MATH 2425, Calculus II

Lab 3: Integration of Rational Functions and Applications

Week of September 7, 2009

Exam dates:
Midterm 1: Friday, September 18, 2009 from 6-8 pm
Midterm 2: Friday, October 23, 2009 from 6-8 pm
Final Exam: Saturday, December 5, 2009 from 3-5:30 pm
If you have a conflict with ANY of the above dates, you must contact the course coordinator no later than Census Date (September 9), by using a form attached to the coordinator's office door (PKH 448) and submitting it together with necessary documentation as indicated on the form.

Study Technique 3: After you complete your homework, read the section(s) in the textbook that will be covered in the next lecture. It is likely that you will not understand everything you read; however, reading the sections before lecture will help you understand what is presented in the lecture.

Social Diffusion

Sociologists sometimes use the phrase social diffusion to describe the way information spreads through a population. The information might be a rumor, a cultural fad, or news about a technical innovation. In a sufficiently large population, the number of people x who have the information is treated as a differentiable function of time t.The rate of diffusion, $\frac{dx}{dt}$, is assumed to be proportional to the number of people who have the information times the number of people who do not. This leads to the differential equation $\frac{dx}{dt} = kx(N - x)$, where N is the number of people in the population.

Problem 1. Suppose t is measured in days, $k = 1/25$, and four people start a rumor at time $t = 0$ in a population of $N = 100$ people.

a. Find x as a function of t by integrating both sides of the equation

$$\frac{1}{x(N-x)}\,dx = k\,dt$$

b. When will half the population have heard the rumor.

c. When will the rumor be spreading the fastest?

Second-order Chemical Reactions

Many chemical reactions are the result of the interaction of two molecules that undergo a change to produce a new product. The rate of the reaction typically depends on the concentration of the two kinds of molecules. If a is the amount of substance A and b is the amount of substance B at time $t = 0$, and if x is the amount of product at time t, then the rate of formation of x may be given by the differential equation

$$\frac{dx}{dt} = k(a - x)(b - x),$$

or

$$\frac{1}{(a - x)(b - x)} \frac{dx}{dt} = k,$$

where k is a constant for the reaction.

Problem 2. Integrate both sides of the equation

$$\frac{1}{(a - x)(b - x)}\, dx = k\, dt$$

to obtain a relation between x and t if

a. $a = b$,

b. $a \neq b$.

Assume in each case that $x = 0$ when $t = 0$.

Approximating π using integrals

It is well known that the number π is "approximately" $\frac{22}{7}$. You may visualize this approximation using integrals as the following problem shows.

Problem 3.

a. Evaluate the integral

$$\int_0^1 \frac{x^4(x - 1)^4}{x^2 + 1}\, dx$$

b. Find out how good the approximation $\pi \approx \frac{22}{7}$ is by expressing $\left(\frac{22}{7} - \pi\right)$ as a percentage of π (you do not have to use integrals to solve this part).