

# Panel Estimates of the Fisher Effect

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## **Abstract**

The relationship between inflation and interest rates represents one of the crucial tests of monetary super-neutrality. To date, there is no consensus among economists with respect to this relationship. Many studies find evidence favorable to a full adjustment of nominal rates of interest to changes in (expected) inflation resulting in no long-run effects on real interest rates consistent with long-run super-neutrality. But a greater number find that the response of nominal interest rates to changes in inflation are insufficient to preclude effects on real interest rates and by extension other real variables. This study uses two new estimators of the long-run Fisher effect that have important advantages over previous estimators employed. The most important of these are the increased power and efficiency associated with the using a cross-section of Fisherian relationships from industrialized economies. The evidence is most consistent with a full Fisher effect supporting long-run super-neutrality.

”When to the sessions of sweet silent thought I summon up remembrance of things past, I sigh the lack of many a thing I sought” - William Shakespeare, Sonnet 30

## 1 Introduction

The relationship between inflation and interest rates represents one of the crucial tests of monetary super-neutrality, Lucas (1980). To date, there is no consensus among economists with respect to this relationship. Some studies find evidence favorable to a full adjustment of nominal rates of interest to changes in (expected) inflation resulting in no long-run effects on real interest rates consistent with long-run super-neutrality.<sup>1</sup> But a number studies find that the response of nominal interest rates to changes in inflation are insufficient to preclude effects on real interest rates and by extension other real variables.<sup>2</sup>

Irving Fisher (1930) described an environment where nominal rates respond one-for-one to changes in the expected decline in the purchasing power of money. When interest income is taxed this Fisher effect is greater than one as suggested by Darby (1975). In the 1970s and 1980s numerous studies produced evidence that the response of nominal interest rates to changes in inflation were less than one. Tanzi (1980) concluded that economic agents suffered from a form of irrationality he termed fiscal illusion. Tobin (1965, 1969) had earlier suggested that inflation was negatively correlated with real interest rates since increases in inflation induced economic agents to shift out of nominal assets and acquire real assets. This Tobin effect offered an explanation for low Fisher effect estimates that did not rely on investor irrationality. An extreme version of this phenomenon was investigated by Carmichael and Stebbing (1983) in which nominal interest rates do not change with changes in inflation leaving real rates as the only equilibrating adjuster. They called this the inverted Fisher hypothesis.

Lucas (1980) points out that the Tobin effect is a short-run phenomenon if one assumes that money is super-neutral in the long run. To see this, examine equation (1) which describes the relationship between nominal interest rates and inflation from a simple IS-LM model augmented with a Lucas AS relationship.

$$\frac{\partial i}{\partial \pi} = \left[ 1 + \frac{L_i}{I_r(L_y + W_p)} \right]^{-1}, \quad (1)$$

In equation (1),  $i$  represents the nominal interest rate,  $\pi$  is the ex-ante inflation rate,  $L_i$  is the interest elasticity of money demand,  $L_y$  is the income elasticity of money demand,  $I_r$  is the interest elasticity of investment demand and  $W_p$  is the price level elasticity of the nominal wage in the labor market. The Tobin effect arises when  $L_i > 0$ . Note that this is true even if one were to maintain a full-employment assumption implying  $W_p = 1$ . Allowing for wage rigidities, i.e.  $W_p < 1$ , would further erode the response of nominal rates to inflation.<sup>3</sup>

In the long run, however,  $L_i = 0$  since output is supply determined and the quantity theory of money holds.<sup>4</sup> Therefore, in the long run, the Fisher effect should reflect fully any changes in inflation.<sup>5</sup>

Mishkin (1992) was one of the first to suggest that due to the apparent non-stationarity of nominal interest rates and inflation a possible source of the low Fisher effect estimates is the spurious regression problem discussed by Granger and Newbold (1974). He correctly pointed out that the proper treatment of the Fisher relation is as a cointegrated system as in Engle and Granger (1987). Mishkin used the Engle-Granger OLS procedure to estimate the Fisher effect but was unable to make any strong conclusions due to the large standard errors of the estimated parameters. Subsequent studies used more efficient estimation procedures and generally found support for a long-run Fisher relation in the U.S. Evans and Lewis (1995) used the Stock and Watson (1993) dynamic OLS (DOLS) estimator and Crowder and Hoffman (1996) used the Johansen (1988) gaussian maximum likelihood estimator (MLE).

The focus of the Fisher hypothesis research has generally been on the U.S.<sup>6</sup> But it is certainly of interest whether nominal interest rates and inflation of other industrialized countries also exhibit a long-run Fisher relation. For example, a necessary condition, but not sufficient, for real interest rates to be equalized internationally is that the Fisher relation holds in each country individually.<sup>7</sup> Rose's (1988) seminal study examined a group of industrialized countries and Koustas and Serletis (1999) for eleven industrialized countries, but neither study found much evidence to support the Fisher hypothesis.

In this study I use data on short-term nominal interest rates and inflation rates over the last three decades collected from nine industrialized nations to examine whether the Fisher relation has empirical support internationally. The methodology follows that of recent studies by employing techniques that are appropriate for non-stationary time series. However, the techniques are extended to take advantage of power and efficiency gains associated with panel estimators. Several of these panel type cointegration estimators have been suggested. I will employ two that are based on asymptotically efficient estimators outside of the panel context. Their extension to panel estimators means that inference is asymptotically normal. The first is the dynamic OLS estimator suggested by Stock and Watson (1993) and Saikkonen (1991) in the context of standard cointegration and by Kao and Chiang (2000) and Mark and Sul (2001) in the context of panel cointegration. The second estimator is that of Johansen (1991) which has been extended to the panel case by Larsson et al (2001).

The next section briefly outlines the implications of the Fisher hypothesis. Section 3 presents the econometric methodology and section 4 presents the empirical results. Section 5 concludes.

## 2 The Fisher Relation

There are several ways to derive the Fisher hypothesis relationship between nominal interest rates and inflation. Earlier I made use of a derivation from an IS-LM-AS model. In this section I will use the results from a Lucas (1978) asset pricing model to derive the Fisher equation and discuss some implications for economic theory and modelling.

Assume a standard asset pricing model where  $k$ -period real and nominal default-free bonds both trade. The intertemporal optimization yields the following relevant Euler equations.

$$U'(C_t) = (1 + \theta)^{-1}(1 + r_t^k)E_t [U'(C_{t+k})] \quad (2)$$

$$\frac{U'(C_t)}{p_t} = (1 + \theta)^{-1}(1 + i_t^k)E_t \left[ \frac{U'(C_{t+k})}{p_{t+k}} \right] \quad (3)$$

Substitution yields,

$$\frac{1}{1 + i_t^k} = \left[ \frac{1}{1 + r_t^k} \right] E_t \left[ \frac{p_t}{p_{t+k}} \right] + (1 + \theta)^{-1} Cov_t \left[ \left( \frac{U'(C_{t+k})}{U'(C_t)} \right), \left( \frac{p_t}{p_{t+k}} \right) \right] \quad (4)$$

where  $i_t^k$  is the  $k$ -period return on the nominal bond,  $r_{t+k}$  is the  $k$ -period return on the real asset,  $U'(C_{t+k})$  is the marginal utility of consumption in period  $t + k$ ,  $p_{t+k}$  is the money price of the consumption bundle,  $E_t$  is the expectations operator conditioned on information available in period  $t$  and  $Cov_t$  is the conditional covariance operator also based on period- $t$  information.

If we assume that preferences are described by the hyperbolic absolute risk aversion (HARA) class and that consumption growth and inflation are jointly log-normal distributed, then equation (4) can be distilled into,

$$i_t^k = E_t \pi_{t+k} + r_t^k + \frac{1}{2} Var_t[\pi_{t+k}] - Cov_t[\pi_{t+k}, q_{t+k}] \quad (5)$$

This form of the Fisher relation specifies that the nominal interest rate is the sum of expected inflation, the real interest rate and an inflation risk premium which depends on the conditional second moments of the joint distribution of  $\pi$  and  $q$ .

Equation (2) implies that the real return is proportional to consumption growth, with the factor of proportionality a function of risk aversion and time preference. Since consumption growth is almost certainly a stationary process, (2) implies that the real rate of interest must also be stationary. If we also assume that the conditional variance of inflation and the conditional covariance of inflation and consumption growth are both stationary, then equation (5) implies that the nominal interest and inflation rates are either both stationary or non-stationary but cointegrated.<sup>8</sup> This latter is the condition that I wish to exploit.

Given the conditions above and the further assumption that expectations are rational, the Fisher relation may be written as,

$$i_t^k = a_0 + a_1\pi_{t+k} + \eta_t \tag{6}$$

where  $\eta_t$  is a possibly autocorrelated stationary residual and  $a_0$  contains the means of the real rate plus the risk premium.<sup>9</sup> The parameter of greatest interest is  $a_1$  the measure of the Fisher effect. From equation (5) the Fisher effect implies  $a_1 = 1$ . But as demonstrated by Darby (1975), when nominal interest income is subject to taxation one might expect  $a_1 > 1$ .<sup>10</sup> Evidence that  $a_1$  is significantly less than one would support the Tobin effect and imply significant non-neutralities associated with monetary policy, even in the long run.

### 3 Empirical Methodology

#### 3.1 Unit Root Tests

The univariate unit root tests used most commonly in the literature are the augmented Dickey-Fuller (*ADF*) (Said and Dickey, 1984) and Phillips-Perron (*PP*) (Phillips and Perron, 1988) tests. It is well known that these univariate unit root tests have notoriously low power against local stationary alternatives and suffer from serious size distortion when the data generating process (DGP) has negative moving average (MA) terms.<sup>11</sup> Elliot, et al. (1996) (ERS) develop a feasible point optimal test that relies on local *GLS* detrending. This test has much greater power than standard *ADF* and *PP* unit root tests. Recently Ng and Perron (1995, 2000) and Perron and Ng (1996), extending the work done by ERS, have developed unit root tests that are based upon the local *GLS* detrending method but also use an autoregressive spectral density estimator of the long-run variance. This class of tests, which they denote the *M*-tests, has much less size distortion in the presence of *MA* errors than the standard tests. This is especially true when one chooses lag truncations based upon a modified information criteria developed by Ng and Perron (2000).

The tests proposed by Ng and Perron are motivated by the DGP in (7),

$$y_t = d_t + u_t, \quad u_t = \rho u_{t-1} + v_t \tag{7}$$

where  $v_t = \varphi(L)e_t = \sum_{j=0}^{\infty} \varphi_j e_{t-j}$ ,  $d_t = \zeta' z_t = \sum_{i=0}^p \zeta_i t^i$  for  $p = 0, 1$ . ERS suggest using a *GLS* detrending method to improve the power of unit root tests. For any series  $\{x_t\}_{t=0}^T$  define  $(x_0^{\bar{\alpha}}, x_t^{\bar{\alpha}}) \equiv (x_0, (1 - \bar{\alpha}L)x_t)$  for some chosen  $\bar{\alpha} = 1 + \bar{c}/T$ . The *GLS* detrended series is defined as,

$$\tilde{y}_t \equiv y_t - \hat{\zeta}' z_t \tag{8}$$

where  $\hat{\zeta}$  minimizes  $S(\bar{\alpha}, \zeta) = (y^{\bar{\alpha}} - \zeta' z_t^{\bar{\alpha}})'(y^{\bar{\alpha}} - \zeta' z_t^{\bar{\alpha}})$ . ERS suggest choosing  $\bar{c} = -7.0$  for  $p = 0$  and  $\bar{c} = -13.5$  for  $p = 1$ . The test recommended by ERS is the  $DF^{GLS}$  statistic given in equation (9).

$$\Delta \tilde{y}_t = \rho \tilde{y}_{t-1} + \sum_{j=1}^k \gamma_j \tilde{y}_{t-j} + e_{tk} \quad (9)$$

Ng and Perron recommend two tests that have similar power to the  $DF^{GLS}$  but that also have superior size properties in the presence of MA errors. These tests are  $MZ_\rho$ ,  $MZ_t$ , and  $MSB$ , collectively referred to as the  $M$  tests. These are defined as,

$$MZ_\rho = (T^{-2} \tilde{y}_t^2 - s_{AR}^2) (2T^{-2} \sum_{t=1}^T \tilde{y}_{t-1}^2)^{-1} \quad (10)$$

$$MSB = \left[ \frac{T^{-2} \sum_{t=1}^T \tilde{y}_{t-1}^2}{s_{AR}^2} \right]^{\frac{1}{2}} \quad (11)$$

and  $MZ_t = MZ_\rho \times MSB$ . All three tests are based on  $s_{AR}^2$ , an autoregressive estimate of the spectral density at frequency zero of  $v_t$ . This estimate is calculated as,

$$s_{AR}^2 = \frac{\hat{\sigma}_k^2}{[1 - \gamma(1)]^2} \quad (12)$$

where  $\gamma(1) = \sum_{i=1}^k \gamma_i$  and  $\hat{\sigma}_k^2 = (T - k)^{-1} \sum_{t=k+1}^T \hat{e}_{tk}^2$  and  $\gamma_i$  and  $\{\hat{e}_{tk}\}$  are taken from estimation of (9) using OLS. The only piece left is to specify a lag truncation parameter  $k$ . Ng and Perron suggest using a modified information criteria ( $MIC$ ) as in (13),

$$MIC(k) = \ln(\hat{\sigma}_k^2) + \frac{C_T(\tau_T(k) + k)}{T - k_{\max}} \quad (13)$$

where  $\tau_T(k) = (\hat{\sigma}_k^2)^{-1} \hat{\rho} \sum_{t=k_{\max}+1}^T \tilde{y}_{t-1}^2$  and  $k_{\max}$  is the largest lag truncation considered. Ng and Perron show that if  $C_T = \ln(T - k_{\max})$  then this reduces to their  $MBIC$  or modified

Bayesian information criteria.

It is still the case that these univariate unit root tests may lack power against stationary alternatives that are close to the non-stationary boundary. One way to overcome the low power associated with univariate unit root tests is to extend the data to allow for a panel testing approach.<sup>12</sup> I apply two common panel unit root tests to the data.

The first is a test suggested by Levin and Lin (1992) based on the model in (14),

$$\Delta y_{it} = \rho_i y_{it-1} + z'_{it} \gamma + u_{it}, \quad i = 1, \dots, N; t = 1, \dots, T, \quad (14)$$

where  $z_{it}$  is the deterministic component and  $u_{it}$  is a stationary process. The Levin and Lin test assumes that  $\rho_i = \rho$  for all  $i$ . Levin and Lin suggest a  $t$ -statistic calculated under the null as

$$t_\rho = \frac{(\hat{\rho} - 1) \sqrt{\sum_{i=1}^N \sum_{t=1}^T \tilde{y}_{i,t-1}^2}}{s_u} \quad (15)$$

where

$$s_u^2 = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \tilde{u}_{it}^2 \quad (16)$$

and  $\tilde{y}_{i,t}$  and  $\tilde{u}_{it}$  are simply  $y_{it}$  and  $u_{it}$  corrected for the deterministic components  $z_{it}$ .

The second panel test is attributed to Im, Pesaran and Shin (1997)(hereafter IPS). It is based on averaging the individual unit root test statistics from standard augmented Dickey-Fuller regressions. Define the  $t$ -bar statistic as

$$\bar{t} = \frac{1}{N} \sum_{i=1}^N t_{\rho_i}, \quad (17)$$

where  $t_{\rho_i}$  is the individual  $t$ -statistic for testing the unit root null hypothesis. IPS suggest a statistic normalized by the expected value and variance of the individual  $t$ -statistics and call this statistic  $t_{IPS}$ . They derive critical values for this statistic via monte carlo simulation.

There are two problems, however, with these panel based unit root tests. First panel unit root tests have serious size distortions when the panel groups are spatially correlated.<sup>13</sup> Second, panel unit root tests have an uninformative alternative hypothesis. Rejection of the null cannot be interpreted as stationarity of all panel groups.<sup>14</sup>

Hansen (1995) has recently suggested a covariate ADF (CADF) test which yields substantial improvements in power relative to the ADF test and has a well defined alternative hypothesis.<sup>15</sup> Consider the simple ADF regression of (18),

$$\Delta y_t = \rho y_{t-1} + \sum_{i=1}^{k-1} \zeta_i \Delta y_{t-i} + v_t. \quad (18)$$

Usually we observe related time series, say  $x_t$ , assuming  $x_t$  is I(1) so that  $\Delta x_t$  is I(0), where the relationship between  $x_t$  and  $y_t$  can be captured as  $e_t = v_t - \Delta x_t' \gamma$  so that the process for  $y_t$  can be alternatively written as,

$$\Delta y_t = \rho y_{t-1} + \Delta x_t' \gamma + \sum_{i=1}^{k-1} \zeta_i \Delta y_{t-i} + e_t. \quad (19)$$

The variance of the error in (19) is  $\sigma_e^2 = \sigma_v^2 - \frac{\sigma_{xv}^2}{\sigma_x^2}$  and will be smaller than the error variance in (18) unless  $\sigma_{xv} = 0$ . Thus hypothesis tests in (19) should have greater power than tests using (18). Hansen demonstrates that the power of the CADF test is inversely related to two factors. The first is the long-run (frequency zero) squared correlation between  $v_t$  and  $e_t$  denoted as  $\eta^2$ .

$$\eta^2 = \frac{\sigma_{ve}^2}{\sigma_e^2 \sigma_v^2} \quad (20)$$

The second factor relating to the power of the CADF test is the variance ratio defined as,

$$\rho^2 = \frac{\sigma_e^2}{\sigma_v^2}. \quad (21)$$

Hansen (1995) demonstrates that with a sample size of 100 observations and an autoregressive root of 0.95 the power for the standard ADF test is 33%. This increases to 51% when  $\eta^2 = 0.7$  and to 90% when  $\eta^2 = 0.3$ . The distribution of the CADF test depends on the value of  $\eta^2$  such that,

$$t_{CADF} = \eta(DF) + (1 - \eta^2)^{1/2}N(0, 1) \quad (22)$$

where  $DF$  represents the Dickey-Fuller distribution and  $N(0, 1)$  is the standard normal.

### 3.2 Efficient Cointegration Estimation

If we assume that nominal interest rates and inflation are best characterized as I(1) processes, then (6) represents a cointegrating regression. The simplest estimator of such a cointegrating equation is an OLS regression of nominal interest rate on inflation. This estimator of the cointegrating parameter was first suggested by Engle and Granger (1987). This estimator has the virtue of simplicity and as Engle and Granger demonstrate, the OLS estimator is superconsistent which eliminates the error-in-variables bias that arises from using actual inflation rather than expected inflation. Superconsistency also allows the econometrician to use the estimated cointegrating parameter as if it were known in subsequent regressions. The problem with this estimator is that it has substantial small sample bias arising from the endogeneity of the regressors and the possible serial correlation of the residuals. Furthermore, the asymptotic distribution of the OLS estimator in a cointegrating model is non-standard making inference on the cointegrating parameter difficult.

Consider the the cointegrated system given in (23) and (24),

$$y_{1t} = \beta y_{2t} + u_{1t} \quad (23)$$

where

$$\Delta y_{2t} = u_{2t} \quad (24)$$

and  $\beta$  is the long-run cointegrating parameter and  $u_t \equiv \text{iid}(0, \Sigma)$  with  $\Sigma$  given as,

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma'_{21} \\ \sigma_{21} & \Sigma_{22} \end{bmatrix}. \quad (25)$$

The maximum likelihood estimator (MLE) of  $\beta$  in (23) is given by,

$$y_{1t} = \beta y_{2t} + \gamma \Delta y_{2t} + u_{1.2t} \quad (26)$$

where  $u_{1.2t} = u_{1t} - \sigma_{12} \Sigma_{22}^{-1} u_{2t}$  and  $\gamma = \Sigma_{22}^{-1} \sigma_{21}$ .<sup>16</sup>

The limiting distribution of this estimator is,

$$T(\beta^* - \beta) \Rightarrow \left( \int_0^1 S_2 S_2' \right)^{-1} \left( \int_0^1 S_2 dS_{1.2}' \right) \quad (27)$$

where  $S_2$  and  $S_{1.2}$  are independent Brownian motions. The distribution in (27) is a Gaussian mixture of normals yielding standard asymptotic inference.

Compare the distribution of the MLE in (27) to the limit distribution of the OLS estimator given below,

$$T(\hat{\beta} - \beta) \Rightarrow \Lambda + \left( \int_0^1 S_2 S_2' \right)^{-1} \left( \int_0^1 S_2 dS_2' \right) \Sigma_{22}^{-1} \sigma_{21} + \left( \int_0^1 S_2 S_2' \right)^{-1} \sigma_{21} \quad (28)$$

where  $\Lambda$  is the distribution of the MLE given in (27). The second term on the right side of (28) is a unit root distribution and the third term on the right side is a bias term arising from the contemporaneous correlation between  $u_{1t}$  and the regressor  $y_{2t}$ . When  $\sigma_{21} = 0$ , the OLS and MLE estimators have equivalent limit distributions.

There are several estimators derived in the cointegration literature to eliminate the biases in the OLS estimator. Phillips and Hansen (1990) suggest a fully modified OLS (FM-OLS) estimator. Let  $\hat{\sigma}_{21}$  be a consistent estimate of  $\sigma_{21}$ , then a modified estimator of  $\beta$  is given by

$$\beta^\ddagger = (y'_{2t}y_{2t})^{-1}(y'_{2t}y_{1t} - T\hat{\sigma}_{21}) \quad (29)$$

which eliminates the bias arising from the third term on the right side of (28). A further modification is necessary in order to remove all the bias from the second term in (28).

$$y_{1t}^+ = y_{1t} - \hat{\sigma}'_{21}\hat{\sigma}_{22}^{-1}\Delta y_{2t} \quad (30)$$

where  $\hat{\sigma}_{22}$  is a consistent estimate of  $\Sigma_{22}$ . The FM-OLS estimator simply replaces  $y_{1t}$  in (29) with  $y_{1t}^+$  and allows standard asymptotic inference.

The dynamic OLS (DOLS) estimator of Saikkonen (1991) and Stock and Watson (1993) uses leads and lags of independent variables to eliminate the correlations of these variables with the errors. Implementation proceeds using the following regression.

$$y_{1t} = \mu_0 + \beta y_{2t} + \delta(L)\Delta y_{2t} + \varepsilon_t \quad (31)$$

where  $\delta(L)$  is a two-sided polynomial in the lag operator such that both leads and lags are included in (31). In general the residuals from (31) will be serially correlated requiring use of an autocorrelation-heteroscedasticity consistent covariance estimator in order to make inference in finite samples.

The other estimator I employ is derived under the assumption that (23) and (24) have a finite order vector autoregressive representation (VAR). The Johansen estimator is the MLE

when the process for  $X_t = \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}$  is given in (32),

$$X_t = \Phi_1 X_{t-1} + \dots + \Phi_k X_{t-k} + \mu + \delta t + \varepsilon_t \quad (32)$$

where  $X_t$  is  $p$ -dimensional vector of variables integrated of order one or less, i.e.,  $I(2)$  and higher orders are ruled out,  $\Phi_j$  are  $(p \times p)$  coefficient matrices,  $\mu$  is a  $(p \times 1)$  vector of constants,  $\delta$  is a  $(p \times 1)$  vector of coefficients on linear trend terms and  $\varepsilon_t$  is a white noise error vector with non-diagonal covariance matrix  $\Omega$ .

The Johansen estimator is calculated from the transformed version of (32) into it's error-correction model (VECM) form as in (33).

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-1} + \mu + \delta t + \varepsilon_t \quad (33)$$

If  $X_t$  is cointegrated, the long-run multiplier matrix  $\Pi = \Phi(1) - I$  can be decomposed into two  $(p \times r)$  matrices such that  $\alpha\beta' = \Pi$ . The  $(p \times r)$  matrix  $\beta$  represents the cointegrating vectors or the long-run equilibria of the system of equations. The  $(p \times r)$  matrix  $\alpha$  is the matrix of error-correction coefficients which measure the rate each variable adjusts to the long-run equilibrium. Maximum likelihood estimation of (33) can be carried out by applying reduced rank regression. Johansen (1988, 1991) suggests first concentrating out the short-run dynamics by regressing  $\Delta X_t$  and  $X_{t-1}$  on  $\Delta X_{t-1}, \Delta X_{t-2}, \dots, \Delta X_{t-k+1}, 1$  and  $t$ , and saving the residuals as  $R_{0t}$  and  $R_{1t}$ , respectively. Next calculate the product moment matrices  $S_{ij} = T^{-1} \sum_{t=1}^T R_{it} R'_{jt}$  and solve the eigenvalue problem  $|\lambda S_{11} - S_{10} S_{00}^{-1} S_{01}| = 0$ . Then order the estimated eigenvalues from largest to smallest  $(\hat{\lambda}_1, \hat{\lambda}_2, \dots, \hat{\lambda}_p)$ . The estimate of  $\beta$ ,  $\hat{\beta}$ , is given by the  $r$ -largest eigenvectors associated with the eigenvalues  $\hat{\lambda}$ . Hypothesis tests on  $\hat{\beta}$  can be conducted using likelihood ratio (LR) tests with standard  $\chi^2$  inference. Let the form

of the linear restrictions on  $\beta$  be given by  $\beta = H\varphi$  where  $H$  is a  $p \times s$  matrix of restrictions and  $\varphi$  is a  $s \times r$  matrix of unknown parameters. The LR test statistic is given by,

$$T \sum_{i=1}^r \ln \left[ \frac{(1 - \tilde{\lambda}_i)}{(1 - \hat{\lambda}_i)} \right] \sim \chi_{r(p-s)}^2 \quad (34)$$

where  $\tilde{\lambda}_i$  are the eigenvalues from the restricted MLE.

The test for cointegration is a test for the number of non-zero eigenvalues. The likelihood ratio statistic testing the rank of  $\Pi$ , or equivalently the number of non-zero eigenvalues, is given by  $-T \sum_{i=r+1}^p \ln(1 - \hat{\lambda}_i)$  and is called the trace statistic by Johansen (1988, 1991).<sup>17</sup>

### 3.3 Panel Cointegration Estimators

Consider the following fixed-effect panel regression:

$$y_{i,t} = \alpha_i + x'_{i,t}\beta + u_{i,t}, i = 1, \dots, N, t = 1, \dots, T, \quad (35)$$

where  $y_{i,t}$  are  $1 \times 1$ ,  $\beta$  is an  $M \times 1$  vector of slope parameters,  $\alpha_i$  are the intercepts, and  $u_{i,t}$  are the stationary disturbance terms. It is assumed that  $x_{i,t}$  are  $M \times 1$  I(1) processes which are themselves not cointegrated such that  $x_{i,t} = x_{i,t-1} + \varepsilon_{i,t}$ . These assumptions imply that (35) represents a system of cointegrating regressions. Let  $w_{i,t} = (u_{i,t}, \varepsilon'_{i,t})'$  and assume that it satisfies the conditions in Kao and Chiang (2000). Then the long-run covariance of  $w_{i,t}$  is,

$$\begin{aligned} \Omega &= \sum_{j=-\infty}^{\infty} E(w_{i,j}w'_{i,0}) \\ &= \Sigma + \Gamma + \Gamma' \\ &= \begin{bmatrix} \Omega_u & \Omega_{u\varepsilon} \\ \Omega_{\varepsilon u} & \Omega_{\varepsilon} \end{bmatrix} \end{aligned} \quad (36)$$

where

$$\Gamma = \sum_{j=1}^{\infty} E(w_{i,j}w'_{i,0}) = \begin{bmatrix} \Gamma_u & \Gamma_{u\varepsilon} \\ \Gamma_{\varepsilon u} & \Gamma_{\varepsilon} \end{bmatrix} \quad (37)$$

and

$$\Sigma = \sum_{j=1}^{\infty} E(w_{i,0}w'_{i,0}) = \begin{bmatrix} \Sigma_u & \Sigma_{u\varepsilon} \\ \Sigma_{\varepsilon u} & \Sigma_{\varepsilon} \end{bmatrix} \quad (38)$$

are partitioned conformably with  $w_{i,t}$ . Then define the one-sided long-run covariance as

$$\begin{aligned} \Delta &= \Sigma + \Gamma \\ &= \sum_{j=1}^{\infty} E(w_{i,0}w'_{i,0}) \end{aligned} \quad (39)$$

$$\Delta = \begin{bmatrix} \Delta_u & \Delta_{u\varepsilon} \\ \Delta_{\varepsilon u} & \Delta_{\varepsilon} \end{bmatrix}.$$

Kao and Chiang derive the limiting distributions for the *OLS*, *FM-OLS* and *DOLS* estimators for the regression specification given in (35). They also investigated the finite sample properties of each estimator through Monte Carlo simulation. They found that (i) the *OLS* estimator has a non-negligible bias, (ii) the *FM-OLS* estimator does not improve on the *OLS* estimator in general, and (iii) the *DOLS* estimator has the best properties of the three.

The *OLS* estimator of  $\beta$  is,

$$\hat{\beta}_{OLS} = \left[ \sum_{i=1}^N \sum_{t=1}^T (x_{it} - \bar{x}_i)(x_{it} - \bar{x}_i)' \right]^{-1} \left[ \sum_{i=1}^N \sum_{t=1}^T (x_{it} - \bar{x}_i)(y_{it} - \bar{y}_i) \right] \quad (40)$$

where a bar over a variable denotes its time average. The *FM-OLS* estimator is constructed by modifying the *OLS* estimates for endogeneity and serial correlation. Let  $\hat{\Omega}_{\varepsilon u}$  and  $\hat{\Omega}_{\varepsilon}$  be

consistent estimates  $\Omega_{\varepsilon u}$  and  $\Omega_\varepsilon$ , respectively. Define

$$\widehat{y}_{i,t}^+ = y_{i,t} - \widehat{\Omega}_{\varepsilon u} \widehat{\Omega}_\varepsilon^{-1} \varepsilon_{i,t} \quad (41)$$

which modifies the dependent variables for endogeneity. The correction for serial correlation is,

$$\widehat{\Delta}_{\varepsilon u}^+ = \widehat{\Delta}_{\varepsilon u} - \widehat{\Delta}_\varepsilon \widehat{\Omega}_\varepsilon^{-1} \widehat{\Omega}_{\varepsilon u} \quad (42)$$

where  $\widehat{\Delta}_{\varepsilon u}$  and  $\widehat{\Delta}_\varepsilon$  are the kernel estimates of  $\Delta_{\varepsilon u}$  and  $\Delta_\varepsilon$ , respectively. This leads to the *FM – OLS* estimator of,

$$\widehat{\beta}_{FM} = \left[ \sum_{i=1}^N \sum_{t=1}^T (x_{it} - \bar{x}_i)(x_{it} - \bar{x}_i)' \right]^{-1} \left[ \sum_{i=1}^N \left( \sum_{t=1}^T (x_{it} - \bar{x}_i) \widehat{y}_{it}^+ - T \widehat{\Delta}_{\varepsilon u}^+ \right) \right]. \quad (43)$$

Finally, the *DOLS* estimator is given as,

$$y_{it} = \alpha_i + x'_{it} \beta + \sum_{j=-q}^q c_{ij} \Delta x_{it+j} + v_{it}. \quad (44)$$

Kao and Chiang derive the asymptotic distributions of the three estimators as follows:

1.  $T\sqrt{N}(\widehat{\beta}_{OLS} - \beta) - \sqrt{N}\delta_{NT} \implies N(0, 6\Omega_\varepsilon^{-1}\Omega_{u, \varepsilon})$ ,
2.  $T\sqrt{N}(\widehat{\beta}_{FM} - \beta) \implies N(0, 6\Omega_\varepsilon^{-1}\Omega_{u, \varepsilon})$ ,
3.  $T\sqrt{N}(\widehat{\beta}_{DOLS} - \beta) \implies N(0, 6\Omega_\varepsilon^{-1}\Omega_{u, \varepsilon})$ ,

where  $\Omega_{u, \varepsilon} = \Omega_u - \Omega_{u\varepsilon}\Omega_\varepsilon^{-1}\Omega_{\varepsilon u}$  and  $\implies$  denotes convergence in distribution. Tests for cointegration within this panel framework have been proposed by Pedroni (1999), McKoskey and Kao (1998) and Kao (1999).

### 3.4 Panel Cointegration Tests

Kao (1999) describes two types of panel cointegration tests. The Dickey-Fuller (DF) type test and the augmented Dickey-Fuller (ADF) test. These tests can be calculated from the residuals from the panel cointegration estimators as:

$$\hat{u}_{i,t} = \rho \hat{u}_{i,t-1} + \nu_{i,t} \quad (45)$$

where the  $\hat{u}_{i,t}$  are the estimated residuals. The null hypothesis of no cointegration can be written as  $H_0 : \hat{\rho} = 1$ . Four tests are considered:

1.  $DF_\rho = \frac{\sqrt{NT}(\hat{\rho}-1)+3\sqrt{3}}{\sqrt{10.2}}$
2.  $DF_t = \sqrt{1.25t_\rho} + \sqrt{1.875N}$
3.  $DF_\rho^* = \frac{\sqrt{NT}(\hat{\rho}-1)+\frac{3\sqrt{N}\hat{\sigma}_v^2}{\hat{\sigma}_{0v}^2}}{\sqrt{3+\frac{7.2\hat{\sigma}_v^4}{\hat{\sigma}_{0v}^4}}}$
4.  $DF_t^* = \frac{t_\rho + \frac{\sqrt{6N}\hat{\sigma}_v}{2\hat{\sigma}_{0v}}}{\sqrt{\frac{\hat{\sigma}_{0v}^2}{2\hat{\sigma}_v^2} + \frac{3\hat{\sigma}_v^2}{10\hat{\sigma}_{0v}^2}}}$

where  $\hat{\sigma}_v^2 = \Sigma_u - \Sigma_{u\varepsilon}\Sigma_\varepsilon^{-1}$  and  $\hat{\sigma}_{0v}^2 = \Omega_u - \Omega_{u\varepsilon}\Omega_\varepsilon^{-1}$ . While  $DF_\rho$  and  $DF_t$  are based on the assumption that the regressors are strictly exogenous with respect to the errors,  $DF_\rho^*$  and  $DF_t^*$  are appropriate when the regressors are endogenous. For the *ADF* test the following augmented regression is run,

$$\hat{u}_{i,t} = \rho \hat{u}_{i,t-1} + \sum_{j=1}^{\kappa} \zeta_j \Delta \hat{u}_{i,t-j} + \nu_{i,t} \quad (46)$$

and the test statistic is calculated as,

$$ADF = \frac{t_{ADF} + \frac{\sqrt{6N}\hat{\sigma}_v}{2\hat{\sigma}_{0v}}}{\sqrt{\frac{\hat{\sigma}_{0v}^2}{2\hat{\sigma}_v^2} + \frac{3\hat{\sigma}_v^2}{10\hat{\sigma}_{0v}^2}}}, \quad (47)$$

where  $t_{ADF}$  is the  $t$ -statistic associated with the hypothesis  $H_0 : \hat{\rho} = 1$  from the estimate given in (46). All of the test statistics have asymptotic standard normal distributions.

The final panel cointegration test considered is one proposed by Larsson et al. (2001) and is based on the Johansen estimator for individual cointegration. Define the Johansen trace test given in 3.3 as  $LR_T \{H(r) | H(p)\}$  where  $\{H(r) | H(p)\}$  denotes the null hypothesis of  $r$  cointegrating relationships versus the alternative of  $p$  cointegrating relationships. Now consider the average  $LR_T$  over all  $i$  panel groups and define the LR-bar statistic as,

$$\overline{LR}_{NT} \{H(r) | H(p)\} = \frac{1}{N} \sum_{i=1}^N LR_{i,T} \{H(r) | H(p)\}. \quad (48)$$

The proposed test statistic is calculated from (49).

$$\Upsilon_{\overline{LR}} \{H(r) | H(p)\} = \frac{\sqrt{N} (\overline{LR}_{NT} \{H(r) | H(p)\} - E(Z_k))}{\sqrt{Var(Z_k)}} \quad (49)$$

where  $E(Z_k)$  is the mean and  $Var(Z_k)$  the variance of the asymptotic trace statistic. Larsson et al. (2001) demonstrate that the statistic defined by (49) has a standard normal distribution.

## 4 Empirical Results

### 4.1 Data

The data used in this study are taken from the International Financial Statistics of the IMF. They consist of monthly observations on short-term nominal interest rates on government debt and consumer price inflation rates, all converted to annualized values. The sample covers the period from January 1960 to December 2000 and include data from the following nations; the United State, the United Kingdom, Germany, Japan, Italy, Belgium, France,

Holland (The Netherlands) and Canada. The data are plotted in Figure 1.

## 4.2 Unit Root Tests

Table 1 displays the univariate unit root tests discussed in section 3.1. The evidence is consistent with all series characterized as unit root processes. The two exceptions are the German and U.S. nominal interest rates.<sup>18</sup> While these tests represent the most powerful univariate tests, they may still lack power relative to multivariate tests. The Levin-Lin test for the panel of inflation rates yields a calculated test statistic of -5.32 implying rejection of the unit root null hypothesis at a less than one percent marginal significance level. However, the IPS test for the inflation rates is -1.05 with a marginal significance level of 15%.<sup>19</sup> Similarly, for the panel of nominal interest rates the Levin-Lin test is -7.57, implying a strong rejection of the unit root hypothesis, but the IPS test is -0.45, consistent with a unit root in all panel members.

The contradictory conclusions of these two tests is not altogether surprising. O'Connell (1998) demonstrates that the Levin-Lin test suffers from extreme size distortions due to spatial correlation. Taylor and Sarno (1998) and Breuer et al. (2001) criticize all of these panel based unit root procedures because of the lack of an informative null hypothesis. Is a rejection of the unit root null in these panel tests taken to mean all series are stationary or only some or only one of the panel members are stationary? Because of these concerns, it is difficult to interpret the Levin-Lin test rejections.

Table 2 presents the covariate augmented Dickey-Fuller (CADF) tests of Hansen (1995). These tests have significantly greater power than standard ADF tests and suffer from size distortions only in the presence of extremely negatively autocorelated errors.<sup>20</sup> As covariates I included first differences of all of the other series used in the study, e.g. the model for the Belgian inflation rate as the dependent variable included first differences of all other inflation

rates and the first differences of the nominal interest rates, including the Belgian nominal rate. The results from table 2 imply that all series are  $I(1)$ . Not only are there no rejections of the unit root null, but the estimated values of  $\eta^2$  and  $\rho^2$  suggest impressive improvements in the power of these tests relative to the standard ADF test. Furthermore, the AIC values from the CADF tests are all significantly smaller than the AIC statistics from the  $DF^{GLS}$  specifications. While not a formal specification test, this does suggest a significantly better fit of the CADF model relative to the univariate models. Given the results from these unit root tests, especially the CADF tests, the evidence is fairly overwhelming in favor of the unit root null hypothesis.

### 4.3 Panel Cointegration Analysis

Establishing the nonstationarity of the data in the previous section means that cointegration methods are appropriate in estimating and testing the Fisher relation. Table 3 displays FM-OLS estimates of the Fisher relation. It includes estimates for each individual Fisher relation and the panel estimates. The FM-OLS estimator is based on a single equation regression specification. While it is natural to specify the nominal interest rate as the dependent variable in such regressions Ng and Perron (1997) suggest that the reverse regression may be more informative and suffer less small sample bias.<sup>21</sup> This suggestion relies on the well-known result that estimator efficiency is improved if the variable with the greater variance is the dependent variable. Since the inflation rates generally have larger variance than nominal interest rates that suggests using them as the dependent variables. Therefore in table 3 I report estimates of the Fisher effect from regressions run in both directions, i.e. inflation on nominal rates and vice versa.<sup>22</sup> Another factor that can affect the estimates is the specification of the deterministic components in the regression model. For that reason I report results from three different specifications, with both a linear trend and constant, with

constant only and with no deterministic variables.<sup>23</sup>

For the most part the Fisher effect estimates in table 3 are statistically consistent with a complete response of nominal interest rates consistent with monetary superneutrality, i.e. the slope estimate is statistically not less than one. The exceptions are the estimates for Japan when nominal interest is the dependent variable and the regression includes both a constant and a trend, the United States when nominal interest is the dependent variable and the regression includes both a constant and a trend and a constant only, and the United Kingdom when the regression includes a constant and a trend regardless of the dependent variable and the specification with nominal interest rate as the dependent variable and a constant in the regression.

Closer examination of the results displayed in table 3 reveal that the estimated Fisher effect is almost always larger in the specifications with inflation as the dependent variable. The exceptions to this all occur in specifications that exclude the deterministic terms. It is interesting to note that the smallest standard errors occur in the models where inflation is the dependent variable and there are no deterministic terms. Overall, these results would seem to be broadly consistent with a full Fisher effect.

The last row of table 3 presents the panel FM-OLS estimates for each of the specifications. Two things are revealed in these panel estimates. First, all of the standard errors are much smaller than their single equation counterparts. Second, two out of the six specifications yield results inconsistent with a full Fisher effect. Both of these occur when the dependent variable is the nominal interest rate and there is at least one deterministic term in the empirical model. In fact the panel Fisher estimate in these two specifications is smaller than any single Fisher estimate from the single equation analysis save that for Japan in column one. Another interesting result from the panel estimates is the large Fisher effect estimates from the specifications using inflation as the dependent variable, all of which are significantly

greater than one consistent with the Darby effect.<sup>24</sup>

Table 4 presents the DOLS estimates for each of the model specifications considered. Like FM-OLS, DOLS is a single equation estimator and so is subject to the same normalization issues. The DOLS Fisher effect estimates are again mostly insignificantly different from one implying general consistency with a full Fisher effect. There are however nine models which yield Fisher effect estimates statistically less than one, almost double the number from the FM-OLS models. All nine, however, occur in models where the nominal interest rate is the dependent variable. This may simply reflect the normalization problem alluded to above. It is still the case that most of the models yield Fisher effect estimates that are not statistically less than unity. The panel DOLS estimates are displayed in the last row of table 4. These estimates are more consistent across deterministic specifications than the FM-OLS but even less consistent across dependent variable specification. All panel DOLS models with nominal interest rate as the dependent variable yield Fisher effect estimates statistically below one, consistent with the Tobin effect. Those models using inflation as the dependent variable yield Fisher effect estimates greater than one, consistent with the Darby effect.

The panel cointegration tests discussed in section 3.4 are displayed in table 5. These tests were calculated using the residuals from the models that included both constant and linear trend in the deterministic specification.<sup>25</sup> All tests statistics lead to a rejection of the null of no cointegration at very high levels of statistical significance.

Tables 6 through 8 present the Johansen cointegration tests and estimates and various hypothesis test statistics for different models based on the deterministic specification.<sup>26</sup> The results in table 6 allow for trends in the underlying data that are eliminated by the cointegrating parameters.<sup>27</sup> The 95% (90%) critical value for the trace statistics presented in column two of table 6, taken from Osterwald-Lenum (1992), is 15.41 (13.33). This leads to rejection of the null of no cointegration in seven of the nine countries. The last row of this

column shows the panel trace statistic. This statistic has a standard normal distribution and so the null of no cointegration is rejected.

The column labelled  $\hat{\beta}$  is the estimated Fisher effect. The next two columns of table 6, labelled  $\beta_i = 0$  and  $\beta_\pi = 0$ , display likelihood ratio (LR) tests of the null hypothesis that either the interest rate or inflation can be excluded from the model. The last column in table 6 presents LR tests of the null that the Fisher effect is equal to unity. Each of the LR tests is distributed  $\chi^2(1)$ . While it is the case all but one of the Fisher effect estimates is less than one, only three are statistically significantly so. The last row of each of these columns displays panel counterparts. If we assume that the individual LR tests are independent, then the panel statistics are distributed as  $\chi^2(9)$  variates. These panel tests strongly reject exclusion of either variable. It is also the case that the panel tests reject a full Fisher effect since the 95% critical value for  $\chi^2$  distribution with nine degrees of freedom is 16.92. Closer examination reveals that this rejection is due to the large statistics for France and the United Kingdom. Excluding these two countries from the panel yields a panel test of the Fisher effect of 12.03 which is distributed  $\chi^2(7)$  which has a 95% critical value of 14.07.

Table 7 displays Johansen results from a model that restricts the constant term to the cointegration vector. This model is directly comparable to the FM-OLS and DOLS models that include a constant only. The 95% (90%) critical value for the trace statistics presented in column two of table 7, taken from Osterwald-Lenum (1992), is 19.96 (17.85). This leads to rejection of the null of no cointegration in only four out of the nine countries at the 5% level of significance and six out of nine at the 10% level. The last row of this column shows the panel trace statistic. This statistic has a standard normal distribution and so the null of no cointegration is rejected at about the 8% level of significance for a one-sided test. Column 3 again displays the estimated Fisher effects and they are mostly less than one. Column 3 presents LR tests of the null that the constant term in the cointegrating vector can be

excluded. This hypothesis is rejected in six of the nine cases. Its panel counterpart in the last row is significant at a very high marginal level of significance. The same is true for the tests of excluding  $i$  or  $\pi$  from the models. Column seven displays the LR tests of the Fisher effect. As in the specification of table 6, there are only three rejections and excluding two of these, France and the U.K., leads to a non-rejection of the panel Fisher hypothesis at conventional levels of significance. The last column of table 7 shows LR tests of the joint hypothesis of excluding the constant and imposing a Fisher effect of one. These tests are distributed  $\chi^2(2)$ . Only three out of nine cases are not rejected.

Table 8 presents the Johansen estimates and tests for a model in which all deterministic terms have been excluded and as such is comparable to the analogous FM-OLS and DOLS models. The 95% (90%) critical value for the trace statistics presented in column two of table 8, taken from Osterwald-Lenum (1992), is 12.53 (10.47). This leads to rejection of the null of no cointegration in only three out of the nine countries at the 5% level of significance and five out of nine at the 10% level. In contrast to the other Johansen estimates, all of the Fisher effect estimates are greater than one in table 8. Furthermore, except for the U.K., all of the exclusion tests can be rejected at the 5% level of significance. The tests for a Fisher effect of unity lead to five rejections out of nine cases. But these rejections are all in favor of Fisher effects greater than one.

## 5 Conclusions

I draw three conclusions from the above empirical analysis. First, the evidence on the unit root characteristics of nominal interest rates and inflation rates among the industrialized nations is consistent with each evolving as an I(1) process over the last forty years. While this result may be at odds with theoretical constraints on nominal interest rates, i.e. zero lower bound, it is entirely consistent with conditions favorable to an equilibrium Fisher re-

relationship between non-stationary inflation rates and nominal interest rates. Second, the estimates of the Fisher effect over the 150 specifications examined yielded results consistent with a full Fisher effect, and thus monetary superneutrality, in 80% of the empirical specifications. Finally, given the wide range of Fisher effect estimates produced using the panel estimators, and these always depending on the deterministic specification and normalization of the regression, it is not clear that the promise of these panel techniques is manifested in the current application.

## Notes

<sup>1</sup>Some examples are Mishkin (1993), Crowder and Hoffman (1996), Lothian and Simaan (1998) and Crowder (1997).

<sup>2</sup>These include MacDonald and Murphy (1989), King and Watson (1997), and Koustas and Serletis (1999).

<sup>3</sup>See Levi and Makin's (1978) equation (6) for further elaboration on this point. A nice derivation of the Tobin effect within a dynamic general equilibrium model is that given by Ahmed and Rogers (1996).

<sup>4</sup>This is simply a recognition that in the steady-state equilibrium the LM curve is vertical.

<sup>5</sup>The astute reader might have already recognized that equation (1) has some serious drawbacks as an explanation for low Fisher effect estimates in light of recent empirical estimates of the parameters in the equation. In the U.S., for example, the interest elasticity of money demand has been estimated in range of 0.0 to -0.2, e.g. Hoffman and Rasche (1991) or Mark and Sul (2001), much too low to generate significant departures from the Fisher effect.

<sup>6</sup>An exception is Crowder (1997), where the long-run Fisher relation is examined for Canada.

<sup>7</sup>It is generally excepted by international economists that real interest rates are not equal internationally. Chung and Crowder (2002) present evidence that the source of rejection of real rate equality is uncovered interest parity and ex-ante PPP.

<sup>8</sup>The assumption of stationarity of both the conditional variance of inflation and the

conditional covariance of inflation and consumption growth has limited empirical support one way or the other. Theoretically there is nothing that prevents these conditional variances from being integrated. One way to model potentially integrated conditional variances is the IGARCH model of Engle and Bollerslev (1986). Therefore, the tests for cointegration employed in the paper can be interpreted as joint tests of the stationarity of real rates and of these conditional variances.

<sup>9</sup>This last point is not as innocuous as it may seem. The constant term in (6) cannot be interpreted as an estimate of the average real rate given the unknown properties of the conditional variance terms in (5).

<sup>10</sup>The size of the Darby effect is unclear as demonstrated by Gandolfi (1982).

<sup>11</sup>Mishkin (1992) and Crowder and Hoffman (1996) demonstrate the relevance of this issue for U.S. inflation rates over the post-war period

<sup>12</sup>See Papell (1998) and the references therein for a discussion of the panel unit root tests as applied to the PPP hypothesis.

<sup>13</sup>O'Connell (1998) showed that the Levin and Lin test suffers from extreme size distortion (rejects a true null too often) when the contemporaneous error terms are correlated across groups (referred to as spatial correlation in the literature). O'Connell further demonstrated that once this spatial correlation was controlled for, the power of these tests dropped significantly.

<sup>14</sup>Taylor and Sarno (1998) have shown that rejection of the null hypothesis in the panel unit root tests cannot be interpreted as stationarity of all the series in the panel. These tests are uninformative about the number of series that are stationary versus the number that are non-stationary. Breuer et al (2001) advocate using a ADF-SUR estimator that overcomes this

problem, but the size and power properties of this test have not been adequately investigated.

<sup>15</sup>Elliot and Jansson (2000) show that these tests can achieve the power envelope under very general conditions

<sup>16</sup>See Phillips and Loretan (1991).

<sup>17</sup>The trace test takes the null hypothesis of at least  $r$  cointegrating relationships in  $X_t$  versus the alternative of  $p$  cointegrating vectors.

<sup>18</sup>The Ng and Perron lag selection procedure requires the econometrician to specify a maximum lag allowed and then uses the modified information criteria to determine the appropriate lag length in a general-to-specific fashion. I set this maximum allowable lag to 18 for the results presented in table 1. While testing the sensitivity of the results to the maximum allowable lag, I found that allowing the maximum to be 64 lags (a value that might be considered excessive) lead to the MBIC of Ng and Perron choosing 47 and 50 lags for the German and U.S. interest rate specifications, respectively. Both tests yielded non-rejections of the unit root null at these large lags.

<sup>19</sup>Each panel estimator included a constant and time trend, allowing for individual specific effects. The number of lags included for each series in the panel is the same as the lag truncation chosen by the Ng and Perron MIC. Results for specifications that included only a constant term yielded identical inference

<sup>20</sup>Hansen (1995) uses Monte Carlo experiments to analyze the power and size of this test and finds that the size distortion associated with negative moving average errors is not significant until the MA(1) parameter is less than -0.5. I estimated simple ARIMA(1,1,1) models for each of the series used in this study to determine the probable size distortions associated with the CADF tests and found that no series had an MA(1) parameter smaller

than -0.36.

<sup>21</sup>Engle and Granger (1987) noted that in cointegrating regressions the estimated slope parameters from reverse regressions will be inverse of each other asymptotically due to the superconsistency of the cointegrating estimator. But in finite samples this result will only hold when the regression  $R^2$  is one.

<sup>22</sup>Note that the reported Fisher effects from the regressions with nominal interest rate as the dependent variable are simply the estimated slope parameter while those from the specifications using inflation as the dependent variable are the inverse of the estimated slope parameter.

<sup>23</sup>In only one specification was the linear trend coefficient statistically significant at conventional significance levels, that for the Netherlands with inflation as the dependent variable. The constant term, however, was significant in all of the regressions that used nominal interest rates as the dependent variable and excluded the linear trend. Otherwise, the constant was insignificant in all other specifications.

<sup>24</sup>Ng and Perron (1997) and Crowder and Wohar (1999) observed similar findings for U.S. data.

<sup>25</sup>Results for the models using different deterministic specifications yielded identical inference and so are not presented. These and all other results can be obtained from the author upon request.

<sup>26</sup>Since the Johansen estimator is a systems estimator, the normalization issue associated with single equation estimators is not relevant.

<sup>27</sup>This is not entirely consistent with the FM-OLS and DOLS models with linear trends

and constants in the empirical models since the latter allow for a trend in the cointegration vector while the former only allows for linear trends in the data. Employing the Johansen specification that allows for linear trends in the cointegration vector yielded results consistent with those presented in table 6. Furthermore, likelihood ratio tests of the significance of the trend terms yielded no rejections of the null hypothesis of excluding such trends.

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Table 1

Unit Root Tests												
	Inflation Rates						Interest Rates					
	ADF	DF <sup>GLS</sup>	MZ <sub>ρ</sub>	MZ <sub>t</sub>	ℓ	AIC	ADF	DF <sup>GLS</sup>	MZ <sub>ρ</sub>	MZ <sub>t</sub>	ℓ	AIC
<b>Belgium</b>	-2.31	-1.54	-5.28	-1.59	16	-2.15	-1.57	-1.27	-3.69	-1.27	1	-1.74
<b>Canada</b>	-2.13	-1.48	-6.26	-1.69	16	-2.10	-2.17	-2.12	-10.69	-2.27	10	-1.13
<b>France</b>	-1.59	-1.25	-3.48	-1.27	15	-2.47	-2.06	-1.77	-6.85	-1.78	1	-0.79
<b>Germany</b>	-2.48	-1.97	-8.59	-2.05	13	-2.13	-3.96	-3.60	-46.63	-4.82	13	-1.79
<b>Italy</b>	-1.44	-1.06	-2.67	-1.08	15	-1.62	-1.08	-1.04	-3.59	-1.17	16	-1.45
<b>Japan</b>	-2.86	-2.24	-10.65	-2.28	12	-0.77	-2.17	-1.93	-7.93	-1.93	1	-2.16
<b>Netherlands</b>	-2.53	-1.18	-2.96	-1.22	13	-1.17	-1.71	-1.08	-3.10	-1.10	1	-2.16
<b>United Kingdom</b>	-2.00	-1.47	-5.13	-1.53	13	-1.31	-2.59	-2.23	-10.76	-2.24	1	-0.72
<b>United States</b>	-2.53	-1.97	-10.20	-2.21	13	-2.63	-2.87	-2.76	-21.30	-3.26	16	-1.33
<b>95% cv</b>	-3.41	-2.91	-17.30	-2.91			-3.41	-2.91	-17.30	-2.91		

NOTES: All regressions included a constant and a linear trend.  $\ell$  denotes the lag truncation used for the augmented regressions. This was chosen using the modified Bayesian information criterion of Ng and Perron (2000). AIC is the standard Akaike information criterion from the  $DF^{GLS}$  specification and is presented for comparison with the CADF tests in table 2.

**Table 2**

<b>CADF Unit Root Tests</b>													
	<b>Inflation Rates</b>							<b>Interest Rates</b>					
	<b>CADF</b>	$\eta^2$	$\rho^2$	<b>95% cv</b>	<b>AIC</b>	$\zeta$		<b>CADF</b>	$\eta^2$	$\rho^2$	<b>95% cv</b>	<b>AIC</b>	$\zeta$
<b>Belgium</b>	-1.13	0.23	0.22	-2.60	-2.53	7.38		-0.94	0.45	0.44	-2.93	-2.55	2.58
<b>Canada</b>	-2.12	0.41	0.41	-2.88	-2.33	2.62		-2.13	0.20	0.23	-2.55	-1.70	6.35
<b>France</b>	-1.03	0.15	0.12	-2.43	-2.76	10.17		-2.02	0.28	0.26	-2.69	-1.74	5.78
<b>Germany</b>	-2.49	0.37	0.31	-2.83	-2.41	5.05		-2.24	0.26	0.23	-2.65	-2.21	6.78
<b>Italy</b>	-1.01	0.23	0.23	-2.61	-1.99	7.72		0.51	0.34	0.35	-2.79	-1.73	3.08
<b>Japan</b>	-1.35	0.29	0.29	-2.71	-1.10	5.60		-0.66	0.30	0.30	-2.72	-3.07	6.57
<b>Netherlands</b>	-2.54	0.39	0.39	-2.85	-1.49	3.50		-1.83	0.22	0.24	-2.59	-3.31	6.23
<b>United Kingdom</b>	-0.88	0.36	0.38	-2.81	-1.66	3.12		-1.27	0.48	0.48	-2.97	-1.46	1.43
<b>United States</b>	-0.85	0.18	0.17	-2.51	-3.13	7.64		-0.50	0.16	0.16	-2.44	-2.04	7.12

NOTES: All regressions included a constant and a linear trend. Critical values are calculated from equation (22) using the estimate of  $\eta^2$ . AIC is the minimized value of the Akaike information criteria and  $\zeta$  is the bandwidth selected using the procedure of Andrews (1991) used to calculate the estimate of the long-run variances.

Table 3

FM-OLS Cointegration Estimates						
	w/Constant and Trend		w/Constant		No Constant	
	$Fisher_i$	$Fisher_\pi$	$Fisher_i$	$Fisher_\pi$	$Fisher_i$	$Fisher_\pi$
<b>Belgium</b>	1.03 (0.36)	1.28 (0.27)	1.00 (0.33)	1.25 (0.28)	1.69 (0.24)	1.69 (0.09)
<b>Canada</b>	0.92 (0.25)	1.01 (0.18)	0.93 (0.26)	0.95 (0.20)	1.51 (0.21)	1.41 (0.08)
<b>France</b>	0.79 (0.15)	0.88 (0.16)	0.71 (0.15)	0.72 (0.19)	1.35 (0.15)	1.27 (0.07)
<b>Germany</b>	1.35 (0.36)	1.45 (0.16)	1.29 (0.36)	1.35 (0.17)	1.78 (0.20)	1.75 (0.06)
<b>Italy</b>	0.82 (0.15)	1.05 (0.22)	0.81 (0.15)	1.00 (0.21)	1.23 (0.11)	1.25 (0.08)
<b>Japan</b>	0.30 (0.13)	1.04 (0.51)	0.72 (0.15)	0.85 (0.32)	1.42 (0.17)	1.39 (0.12)
<b>Netherlands</b>	1.14 (0.32)	1.41 (0.23)	0.58 (0.30)	1.67 (0.30)	1.68 (0.23)	1.69 (0.09)
<b>United Kingdom</b>	0.55 (0.15)	0.58 (0.29)	0.49 (0.16)	0.53 (0.33)	1.19 (0.18)	1.08 (0.11)
<b>United States</b>	0.59 (0.12)	0.70 (0.17)	0.58 (0.12)	0.66 (0.17)	1.24 (0.11)	1.19 (0.06)
<i>Panel</i> FM-OLS	0.45 (0.02)	1.63 (0.02)	0.42 (0.02)	1.72 (0.02)	1.10 (0.02)	1.52 (0.02)

NOTES:  $Fisher_i$  is the estimate of the Fisher effect from the empirical model where nominal interest rate is the dependent variable.  $Fisher_\pi$  is the estimate of the Fisher effect from the empirical model where inflation is the dependent variable, i.e. the inverse of the slope estimate in this *reverse* regression. Numbers in parentheses represent asymptotically valid standard errors.

Table 4

DOLS Cointegration Estimates						
	w/Constant and Trend		w/Constant		No Constant	
	$Fisher_i$	$Fisher_\pi$	$Fisher_i$	$Fisher_\pi$	$Fisher_i$	$Fisher_\pi$
<b>Belgium</b>	0.54 (0.55)	1.92 (0.26)	0.50 (0.51)	1.92 (0.30)	1.40 (0.38)	1.79 (0.10)
<b>Canada</b>	0.71 (0.22)	1.69 (0.22)	0.64 (0.25)	1.92 (0.26)	1.32 (0.27)	1.67 (0.11)
<b>France</b>	0.67 (0.20)	1.35 (0.32)	0.56 (0.21)	1.47 (0.38)	1.19 (0.28)	1.49 (0.15)
<b>Germany</b>	0.99 (0.19)	1.72 (0.12)	0.92 (0.19)	1.67 (0.14)	1.63 (0.17)	1.82 (0.05)
<b>Italy</b>	0.53 (0.19)	1.25 (0.29)	0.48 (0.22)	1.47 (0.30)	1.05 (0.23)	1.35 (0.12)
<b>Japan</b>	0.13 (0.16)	1.92 (0.45)	0.38 (0.20)	0.96 (0.31)	0.99 (0.33)	1.35 (0.11)
<b>Netherlands</b>	0.52 (0.31)	1.69 (0.25)	0.27 (0.25)	2.44 (0.36)	1.36 (0.52)	1.82 (0.10)
<b>United Kingdom</b>	0.43 (0.16)	1.03 (0.42)	0.36 (0.18)	1.23 (0.51)	0.97 (0.27)	1.27 (0.17)
<b>United States</b>	0.56 (0.22)	1.49 (0.25)	0.54 (0.22)	1.59 (0.27)	1.15 (0.23)	1.41 (0.10)
<i>Panel DOLS</i>	0.52 (0.09)	1.41 (0.13)	0.47 (0.09)	1.49 (0.15)	0.52 (0.11)	1.41 (0.06)

NOTES:  $Fisher_i$  is the estimate of the Fisher effect from the empirical model where nominal interest rate is the dependent variable.  $Fisher_\pi$  is the estimate of the Fisher effect from the empirical model where inflation is the dependent variable, i.e. the inverse of the slope estimate in this *reverse* regression. Numbers in parentheses represent asymptotically valid standard errors.

**Table 5**

<b>Panel Cointegration Tests</b>					
	$DF_{\rho}$	$DF_t$	$DF_{\rho}^*$	$DF_t^*$	$ADF$
FM-OLS $_i$	-102.74	-56.16	-182.18	-34.85	-5.24
FM-OLS $_{\pi}$	-84.28	-52.13	-151.57	-31.19	-5.15
DOLS $_i$	-23.88	-18.35	-46.41	-11.14	-3.29
DOLS $_{\pi}$	-12.23	-9.30	-25.83	-6.20	-4.09

NOTES: All statistics are asymptotically distributed as standard normal variates.

**Table 6**

<b>Johansen Cointegration Estimates - Linear Trends in Data</b>					
	$r = 0$	$\hat{\beta}$	$\beta_i = 0$	$\beta_{\pi} = 0$	$\beta_{\pi} = \beta_i$
<b>Belgium</b>	11.96	1.95	0.71	2.78	0.68
<b>Canada</b>	25.29	0.60	13.35	4.14	2.62
<b>France</b>	20.60	0.52	15.44	5.56	6.15
<b>Germany</b>	41.23	0.86	32.11	16.07	0.95
<b>Italy</b>	21.36	0.69	12.09	7.93	2.06
<b>Japan</b>	11.88	0.75	6.23	9.10	0.69
<b>Netherlands</b>	18.70	0.07	6.17	0.04	4.81
<b>United Kingdom</b>	16.84	0.15	7.81	0.41	7.81
<b>United States</b>	18.52	0.78	3.03	2.35	0.22
$\Upsilon_{LR}\{H(r)   H(p)\}$	2.60		96.94	48.38	25.99

NOTES: The column labelled  $r = 0$  is the trace statistic of the null hypothesis of no cointegration. The 95% critical value for this is 15.41 taken from Osterwald-Lenum (1992) table 1.  $\hat{\beta}$  is the estimate of the Fisher effect.  $\beta_i = 0$  is the likelihood ratio test of the null that the nominal interest rate can be excluded from the model and is distributed as a  $\chi^2(1)$  variate.  $\beta_{\pi} = 0$  is the likelihood ratio test of the null that inflation can be excluded from the model and is distributed as a  $\chi^2(1)$  variate.  $\beta_{\pi} = \beta_i$  is the likelihood ratio test of the null that the Fisher effect is 1.0 and is distributed as a  $\chi^2(1)$  variate.  $\Upsilon_{LR}\{H(r) | H(p)\}$  is the panel trace test and is distributed as a standard normal variate. The statistics on the last row are the panel counterparts to the individual group statistics. Under the assumption that the LR tests are independent the panel tests of  $\beta_i = 0$ ,  $\beta_{\pi} = 0$  and  $\beta_{\pi} = \beta_i$  are distributed as  $\chi^2(9)$ .

Table 7

Johansen Cointegration Estimates - Constant in Cointegrating Vector							
	$r = 0$	$\hat{\beta}$	$\beta_0 = 0$	$\beta_i = 0$	$\beta_\pi = 0$	$\beta_\pi = \beta_i$	$\beta_\pi = \beta_i$ and $\beta_0 = 0$
<b>Belgium</b>	12.02	1.97	0.02	0.70	2.83	0.70	4.49
<b>Canada</b>	25.32	0.60	8.00	13.34	8.00	2.61	12.09
<b>France</b>	20.76	0.52	10.51	15.34	5.51	6.19	11.31
<b>Germany</b>	41.25	0.86	17.97	32.09	16.07	0.95	28.79
<b>Italy</b>	21.37	0.69	4.53	12.09	7.94	2.06	6.43
<b>Japan</b>	12.90	0.74	1.92	6.37	8.26	0.76	2.92
<b>Netherlands</b>	18.88	0.07	6.58	6.33	0.04	4.88	10.62
<b>United Kingdom</b>	16.94	0.15	8.58	7.73	0.40	7.79	9.08
<b>United States</b>	18.59	0.79	1.02	3.06	2.40	0.21	5.63
$\Upsilon_{LR} \{H(r)   H(p)\}$	1.42		59.13	97.05	51.45	26.15	91.36

NOTES: The column labelled  $r = 0$  is the trace statistic of the null hypothesis of no cointegration. The 95% critical value for this is 19.96 taken from Osterwald-Lenum (1992) table 1\*.  $\hat{\beta}$  is the estimate of the Fisher effect.  $\beta_0 = 0$  is the likelihood ratio test of the null that the constant can be excluded from the model and is distributed as a  $\chi^2(1)$  variate.  $\beta_i = 0$  is the likelihood ratio test of the null that the nominal interest rate can be excluded from the model and is distributed as a  $\chi^2(1)$  variate.  $\beta_\pi = 0$  is the likelihood ratio test of the null that inflation can be excluded from the model and is distributed as a  $\chi^2(1)$  variate.  $\beta_\pi = \beta_i$  is the likelihood ratio test of the null that the Fisher effect is 1.0 and is distributed as a  $\chi^2(1)$  variate.  $\beta_\pi = \beta_i$  and  $\beta_0 = 0$  is the likelihood ratio test of the null that the Fisher effect is 1.0 and that the constant can be excluded and is distributed as a  $\chi^2(2)$  variate.  $\Upsilon_{LR} \{H(r) | H(p)\}$  is the panel trace test and is distributed as a standard normal variate. The statistics on the last row are the panel counterparts to the individual group statistics. Under the assumption that the LR tests are independent the panel tests of  $\beta_0 = 0$ ,  $\beta_i = 0$ ,  $\beta_\pi = 0$  and  $\beta_\pi = \beta_i$  are distributed as  $\chi^2(9)$  and the panel hypothesis  $\beta_\pi = \beta_i$  and  $\beta_0 = 0$  is distributed as  $\chi^2(18)$ .

Table 8

Johansen Cointegration Estimates - No Deterministic Terms					
	$r = 0$	$\hat{\beta}$	$\beta_i = 0$	$\beta_\pi = 0$	$\beta_\pi = \beta_i$
<b>Belgium</b>	7.76	1.70	6.13	6.98	4.47
<b>Canada</b>	12.95	1.46	9.41	9.24	4.09
<b>France</b>	9.09	1.21	6.19	5.15	0.79
<b>Germany</b>	19.98	1.66	17.41	16.19	10.82
<b>Italy</b>	13.23	1.20	11.10	11.06	0.00
<b>Japan</b>	10.60	1.23	6.43	7.06	1.00
<b>Netherlands</b>	6.59	1.78	4.77	6.00	4.04
<b>United Kingdom</b>	5.99	1.26	2.06	2.59	0.50
<b>United States</b>	10.85	1.40	7.62	9.06	4.61
$\Upsilon_{LR}\{H(r)   H(p)\}$	1.30		71.12	73.33	30.32

NOTES: See notes to table 6. Trace test 95% critical value of 12.53 from table 0 in Osterwald-Lenum (1992).

# Nominal Interest Rates and Inflation Rates

January 1960 to December 2000

