969)
IT/CA	0.75	0.2059	3.5158	0.3265	0.9645	1.7766	1.7392	0.4566
	(0.7989)	(0.0565)	(0.4754)	(0.1338)	(0.5885)	(0.6200)	(0.1872)	(0.7958)
US/FR	0.62	0.2443	1.5127	0.4432	0.3883	1.2524	0.2603	0.8847
	(0.0039)	(0.0005)	(0.8243)	(0.0001)	(0.0007)	(0.7404)	(0.6099)	(0.6425)
UK/FR	0.11	0.8004	3.6716	0.2242	0.1009	2.8518	0.8198	0.6160
	(0.0026)	(0.0648)	(0.4522)	(0.0311)	(0.1853)	(0.4150)	(0.3652)	(0.7349)
CA/FR	0.14	0.6989	4.2224	0.1377	2.7532	1.7645	2.4579	3.2987
	(0.0093)	(0.0931)	(0.3767)	(0.0105)	(0.9398)	(0.6228)	(0.1169)	(0.1921)
JP/FR	0.48	0.3033	2.1432	-0.3901	127.22	1.2475	0.8957	1.5348
	(0.4434)	(0.1208)	(0.7094)	(0.7190)	(293.97)	(0.7416)	(0.3439)	(0.4642)
GE/FR	0.08	0.8752	5.1450	0.5112	-4.0668	2.9261	2.2189	2.3269
	(0.0033)	(0.1721)	(0.2727)	(0.7792)	(7.6425)	(0.4017)	(0.1363)	(0.3124)
(73:I~90:II)	0.08	0.8864	3.9876	0.8204	0.4413	3.0440	0.9436	1.3531
	(0.0026)	(0.1561)	(0.4076)	(1.5587)	(1.5096)	(0.3848)	(0.3313)	(0.5083)
IT/FR	0.24	0.5178	1.6998	523.14	0.6511	1.6692	0.0306	0.3552
	(0.0464)	(0.1041)	(0.7907)	(65593)	(0.3211)	(0.6437)	(0.8611)	(0.8372)
US/IT	0.31	0.4301	2.8384	0.3189	1.7063	1.7928	1.0456	1.1859
	(0.1258)	(0.1291)	(0.5852)	(0.0112)	(0.4175)	(0.6164)	(0.3065)	(0.5526)
UK/IT	0.09	0.8512	3.3855	0.8150	1.9965	1.0638	2.3217	0.3902
	(0.0040)	(0.1629)	(0.4954)	(0.4807)	(1.3971)	(0.7858)	(0.1275)	(0.8227)
CA/IT	0.09	0.8575	4.9664	696.81	0.2541	4.1534	0.8130	4.0083
	(0.0032)	(0.1398)	(0.2907)	(61899)	(0.1406)	(0.2453)	(0.3672)	(0.1347)
FR/IT	0.98	0.1619	3.1431	0.5395	0.8451	2.2176	0.9255	2.4558
	(1.7330)	(0.0551)	(0.5341)	(0.0068)	(0.3644)	(0.5284)	(0.3360)	(02929)
GE/IT	0.15	0.6977	0.5188	-0.1207	0.6956	0.5167	0.0021	0.0801
	(0.0196)	(0.1945)	(0.9716)	(3.3810)	(0.9064)	(0.9151)	(0.9634)	(0.9607)
(73:I~90:II)	0.13	0.7432	0.9648	33.671	-2.0022	0.0691	0.8957	0.5146
	(0.0109)	(0.1593)	(0.9150)	(7745.5)	(7.5968)	(0.9952)	(0.3439)	(0.7731)
JP/IT	0.72	0.2129	1.6755	0.4997	-1.9082	0.6787	0.9968	0.5445
	(0.6756)	(0.0535)	(0.7951)	(0.9985)	(3.1687)	(0.8781)	(0.3180)	(0.7616)

Table 3 (continued)

Currencies	Half-life (a)	b_r (b)	J_r (c)	$b_{u, hs}$ (d)	$b_{u,ga}$ (e)	J_u (f)	LR (g)	LR1 (h)
US/GE	0.26	0.4893	5.4271	0.1796	2.5235	2.4944	2.9327	2.0976
	(0.0623)	(0.1092)	(0.2462)	(0.0991)	(0.5931)	(0.4762)	(0.0868)	(0.3503)
(73:I~90:II)	0.24	0.5183	5.0626	-1.1395	3.3927	4.4012	0.6614	0.2164
	(0.0549)	(0.1235)	(0.2809)	(4.0909)	(2.8308)	(0.2212)	(0.4160)	(0.3301)
UK/GE	0.18	0.6280	2.1148	0.1259	0.7416	2.0431	0.0717	1.5046
	(0.0455)	(0.2542)	(0.7146)	(0.1271)	(0.5005)	(0.5635)	(0.7888)	(0.4712)
(73:I~90:II)	0.15	0.6833	2.0224	654.06	0.1802	1.5121	0.5103	0.9025
	(0.0231)	(0.2027)	(0.7316)	(61983)	(0.4877)	(0.6794)	(0.4750)	(0.6368)
CA/GE	0.30	0.4399	2.6439	0.5332	2.4444	2.4751	0.1688	1.9474
	(0.1272)	(0.1430)	(0.6190)	(0.2130)	(0.9530)	(0.4798)	(0.6811)	(0.3776)
(73:I~90:II)	0.25	0.5019	6.7973	0.9761	2.9121	6.0932	0.7041	2.6732
	(0.0822)	(0.1606)	(0.1469)	(6.1268)	(2.3559)	(0.1071)	(0.4014)	(0.2627)
FR/GE	0.55	0.2687	3.3404	0.0561	0.1762	3.0777	0.2627	1.5551
	(0.6163)	(0.1090)	(0.5025)	(0.0344)	(0.2603)	(0.3797)	(0.6082)	(0.4595)
(73:I~90:II)	0.47	0.3064	3.4510	0.0224	0.4879	1.7541	1.6969	1.2838
	(0.4338)	(0.1226)	(0.4853)	(0.0185)	(0.7029)	(0.6249)	(0.1926)	(0.5262)
IT/GE	0.17	0.6345	2.2022	0.1875	1.7592	2.2015	0.0007	1.2005
	(0.0725)	(0.4270)	(0.6986)	(0.1130)	(0.4367)	(0.5316)	(0.9788)	(0.5486)
(73:I~90:II)	0.14	0.6925	0.6938	0.7158	0.8009	0.6155	0.0783	0.0607
	(0.0192)	(0.1824)	(0.9520)	(3.6612)	(0.1926)	(0.8928)	(0.7796)	(0.9701)
JP/GE	0.27	0.4762	2.0422	0.0461	0.8860	1.4674	0.5748	0.1699
	(0.0380)	(0.0594)	(0.7279)	(0.0106)	(0.7989)	(0.6897)	(0.4483)	(0.9185)
(73:I~90:II)	0.25	0.5036	1.0722	-0.5070	0.7206	1.0414	0.0308	0.2226
	(0.0801)	(0.1590)	(0.8986)	(13.512)	(0.3724)	(0.7912)	(0.8606)	(0.8946)

Notes: For the unrestricted estimation, $b_{u, hs}$ is the estimate for the speed of adjustment coefficient from the Hansen and Sargent equations, and $b_{u, ga}$ is the estimate for the coefficient obtained from the gradual adjustment equation.
For column (a): Half-life in years.
For columns (a), (b), (d), and (e): Standard errors are in parentheses.
For columns (c), (f), (g), and (h): P-values are in parentheses.

Table 4. System Method Results for CPI-based Real Exchange Rates

Currencies	Half-life (a)	b_r (b)	J_r (c)	$b_{u, hs}$ (d)	$b_{u,ga}$ (e)	J_u (f)	LR (g)	LR1 (h)
US/JP	1.19	0.1351	6.1738	1949.7	3.7470	5.0784	1.0954	3.8260
	(3.4288)	(0.0605)	(0.1865)	(64310)	(2.1497)	(0.1661)	(0.2952)	(0.1476)
UK/JP	0.82	0.1896	2.3271	558.28	-0.2171	2.1092	0.2179	0.1517
	(1.7569)	(0.0942)	(0.6758)	(36530)	(0.2369)	(0.5501)	(0.6406)	(0.9269)
CA/JP	1.66	0.0989	7.0424	0.2427	0.9030	6.7110	0.3314	5.2770
	(5.5390)	(0.0361)	(0.1336)	(0.3716)	(0.7310)	(0.0817)	(0.5648)	(0.0714)
FR/JP	1.29	0.1258	1.9740	0.1624	0.1558	1.8241	0.1499	1.2650
	(5.8398)	(0.0819)	(0.7405)	(0.4519)	(0.1222)	(0.6096)	(0.6986)	(0.5312)
GE/JP	1.19	0.1356	5.3413	0.3028	0.0102	4.5505	0.7908	3.5360
	(17.932)	(0.3202)	(0.2540)	(0.1936)	(0.3029)	(0.2078)	(0.3738)	(0.1706)
(73:I~90:II)	1.36	0.1199	2.8838	0.2248	1.3528	2.4896	0.3942	0.5485
	(9.3155)	(0.1120)	(0.5774)	(0.1286)	(0.5503)	(0.4771)	(0.5301)	(0.7601)
IT/JP	0.76	0.2027	1.8546	-0.1865	0.6647	1.0068	0.8478	1.2603
	(2.1676)	(0.1998)	(0.7624)	(0.1547)	(0.1265)	(0.7996)	(0.3571)	(0.5325)
JP/US	1.16	0.1383	2.8681	0.0899	1.2967	1.0057	1.8624	1.1122
	(3.0318)	(0.0577)	(0.5801)	(0.0105)	(1.1205)	(0.7998)	(0.1723)	(0.5734)
UK/US	1.04	0.1535	5.4605	3.3169	-2.7018	4.2778	1.1827	0.6494
	(1.6176)	(0.0432)	(0.2432)	(4.6182)	(3.3162)	(0.2329)	(0.2768)	(0.7227)
CA/US	1.38	0.1179	1.0369	1.1506	1.1020	1.0238	0.0131	0.7055
	(5.6261)	(0.0641)	(0.9042)	(1.5315)	(0.3963)	(0.7954)	(0.9088)	(0.7027)
FR/US	1.57	0.1045	0.7785	0.1020	1.1107	0.6121	0.1664	0.4167
	(9.2533)	(0.0718)	(0.9413)	(0.1853)	(0.6502)	(0.8936)	(0.6833)	(0.8119)
GE/US	1.57	0.1044	2.1670	0.0479	0.0551	2.1317	0.0353	1.3882
	(5.2483)	(0.0406)	(0.7051)	(0.0378)	(0.2605)	(0.5455)	(0.8509)	(0.4995)
(73:I~90:II)	0.94	0.1682	1.9856	-2584.3	-0.2940	1.3685	0.6171	1.0587
	(1.9142)	(0.0690)	(0.7384)	(16723)	(0.9954)	(0.7129)	(0.4321)	(0.5889)
IT/US	0.25	0.5028	1.1190	0.1195	0.7706	1.0497	0.0693	0.3209
	(0.1342)	(0.2643)	(0.8912)	(0.2691)	(0.9636)	(0.7892)	(0.7923)	(0.8517)
US/UK	2.76	0.0608	1.3352	0.0614	0.8852	1.0334	0.3018	0.1545
	(3.7913)	(0.0054)	(0.8553)	(0.0059)	(0.1942)	(0.7931)	(0.5827)	(0.9256)
JP/UK	2.57	0.0652	4.2183	0.0586	0.7119	2.8596	1.3587	2.6621
	(0.0565)	(0.0001)	(0.3772)	(0.0251)	(0.7019)	(0.4137)	(0.2437)	(0.2642)
CA/UK	2.57	0.0653	5.8054	0.0744	0.9760	5.2874	0.5180	4.0009
	(0.2250)	(0.0004)	(0.2141)	(0.0097)	(0.2299)	(0.1519)	(0.4716)	(0.1352)
FR/UK	2.27	0.0735	6.1451	0.0631	1.2141	5.4676	0.6775	1.2051
	(0.0389)	(0.0001)	(0.1885)	(0.0011)	(0.3871)	(0.1406)	(0.4104)	(0.5474)
GE/UK	2.53	0.0662	1.4915	0.0655	0.0505	0.9609	0.5306	0.3785
	(0.5932)	(0.0011)	(0.8281)	(0.0003)	(0.0444)	(0.8107)	(0.4663)	(0.8275)
(73:I~90:II)	1.76	0.0937	3.3757	0.1604	0.2483	3.0385	0.3372	2.2237
	(8.66)	(0.0476)	(0.4970)	(0.0266)	(0.1455)	(0.3857)	(0.5614)	(0.3289)
IT/UK	2.58	0.0649	3.1902	3.0254	0.4809	2.1359	1.0543	1.5759
	(0.2294)	(0.0004)	(0.5265)	(3.5186)	(0.6287)	(0.5446)	(0.3045)	(0.4547)

Table 4 (continued)

Currencies	Half-life (a)	b_r (b)	J_r (c)	$b_{u, hs}$ (d)	$b_{u,ga}$ (e)	J_u (f)	LR (g)	LR1 (h)
US/CA	2.95	0.0571	2.0478	0.5779	4.5191	0.6652	1.3826	0.2479
	(9.3785)	(0.0110)	(0.7269)	(1.8963)	(3.1983)	(0.8813)	(0.2396)	(0.8834)
UK/CA	2.57	0.0651	1.8403	9.9071	3.9390	0.9687	0.8716	1.1620
	(7.7260)	(0.0136)	(0.7651)	(74.773)	(2.6286)	(0.8088)	(0.3505)	(0.5593)
JP/CA	0.40	0.3528	3.6762	0.1878	0.0677	3.2073	0.4689	0.6533
	(0.3965)	(0.1885)	(0.4515)	(102.94)	(1.8868)	(0.3607)	(0.4934)	(0.7213)
FR/CA	1.15	0.1399	1.4873	0.0171	22.087	1.4843	0.0030	1.0344
	(0.8961)	(0.0177)	(0.8288)	(0.2096)	(17.522)	(0.6858)	(0.9563)	(0.5961)
GE/CA	1.06	0.1504	2.1491	0.0657	15.2830	1.4444	0.7047	1.4612
	(1.5327)	(0.0383)	(0.7083)	(0.4728)	(14.210)	(0.6951)	(0.4012)	(0.4816)
(73:I~90:II)	1.00	0.1589	3.3144	0.2914	13.317	1.8195	1.4949	2.7262
	(1.7623)	(0.0525)	(0.5066)	(0.3000)	(3.1861)	(0.6106)	(0.2214)	(0.2558)
IT/CA	1.02	0.1567	2.4382	-3.0380	1.1590	1.3424	1.0958	0.4620
	(1.3966)	(0.0399)	(0.6557)	(287.22)	(2.4265)	(0.7190)	(0.2951)	(0.7937)
US/FR	0.95	0.1672	1.4751	0.2897	0.6899	0.0188	1.4563	0.1627
	(2.3143)	(0.0818)	(0.8310)	(0.0013)	(1.8124)	(0.9993)	(0.2275)	(0.9218)
UK/FR	0.21	0.5574	3.5303	0.1749	0.8604	1.8452	1.6851	0.1647
	(0.0634)	(0.1983)	(0.4732)	(0.1295)	(0.2260)	(0.6051)	(0.1942)	(0.9209)
CA/FR	0.20	0.5786	3.7024	0.1716	0.8056	2.3339	1.3685	0.1306
	(0.1149)	(0.4282)	(0.4477)	(0.1301)	(0.3449)	(0.5061)	(0.2420)	(0.9367)
JP/FR	2.33	0.0717	6.9931	0.5252	1.0896	5.2276	1.7655	3.2547
	(6.1011)	(0.0145)	(0.1362)	(5.9762)	(1.3579)	(0.1558)	(0.1839)	(0.1964)
GE/FR	0.37	0.3766	4.4322	0.7958	0.7497	4.4180	0.0142	1.8734
	(0.2527)	(0.1539)	(0.3506)	(0.3533)	(0.2773)	(0.2197)	(0.9051)	(0.3919)
(73:I~90:II)	0.41	0.3412	3.8233	0.4274	-0.1016	2.6823	1.1410	1.3258
	(0.4062)	(0.1704)	(0.4304)	(0.8129)	(1.0753)	(0.4432)	(0.2854)	(0.5153)
IT/FR	0.77	0.2024	0.8198	2.5119	3.7747	0.7964	0.0234	0.1417
	(12.669)	(0.8456)	(0.9357)	(2973.3)	(4.5792)	(0.8503)	(0.8784)	(0.9316)
US/IT	1.45	0.1128	1.3408	0.0418	1.3105	1.3105	0.0303	0.1248
	(8.0557)	(0.0797)	(0.8544)	(0.0226)	(0.7266)	(0.7266)	(0.8618)	(0.9395)
UK/IT	0.49	0.3036	2.6693	0.2762	0.6739	2.2634	0.4056	1.1864
	(0.4920)	(0.1345)	(0.6145)	(0.7971)	(0.5354)	(0.5195)	(0.5242)	(0.5525)
CA/IT	0.93	0.1692	3.1206	1.8198	0.1591	2.6676	0.4530	0.6539
	(2.3343)	(0.0858)	(0.5378)	(3.8271)	(0.2528)	(0.4457)	(0.5009)	(0.7211)
FR/IT	1.09	0.1462	2.1628	-0.0585	-47.687	0.3665	1.7963	0.6956
	(3.4976)	(0.0797)	(0.7058)	(0.0075)	(54.134)	(0.9470)	(0.1801)	(0.7062)
GE/IT	0.69	0.2225	6.2024	1.6433	0.7546	3.7277	2.4747	3.7191
	(1.2414)	(0.1142)	(0.1845)	(9.3041)	(0.4395)	(0.2923)	(0.1156)	(0.1557)
(73:I~90:II)	0.53	0.2789	1.2579	0.8764	0.7210	0.4320	0.8259	0.9309
	(1.0483)	(0.2115)	(0.8684)	(3.1149)	(0.2140)	(0.9335)	(0.3634)	(0.6278)
JP/IT	0.98	0.1624	5.7687	0.0792	0.6338	4.1081	1.6606	3.8883
	(0.8842)	(0.0284)	(0.2171)	(0.0319)	(1.2526)	(0.2500)	(0.1975)	(0.1431)

Table 4 (continued)

Currencies	Half-life (a)	b_r (b)	J_r (c)	$b_{u, hs}$ (d)	$b_{u,ga}$ (e)	J_u (f)	LR (g)	LR1 (h)
US/GE	1.09	0.1469	2.5382	0.6637	1.2536	1.4988	1.0394	0.1212
	(3.1887)	(0.0738)	(0.6378)	(209.21)	(0.6651)	(0.6825)	(0.3079)	(0.9412)
(73:I~90:II)	0.80	0.1945	1.5798	0.1641	1.9877	1.3786	0.2012	0.3480
	(0.8648)	(0.0505)	(0.8124)	(0.8533)	(0.5436)	(0.7105)	(0.6537)	(0.8403)
UK/GE	0.52	0.2836	2.0590	0.6985	0.7204	0.7304	1.3286	0.7843
	(0.5801)	(0.1242)	(0.7249)	(18.874)	(0.6872)	(0.8660)	(0.2490)	(0.6756)
(73:I~90:II)	0.47	0.3094	1.9355	-1.7634	0.1206	1.3331	0.6024	0.6804
	(0.5123)	(0.1500)	(0.7476)	(0.5604)	(0.1036)	(0.7213)	(0.4376)	(0.7116)
CA/GE	0.82	0.1901	4.7630	0.7505	1.3543	2.9728	1.7902	2.9589
	(0.6842)	(0.0911)	(0.3124)	(2.8707)	(5.1176)	(0.3958)	(0.1890)	(0.2277)
(73:I~90:II)	0.70	0.2201	4.2978	0.3512	0.3559	3.9716	0.3262	2.5822
	(1.1539)	(0.1023)	(0.3671)	(0.0544)	(0.3868)	(0.2645)	(0.5679)	(0.2749)
FR/GE	0.94	0.1689	4.7637	0.1216	0.4864	4.4388	0.3249	0.4506
	(1.8089)	(0.0661)	(0.3124)	(0.2866)	(0.2665)	(0.2178)	(0.5686)	(0.7982)
(73:I~90:II)	0.74	0.2090	2.2867	0.3087	0.8683	1.5180	0.7687	0.3891
	(0.8484)	(0.0631)	(0.6831)	(0.5935)	(0.5818)	(0.6781)	(0.3806)	(0.8232)
JP/GE	1.19	0.1356	0.2614	0.3752	-6.4276	0.1321	0.1293	0.0695
	(3.4385)	(0.0614)	(0.9921)	(2.6772)	(17.133)	(0.9877)	(0.7191)	(0.9658)
(73:I~90:II)	0.80	0.1947	7.7267	0.2025	-8.4221	6.5928	1.1339	5.1531
	(2.6606)	(0.1559)	(0.1021)	(0.4287)	(29.946)	(0.0860)	(0.2869)	(0.0760)
IT/GE	0.36	0.3785	1.3102	0.9175	-3.5808	0.9339	0.3763	0.1389
	(0.2882)	(0.1790)	(0.8596)	(3.6005)	(3.8845)	(0.8172)	(0.5395)	(0.9329)
(73:1~90:II)	0.32	0.4151	2.2733	0.6759	-2.6007	0.8772	1.3961	1.1734
	(0.1979)	(0.1762)	(0.6856)	(2.8195)	(3.0391)	(0.8309	(0.2373)	(0.5561)

Notes: For the unrestricted estimation, $b_{u, hs}$ is the estimate for the speed of adjustment coefficient from the Hansen and Sargent equations, and $b_{u, ga}$ is the estimate for the coefficient obtained from the gradual adjustment equation.
For column (a): Half-life in years.
For columns (a), (b), (d), and (e): Standard errors are in parentheses.
For columns (c), (f), (g), and (h): P-values are in parentheses.

Table 5. System Method Results for GDP deflator-based Real Exchange Rates

Currencies	Half-life (a)	b_r (b)	J_r (c)	$b_{u, hs}$ (d)	$b_{u,ga}$ (e)	J_u (f)	LR (g)	LR1 (h)
US/JP	0.83	0.1876	4.9088	0.4096	0.0189	2.2986	2.6102	2.9972
	(2.1467)	(0.1111)	(0.2967)	(0.1470)	(0.1330)	(0.5127)	(0.1062)	(0.2234)
UK/JP	0.49	0.2987	2.4589	1.5632	0.8501	0.9685	1.4904	0.6915
	(0.4096)	(0.1056)	(0.6520)	(1.5698)	(0.9983)	(0.8088)	(0.2221)	(0.7076)
CA/JP	0.79	0.1978	0.6574	1.9231	6.4257	0.3883	0.2691	0.2436
	(0.8465)	(0.0523)	(0.9564)	(1.0469)	(16.015)	(0.9426)	(0.6039)	(0.8853)
FR/JP	0.98	0.1625	3.2477	2.4014	0.7707	2.5425	0.7053	0.3638
	(2.9923)	(0.0963)	(0.5172)	(6.9302)	(0.4269)	(0.4676)	(0.4010)	(0.8336)
GE/JP	1.20	0.1344	2.3715	0.8161	1.6872	1.9147	0.4568	0.2595
	(3.5963)	(0.0624)	(0.6677)	(0.7627)	(0.9392)	(0.5902)	(0.4991)	(0.8783)
(73:I~90:II)	0.91	0.1734	4.0708	0.1460	0.7949	3.8065	0.2643	2.2510
	(1.9044)	(0.0759)	(0.3965)	(0.3073)	(0.1196)	(0.2831)	(0.6071)	(0.3244)
IT/JP	0.72	0.2145	3.0014	0.9963	0.7785	2.6664	0.3350	0.5855
	(1.3384)	(0.1087)	(0.5575)	(1.2445)	(0.8298)	(0.4459)	(0.5627)	(0.7462)
JP/US	0.72	0.2136	1.6160	4558.4	-0.3416	0.7416	0.8744	0.3668
	(0.8143)	(0.0652)	(0.8058)	(23229)	(0.3728)	(0.8633)	(0.3497)	(0.8324)
UK/US	0.31	0.4293	2.6708	0.6415	2.1422	2.0267	0.6441	0.2341
	(0.1984)	(0.2021)	(0.6143)	(0.1171)	(0.2286)	(0.5668)	(0.4222)	(0.8895)
CA/US	1.00	0.1585	3.2922	0.7961	-0.0917	2.9578	0.3344	0.8305
	(1.4094)	(0.0418)	(0.5101)	(0.6799)	(0.0945)	(0.3981)	(0.5630)	(0.6601)
FR/US	0.64	0.2383	2.3665	0.1354	0.2121	1.2031	1.1634	0.7600
	(0.7105)	(0.0827)	(0.6686)	(0.6180)	(0.8073)	(0.7522)	(0.2807)	(0.6838)
GE/US	0.63	0.2394	3.5216	0.6966	0.8197	2.2731	1.2485	0.6610
	(0.9572)	(0.1132)	(0.4745)	(0.2931)	(0.1449)	(0.5176)	(0.2638)	(0.7185)
(73:I~90:II)	0.64	0.2369	0.7039	0.6378	0.0807	0.1984	0.5055	0.5551
	(1.3450)	(0.1534)	(0.9508)	(0.3795)	(0.0426)	(0.9778)	(0.4771)	(0.7576)
IT/US	0.66	0.2317	3.4026	6.2793	-1.1198	3.4017	0.0009	1.7606
	(0.7987)	(0.0844)	(0.4928)	(23.570)	(0.8060)	(0.3333)	(0.9760)	(0.4146)
US/UK	1.22	0.1328	3.2272	3.5741	-0.0498	1.9588	1.2684	1.4039
	(2.4320)	(0.0460)	(0.5205)	(14.744)	(0.0767)	(0.5809)	(0.2601)	(0.4956)
JP/UK	0.24	0.5112	4.5418	0.7061	0.6281	2.1117	2.4301	4.3503
	(0.0174)	(0.0370)	(0.3376)	(0.4596)	(0.3133)	(0.5495)	(0.1190)	(0.1135)
CA/UK	1.01	0.1579	4.2186	0.5741	-0.0947	3.7233	0.4953	2.5972
	(1.5397)	(0.0451)	(0.3772)	(18.388)	(0.1035)	(0.2929)	(0.4815)	(0.2729)
FR/UK	0.56	0.2667	2.1831	0.3869	0.2286	2.1807	0.0024	1.0023
	(0.5422)	(0.0934)	(0.7021)	(0.1728)	(0.0907)	(0.5357)	(0.9609)	(0.6058)
GE/UK	0.70	0.2196	4.0965	0.0843	0.1894	3.8182	0.2783	0.3377
	(0.4876)	(0.0429)	(0.3931)	(0.2207)	(0.1505)	(0.2818)	(0.5978)	(0.8446)
(73:I~90:II)	0.58	0.2575	0.5743	0.7230	0.1140	0.1789	0.3954	0.2470
	(1.1457)	(0.1745)	(0.9658)	(0.2074)	(0.0812)	(0.9809)	(0.5294)	(0.8838)
IT/UK	1.16	0.1384	3.3461	49.497	0.1367	2.8353	0.5108	0.5830
	(3.6066)	(0.0688)	(0.5017)	(359.66)	(0.7474)	(0.4177)	(0.4747)	(0.7471)

Table 5 (continued)

Currencies	Half-life (a)	b_r (b)	J_r (c)	$b_{u, hs}$ (d)	$b_{u,ga}$ (e)	J_u (f)	LR (g)	LR1 (h)
US/CA	0.19	0.5932	7.7061	1.2448	0.0798	5.4003	2.3058	4.6131
	(0.0377)	(0.1585)	(0.1029)	(5.7849)	(0.1079)	(0.1447)	(0.5802)	(0.0996)
UK/CA	0.28	0.4594	2.4617	0.3653	0.4402	1.7524	0.7093	1.2915
	(0.1068)	(0.1435)	(0.6514)	(0.0135)	(0.1237)	(0.6253)	(0.3996)	(0.5242)
JP/CA	0.46	0.3160	3.5871	0.4704	0.2425	2.9524	0.6347	1.2011
	(0.2963)	(0.0937)	(0.4647)	(1.2658)	(0.1231)	(0.3990)	(0.4256)	(0.5485)
FR/CA	0.47	0.3103	6.4468	0.1909	-1.7359	4.6651	1.7817	3.4263
	(0.0114)	(0.0034)	(0.1681)	(0.3665)	(0.2935)	(0.1980)	(0.1819)	(0.1803)
GE/CA	0.29	0.4549	3.0699	0.0313	0.1129	1.1905	1.8794	0.1772
	(0.0355)	(0.0458)	(0.5461)	(0.0260)	(0.1213)	(0.7552)	(0.1704)	(0.9152)
(73:I~90:II)	0.21	0.5534	5.6864	2.9685	0.3635	4.5665	1.1199	2.9883
	(0.0666)	(0.2014)	(0.2238)	(5.8580)	(0.2524)	(0.2064)	(0.2899)	(0.2244)
IT/CA	0.35	0.3886	7.3965	2.6228	0.1318	4.4772	2.9193	4.2581
	(0.1268)	(0.0872)	(0.1163)	(8.8875)	(0.1216)	(0.2143)	(0.0875)	(0.1190)
US/FR	0.69	0.2235	4.2316	0.4971	0.0841	4.0893	0.1423	1.3842
	(0.9014)	(0.0842)	(0.3755)	(0.4864)	(0.0956)	(0.2519)	(0.7060)	(0.5005)
UK/FR	0.16	0.6578	3.7001	28.585	0.0921	3.5531	0.1470	0.3819
	(0.0119)	(0.0848)	(0.4480)	(39.607)	(0.0664)	(0.3139)	(0.7014)	(0.8261)
CA/FR	0.20	0.5708	0.4364	0.1513	0.3481	0.3185	0.1179	0.2193
	(0.0505)	(0.1765)	(0.9793)	(3.8506)	(0.9092)	(0.9565)	(0.7313)	(0.8961)
JP/FR	1.48	0.1102	1.3512	1.2438	1.7807	1.1257	0.2255	0.1630
	(10.440)	(0.0959)	(0.8526)	(6.1055)	(4.4802)	(0.7708)	(0.6348)	(0.9217)
GE/FR	0.47	0.3085	1.6911	0.2132	1.3447	0.7821	0.9090	0.4307
	(0.5226)	(0.1514)	(0.7923)	(0.4161)	(0.9176)	(0.8537)	(0.3403)	(0.8062)
(73:I~90:II)	0.44	0.3234	1.1159	0.1470	0.0832	0.4403	0.6756	0.8386
	(0.3937)	(0.1355)	(0.8917)	(0.2526)	(0.1243)	(0.9317)	(0.4111)	(0.6575)
IT/FR	0.45	0.3207	2.3232	0.6138	1.8593	1.4132	0.9100	0.4847
	(0.4797)	(0.1601)	(0.6765)	(0.3933)	(0.6632)	(0.7024)	(0.3401)	(0.7847)
US/IT	0.58	0.2610	5.3563	6.5224	0.1518	3.4220	1.9343	2.8638
	(0.9204)	(0.1452)	(0.2526)	(5.7945)	(0.1821)	(0.3310)	(0.1642)	(0.2388)
UK/IT	0.40	0.3532	3.5974	5386.1	-0.0979	3.2491	0.3483	1.0512
	(0.1558)	(0.0744)	(0.4632)	(31207)	(0.0956)	(0.3547)	(0.5551)	0.5912)
CA/IT	0.31	0.4272	0.5997	6.8931	28.647	0.4336	0.1661	0.3933
	(0.2208)	(0.2205)	(0.9630)	(40.799)	(55.341)	(0.9332)	(0.6836)	(0.8214)
FR/IT	1.07	0.1501	4.4077	0.1697	-1.2122	3.7607	0.6470	0.1464
	(0.2135)	(0.0053)	(0.3536)	(19.659)	(0.3742)	(0.2884)	(0.4211)	(0.9294)
GE/IT	0.39	0.3615	6.1783	0.2766	0.1628	3.7831	2.3952	2.8920
	(0.0846)	(0.0441)	(0.1862)	(0.0714)	(0.3104)	(0.2858)	(0.1217)	(0.2355)
(73:I~90:II)	0.29	0.4551	3.6455	0.4196	-0.3916	1.9275	1.7180	0.8720
	(0.1364)	(0.1763)	(0.4560)	(0.3311)	(0.7137)	(0.5875)	(0.1889)	(0.6466)
JP/IT	0.91	0.1738	1.7915	0.0424	0.6038	1.5594	0.2321	0.3769
	(6.3423)	(0.2547)	(0.7740)	(0.2562)	(0.5807)	(0.6686)	(0.6299)	(0.8282)

Table 5 (continued)

Currencies	Half-life (a)	b_r (b)	J_r (c)	$b_{u, hs}$ (d)	$b_{u,ga}$ (e)	J_u (f)	LR (g)	LR1 (h)
US/GE	0.41	0.3437	5.3330	5.3996	0.5233	3.0099	2.3231	2.6003
	(0.3278)	(0.1413)	(0.2548)	(13.868)	(0.6821)	(0.3901)	(0.1274)	(0.2724)
(73:I~90:II)	0.32	0.4193	2.9811	0.4013	0.4557	1.0434	1.9377	0.9010
	(0.1197)	(0.1110)	(0.5609)	(0.1263)	(0.0647)	(0.7907)	(0.1639)	(0.6373)
UK/GE	0.33	0.4075	2.9812	0.2775	0.1875	2.4343	0.5469	0.7519
	(0.1467)	(0.1214)	(0.5609)	(0.0152)	(0.0939)	(0.4872)	(0.4595)	(0.6866)
(73:I~90:II)	0.27	0.4701	4.4270	0.1773	0.7623	3.9573	0.4697	2.5680
	(0.1108)	(0.1639)	(0.3512)	(0.2409)	(0.1754)	(0.2661)	(0.4931)	(0.2769)
CA/GE	0.57	0.2624	2.0318	0.2751	0.1471	1.2548	0.7770	0.5655
	(1.2323)	(0.2005)	(0.7298)	(0.1840)	(0.2116)	(0.7398)	(0.3780)	(0.7537)
(73:I~90:II)	0.47	0.3081	3.6157	0.8419	0.0573	3.2325	0.3832	0.7320
	(0.2549)	(0.0735)	(0.4605)	(0.2916)	(0.0596)	(0.3571)	(0.5358)	(0.6935)
FR/GE	0.70	0.2196	1.0012	1.0436	-0.2978	0.7588	0.2424	0.0001
	(1.3209)	(0.1162)	(0.9096)	(1.8107)	(0.0841)	(0.8592)	(0.6224)	(0.9998)
(73:I~90:II)	0.50	0.2937	3.6317	0.1004	-0.8021	2.9925	0.6392	2.4225
	(0.1611)	(0.0391)	(0.4581)	(0.7639)	(0.6220)	(0.3927)	(0.4240)	(0.2978)
JP/GE	0.72	0.2131	2.1843	0.6311	0.0454	1.2434	0.9409	1.9598
	(0.0919)	(0.0073)	(0.7018)	(0.0967)	(0.0339)	(0.7426)	(0.3320)	(0.3753)
(73:I~90:II)	0.63	0.2398	1.2737	0.3349	-0.1662	0.8311	0.4426	0.5491
	(0.7895)	(0.0939)	(0.8658)	(0.8858)	(0.9942)	(0.8420)	(0.5058)	(0.7599)
IT/GE	0.30	0.4349	3.4296	13.113	-0.7021	3.4137	0.0159	3.1790
	(0.0318)	(0.0342)	(0.4886)	(33.293)	(0.4229)	(0.3321)	(0.8996)	(0.2040)
(73:I~90:II)	0.30	0.4395	0.9591	-2.4403	2.2982	0.4045	0.5546	0.7880
	(0.2385)	(0.2671)	(0.9159)	(4.2355)	(2.3554)	(0.9393)	(0.4564)	(0.6743)

Notes: For the unrestricted estimation, $b_{u, hs}$ is the estimate for the speed of adjustment coefficient from the Hansen and Sargent equations, and $b_{u, ga}$ is the estimate for the coefficient obtained from the gradual adjustment equation.
For column (a): Half-life in years.
For columns (a), (b), (d), and (e): Standard errors are in parentheses.
For columns (c), (f), (g), and (h): P-values are in parentheses.