MATH 2425

CALCULUS II

Lab 8: Week of October 26, 2009

Power Series and Taylor Series

Problem 1. Assume that $\sum |a_k|$ converges. Prove that the radius of convergence for $\sum a_k x^k$ is at least 1. *Hint: show that the power series is convergent at* $x = \pm 1$.

Problem 2. What is wrong with the following computation for $x > 0$?

$$2 < (1 + x + x^2 + x^3 + \cdots) + \left(1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \cdots\right) = \frac{1}{1-x} + \frac{1}{1-\frac{1}{x}} = 1.$$

Problem 3. Suppose that the power series $\sum_{k=0}^{\infty} c_k x^k$ converges when $x = -4$ and diverges when $x = 7$. Which of the following are true, false, or not possible to answer? Give a reason for each answer.

1. The power series converges when $x = 10$.
2. The power series converges when $x = 3$.
3. The power series converges when $x = 1$.
4. The power series converges when $x = -6$.

Problem 4. Taylor's remainder formula can be used to derive some handy inequalities. Derive the following

1. $|\ln(1+x) - x| \le \frac{1}{2}x^2$ for $x \ge 0$.
2. $|\sin x - x| \le \frac{1}{6}|x|^3$ for all x.

Problem 5. Why doesn't $x^{1/3}$ have a Taylor series about $x = 0$?