**MATH 2425, Calculus II**
**Lab 6: Problem Solving Activity Week of 5 October 2009**

Fun With Series

**1.** Which of the following are always true and which are sometimes false? If the answer is sometimes false, give an example of a series that demonstrates this. If the answers is always true, indicate why by referencing a theorem or an idea in the book.

**a.** If $\sum a_k$ converges, then $\lim_{k\to\infty} a_k = 0$.

**b.** If $\lim_{k\to\infty} a_k = 0$ then $\sum a_k$ converges.

**c.** If $\sum a_k$ diverges and $\sum b_k$ diverges then $\sum(a_k + b_k)$ diverges.

**d.** If $\sum a_k$ converges and $\sum b_k$ diverges then $\sum(a_k + b_k)$ diverges.

**e.** $\sum_{k=0}^{\infty} r^k = 1/(1-r)$ whenever $r \neq 1$.

**f.** If the partial sums $S_n = \sum_{k=1}^{n} a_k$ are bounded, then $\sum a_k$ converges

**g.** If $a_k \geq 0$ for all $k$ and the partial sums are bounded, then $\sum a_k$ converges

**h.** If $\sum a_k$ converges and $\sum b_k$ converges then $\sum(a_k b_k) = (\sum a_k)(\sum b_k)$

**2.** Is there a value $r$ such that $\sum_{k=0}^{\infty} r^k = \frac{7}{8}$? What about $\sum_{k=0}^{\infty} r^k = \frac{1}{2}$? Explain!

**3.** Compute (note that $r \to -1^+$, so $r > -1$)

$$\lim_{r\to(-1)^+} \left( \sum_{k=0}^{\infty} r^k \right)$$

Does it make sense to compute

$$\sum_{k=0}^{\infty} \left( \lim_{r\to(-1)^+} r^k \right)?$$