A note on "Accuracy of Computed Eigenvectors via Optimizing a Rayleigh Quotient"

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Proof of Theorem 2.2 in [2] can be easily modified to improve [2, (2.6)] to

$$\|\sin\Theta(U_k,\widetilde{U}_k)\|_{\rm F}\le\frac{\epsilon}{\sqrt{\lambda_k-\lambda_{k+1}}} \tag{2.6'}$$

which is [1, Theorem 4] and can also be derived from some of the estimates in [3, Chapter 5].

The modification goes simply by improving [2, (2.8)] and [2, (2.9)] to, respectively,

$$\operatorname{trace}((I_k-YY^*)\Lambda_k)\ge\lambda_k\operatorname{trace}(I_k-YY^*), \tag{2.8'}$$

$$\operatorname{trace}(Q^*ZZ^*Q\Omega)\le\mu_1\operatorname{trace}(Q^*ZZ^*Q). \tag{2.9'}$$

They are true because $I_k-YY^*\succeq 0$, $\Lambda_k\succeq\lambda_k I_k$, $Q^*ZZ^*Q\succeq 0$, and $\Omega\preceq\mu_1 I_\ell$, and thus

$$(I_k-YY^*)^{1/2}\Lambda_k(I_k-YY^*)^{1/2}\succeq\lambda_k\,(I_k-YY^*),$$
$$(Q^*ZZ^*Q)^{1/2}\Omega(Q^*ZZ^*Q)^{1/2}\preceq\mu_1\,(Q^*ZZ^*Q).$$

They imply (2.8') and (2.9').

References

[1] J. Kovač-Striko and K. Veselić, *Some remarks on the spectra of Hermitian matrices*, 145 (1991), pp. 221–229.

[2] R.-C. Li, *Accuracy of computed eigenvectors via optimizing a Rayleigh quotient*, 44 (2004), pp. 585–593.

[3] H. F. Weinberger, *Variational Methods for Eigenvalue Approximation*, vol. 15 of CBMS-NSF Regional Conference Series in Applied Mathematics, SIAM, Philadelphia, 1974.

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