MATH 2425, Calculus II

Lab 7: Convergence Tests for Series

Week of October 12-16, 2009

Limit Comparison Test

1. Suppose that $a_n > 0$ and $b_n > 0$ for $n \geq N$ (N an integer). Which of the following are always true? If the answer is not always true, give an example of a series for which it is false. If the answer is always true, indicate why and refer to a statement written in the book or stated in class.

a. If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ converges then $\sum a_n$ converges.

b. If $\lim_{n\to\infty} \frac{b_n}{a_n} = \infty$ and $\sum b_n$ converges then $\sum a_n$ converges.

c. If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ diverges then $\sum a_n$ diverges.

The Ratio and Root Tests

2. Give an example for a series $\sum a_n$ with $a_n \geq 0$ for $n \geq N$ (N an integer) for which

a. $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ and $\sum a_n$ is convergent.

b. $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ and $\sum a_n$ is divergent.

c. $\lim_{n\to\infty} \sqrt[n]{a_n} = 1$ and $\sum a_n$ is convergent.

d. $\lim_{n\to\infty} \sqrt[n]{a_n} = 1$ and $\sum a_n$ is divergent.

3. For the following series use the Ratio and the Root Tests to check if the series is convergent. If neither of the tests provides information about convergence use another test to solve the problem.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$

c. $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^2}$

d. $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^n}$