

MATH 2425  
CALCULUS II  
Lab 4: Week of September 21, 2009  
Improper Integrals

**Problem 1.** Evaluate each of the following integrals:

a)  $F(t) = \int_1^t \frac{1}{x^2} dx$

b)  $G(t) = \int_1^t \frac{1}{x} dx$

c)  $H(t) = \int_1^t \frac{1}{\sqrt{x}} dx$

**Problem 2.** Evaluate each of the following limits for functions given in **Problem 1**:

a)  $\lim_{t \rightarrow \infty} F(t)$

b)  $\lim_{t \rightarrow \infty} G(t)$

c)  $\lim_{t \rightarrow \infty} H(t)$

**Problem 3.** Let  $J(t) = \int_1^t \frac{1}{x^\alpha} dx$ , where  $\alpha$  is any real number. Investigate for which values of  $\alpha$  does  $\lim_{t \rightarrow \infty} J(t)$  converge, and for which values of  $\alpha$  does  $\lim_{t \rightarrow \infty} J(t)$  diverge. In case of convergence, what does  $\lim_{t \rightarrow \infty} J(t)$  represent.

**Problem 4.** Evaluate each of the following integrals:

a)  $W(t) = \int_t^1 \frac{1}{x^2} dx$

b)  $X(t) = \int_t^1 \frac{1}{x} dx$

c)  $Y(t) = \int_t^1 \frac{1}{\sqrt{x}} dx$

**Problem 5.** Evaluate each of the following limits for functions given in **Problem 4**:

a)  $\lim_{t \rightarrow 0^+} W(t)$

b)  $\lim_{t \rightarrow 0^+} X(t)$

c)  $\lim_{t \rightarrow 0^+} Y(t)$

**Problem 6.** Let  $K(t) = \int_t^1 \frac{1}{x^\beta} dx$ , where  $\beta$  is any real number. Investigate for which values of  $\beta$  does  $\lim_{t \rightarrow 0^+} K(t)$  converge, and for which values of  $\beta$  does  $\lim_{t \rightarrow 0^+} K(t)$  diverge. In case of convergence, what does  $\lim_{t \rightarrow 0^+} K(t)$  represent.

**Problem 7.** Assume that  $\alpha \neq 0$ , and let  $\beta = 1/\alpha$  (which must not be zero, either). Find out a relationship between  $\lim_{t \rightarrow \infty} J(t)$  and  $\lim_{t \rightarrow 0^+} K(t)$ .