MATH 1426 (Sec 101, 301 & 302) INFORMATION SHEET FOR MIDTERM 2 SPRING 2006

MIDTERM 2 is in PKH 319, 6:00-8:00 pm on Fri Mar 31

- See above for date, time and location of MIDTERM 2.
- The material covered on MIDTERM 2 is the same as that covered on the homework and on the worksheets through §4.3 inclusive. (Homework is listed over the page.)
- Ken and I are available in our office hours (see syllabi).
- This MIDTERM will be, in part, multiple choice. Half of the points will be for the multiple-choice part, and the other half for the show-your-work part. There will be 5 choices of answer per multiple-choice question and, for each, only one answer will be the correct one. You should do rough work on the MIDTERM or on paper provided by me. You should bring with you to the MIDTERM a scantron form,

  882-ES or 882-E, a number-2 pencil, an allowed graphing calculator & photo ID.
- A good (& usually effective!) way to review is to go over the homework problems you have not already done & make sure you understand all the homework well, the worksheets well and the quizzes well. I also recommend that you look through Midterm 2 from Fall 2003 & Spring 2004 which are posted at

  www.uta.edu/faculty/retakh/1426/exams.html

  I recommend that you time yourself while working on those tests, without access to your notes. In addition, this information sheet provides some more practice problems; you will spend Wed Mar 29 working on these and other questions individually (or in groups) in class. These practice questions do NOT form a model for the midterm. These questions are intended only to help you identify any gaps in your understanding. In the last 24 hours before the midterm, reread ALL the homework problems including the worksheet questions, skim through the lecture notes, & go over the quizzes and these practice questions again.
- Learn some basic trigonometric substitutions, quadratic formula, pythagorean formula & formulae from this class. Learn some simple graphs, such as those given on Page 27 of your book.
- Bring with you to the MIDTERM a working calculator (with working batteries!), satisfying the criteria on the first-day handout, & some form of photo ID (e,g either a driver license or your UTA ID). I will ask to see the ID when you turn in your MIDTERM to me. In particular, calculators with keyboards or with internet capability are not permitted on the MIDTERM. Cell phones should be out of sight and switched off.
- Try to keep your eyes on your own work during the MIDTERM.
- If you wish to leave the room during the MIDTERM, you should ask permission first & turn in your MIDTERM to me. Only in exceptional circumstances will I let you continue the MIDTERM should you return. (So it is better to be 3 minutes late to the MIDTERM, rather than ask to go to the bathroom during the MIDTERM.) If you finish early but prefer to stay in the room, then you should NOT get out any work, book nor item, no matter what the subject matter is. Should you wish to leave the MIDTERM early, then you may.
- It is your responsibility to be on time.

PRACTICE QUESTIONS

Questions 2-15 are questions occurring on previous midterms of mine.

1. Rework practice questions from information sheet for Midterm 1.

2. [§3.4] Suppose that a bacteria population starts with 600 cells of bacteria and triples every hour.

   (a) What is the population after one hour?
   (b) What is the population after two hours?
   (c) What is the population after three hours?
   (d) What is the population after t hours?
   (e) Find the rate of increase of the bacteria population after five hours.

3. [§3.3, §3.5] Functions f and g and their first and second derivatives, f', g', f'', g'', are defined on $(-\infty, \infty)$, and, at $0, 1, 2$, they take on the values given in the following table.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$	$f''(x)$	$g''(x)$
0	5	3	2	-10	7	3
1	0	3	2	-4	-3	4
2	-8	7	6	3	4	3

   (a) Let $G(x) = x^4 f(x)$. Compute $G'(2)$ if it exists; if it does not exist, explain why not.
   (b) Let $H(x) = g(f(x))$. Compute $H'(1)$ if it exists; if it does not exist, explain why not.
   (c) Let $F(x) = e^{g(x)}$. Compute $F'(0)$ if it exists; if it does not exist, explain why not.
   (d) Let $K(x) = \ln f(x)$. Compute $K'(0)$ if it exists; if it does not exist, explain why not.
   (e) Let $J(x) = \sin(x + g(x) - 3)$. Compute $J'(0)$ if it exists; if it does not exist, explain why not.
   (f) Let $M(x) = \ln f(x)$. Compute $M'(2)$ if it exists; if it does not exist, explain why not.

4. [§3.5: 50,51] The functions f and g and their first and second derivatives, f', g', f'', g'', are defined on $(-\infty, \infty)$, and, at $0, 1, 2$, they take on the values given in the following table.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$	$f''(x)$	$g''(x)$
0	5	-1	2	-4	7	3
1	-3	3	1	-10	-3	4
2	8	7	6	3	4	-5

   (a) Let $H(x) = e^{g(x)}$. Compute $H'(2)$ if it exists; if it does not exist, explain why not.
   (b) Let $H(x) = e^{g(x)}$ (as in (a)). Compute $H''(2)$ if it exists; if it does not exist, explain why not.
   (c) Let $K(x) = \ln f(x)$. Compute $K'(1)$ if it exists; if it does not exist, explain why not.

5. [§3.2: 31-34, §3.5: 53-55, §3.7: 59,60] The functions f and g and their first and second derivatives, f', g', f'', g'', are defined on $(-\infty, \infty)$, and, at $0, 1, 2$, they take on the values given in the following table.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$	$f''(x)$	$g''(x)$
0	2	-3	-4	7	-2	0
1	-4	1	10	-2	-4	-5
2	6	5	5	3	3	4

   (a) If $F(x) = \dfrac{f(x)}{x}$, compute $F'(2)$. (b) If $G(x) = (g(x))^3$, compute $G''(2)$.
   (c) If $H(x) = f(x)e^{3x}$, compute $H'(0)$. (d) If $K(x) = \tan^{-1}(f(x)) = \arctan(f(x))$, compute $K'(1)$.

6. Some functions and their corresponding derivatives are shown. Give the steps needed to compute the derivatives (NOT by using a limit) and state which rules you use.

(a) $f(x) = e^{\sin x}$ $\qquad f'(x) = (\cos x)e^{\sin x}$

(b) $f(x) = \dfrac{5x+2}{7\tan x}$ $\qquad f'(x) = \dfrac{5\cos x \sin x - 5x - 2}{7\sin^2 x}$

(c) $f(x) = 2^{x^2}$ $\qquad f'(x) = (\ln 2)x2^{x^2+1}$

(d) $4\cos x \sin y = e^y$ $\qquad \dfrac{dy}{dx} = \dfrac{4\sin x \sin y}{4\cos x \cos y - e^y}$

7. [§3.6] Given that $x^y = y^x$, compute $\dfrac{dy}{dx}$ at the point $(7, 7)$.

8. [§3.6] Given that $x^{\cos y} = y^{\sin x}$ and that $\cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}} = \sin\dfrac{\pi}{4}$, compute y' at the point $\left(\dfrac{\pi}{4}, \dfrac{\pi}{4}\right)$.

(a) 0.681 (b) 1.468 (c) $\dfrac{4+\pi\ln\pi-\pi\ln 4}{4-\pi\ln\pi+\pi\ln 4}$ (d) $\dfrac{4-\pi\ln\pi+\pi\ln 4}{4+\pi\ln\pi-\pi\ln 4}$ (e) none of these.

9. [§3.7, W6] A block of ice, in the shape of a cube, originally having volume 2,000 cm^3, is melting in such a way that the length of each of its edges is decreasing at the rate of 3 cm/hr. Assuming that the block of ice maintains its cubical shape, find the rate of change of the surface area of the cube at the time the volume is 1728 cm^3. In so doing, sketch a picture of the situation, labelling relevant items, and lay out your work very clearly (as is usually requested).

10. [§3.7] A balloon is rising vertically at a constant speed of 5 meters per second. A dog is running along a straight line at 15 meters per second, chasing the balloon, and overshoots it. When the dog passes under the balloon, the balloon is 45 meters above the dog. How fast is the distance between the dog and the balloon increasing three seconds after the dog passes under the balloon?

(a) 12.5 m/s (b) 13 m/s (c) 13.5 m/s (d) $\dfrac{22}{\sqrt{3}}$ m/s (e) none of these.

11. [§3.8] Suppose that we know that a function g has derivative $g'(x) = \sqrt{x^2+16}$ for all x, and that $g(3) = -2$. Use a tangent-line approximation to estimate the value of $g(3.05)$.

(a) -2.01 (b) -1.75 (c) -1.95 (d) -1.9 (e) none of these.

12. [§3.8] The edge of a cube was found to be 20 cm, with a possible error in measurement of 0.2 cm. What is the maximal possible error in computing the volume of the cube?

(a) 240 cm^3 (b) 270 cm^3 (c) 0.008 cm^3 (d) too negligible to compute (e) none of these.

13. [§3.6] If $f(x) = 3 + x + e^x$ and $g = f^{-1}$, then $g'(4)$ is

(a) 0 (b) $\dfrac{1}{2}$ (c) 1 (d) does not exist (e) not enough information given.

14. [§4.1] Find the absolute maximum of $f(x) = \begin{cases} 7 & \text{if } 0 \le x < 2 \\ 2x+3 & \text{if } 2 \le x \le 5. \end{cases}$.

(a) 0 (b) 2 (c) 5 (d) 7 (e) none of these.

15. [§4.3] Sketch the graph of one function f with all the following properties:

$$\begin{array}{lll} f'(x) < 0 & \text{for } x < -1, & f''(x) > 0 \quad \text{for } x < 2, \\ f'(x) < 0 & \text{for } x > 3, & f''(x) < 0 \quad \text{for } x > 2. \\ f'(x) > 0 & \text{for } x \in (-1, 3), & \end{array}$$

(Note: you are NOT asked to find a formula for f, and there could be more than one correct answer to this question, or there could be no such f.)

If you claim that there is no such f, then you should justify your claim.

Such a function described above could have relative extrema and/or inflection points; some forced by the stated conditions, and possibly some additional ones put in by you.

Give the x-coordinates where such a function **must** have relative extrema (not ones added in by you).

Give the x-coordinates where such a function **must** have inflection points (not ones added in by you).

16. Suppose f is a function with the property that $f'(x) = \cos(x^2)$. Find $g'(x)$, where $g(x) = f(x^2)$.

(a) $g'(x) = 2x\cos(x^4)$ (b) $g'(x) = \cos(x^4)$ (c) $g'(x) = \sin(x^4)$ (d) undefined (e) none of these.

17. Suppose f is a function with the property that $f'(x) = \cos(x^2)$. Find $h'(x)$, where $h(x) = f(\frac{1}{x})$.

(a) $h'(x) = -\dfrac{1}{x^2}$ (b) $h'(x) = \cos\left(\dfrac{1}{x^2}\right)$ (c) $h'(x) = -\dfrac{1}{x^2}\cos\left(\dfrac{1}{x^2}\right)$ (d) undefined (e) none of these.

18. Let functions $f(x)$ and $g(x)$ be as in Question 4. Let $P(x) = \ln(f(-g(x)))$. Compute $P'(0)$ if it exists; if it does not, explain why not

(a) $\dfrac{3}{4}$ (b) $-\dfrac{4}{3}$ (c) $\dfrac{3}{5}$ (d) does not exist (e) not enough information given.

19. If a function f has derivative $f'(x) = (1-x)^{\frac{2}{5}} - \frac{2x}{5}(1-x)^{-\frac{3}{5}}$, then f has a relative maximum at

(a) 0 (b) 1 (c) $\dfrac{5}{7}$ (d) does not exist (e) not enough information given.

20. If a function f has derivative $f'(x) = x^2(x-2)(5x-6)$, then f has a relative minimum at

(a) 0 (b) $\dfrac{6}{5}$ (c) 2 (d) does not exist (e) not enough information given.

21. If $s(t) = 1 - 2t - t^2$, then the absolute minimum of s over the interval $-4 \leq t \leq 1$ is

(a) -7 (b) -2 (c) -1 (d) 2 (e) does not exist.

22. If $h(y) = 2y^3 + 3y^2 + 4$, then the absolute minimum of h over the interval $-2 \leq y \leq 1$ is

(a) 0 (b) 4 (c) 5 (d) 9 (e) does not exist.

23. All the critical numbers of $g(t) = 5t^{\frac{2}{3}} + t^{\frac{5}{3}}$ are (a) -2 & 0 (b) -2 & 1 (c) 0 & 1 (d) -2 (e) 1.

24. A television camera is positioned 4,000 ft from the base of a rocket-launching pad. A rocket rises vertically and its speed is 600 ft/sec when it has risen 3,000 ft. How fast is the distance from the television camera to the rocket changing at that moment?

(a) 60 ft/sec (b) 360 ft/sec (c) 720 ft/sec (d) $6,000$ ft/sec (e) not enough information given.

25. If $x\sin y + \cos 2y = \cos y$, then the value of $\dfrac{dy}{dx}$ at $\left(1, \dfrac{\pi}{2}\right)$ is

(a) -1 (b) 0 (c) 1 (d) does not exist (e) not enough information given.

26. If $f(x) = x^{\sin x}$ for $x > 0$, then $f'\left(\frac{\pi}{2}\right)$ is

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) does not exist (e) not enough information given.

27. If $g(t) = t^{e^t}$, then $g'(1)$ is

(a) 0 (b) 1 (c) e (d) does not exist (e) not enough information given.

28. If $y = x^{f(x)}$, $x > 0$, for some differentiable function f, then $\frac{dy}{dx}$ is

(a) $f(x)x^{f(x)-1}$ (b) $f'(x)(\ln x)x^{f(x)}$ (c) $f'(x)(\ln x)x^{f(x)} + f(x)x^{f(x)-1}$ (d) does not exist (e) not enough information given.

29. If $y = f(x)^x$, for some positive-valued differentiable function f, then $\frac{dy}{dx}$ is

(a) $f(x)^x \ln(f(x))$ (b) $f(x)^x \ln(f(x)) + xf'(x)f(x)^{x-1}$ (c) $xf'(x)f(x)^{x-1}$ (d) does not exist (e) not enough information given.

30. Find the smallest and largest values of $x^4 - 2x^5 + 5$ on $[0, 1]$.

(a) 4, 5.00512 (b) 4, 5 (c) 3.9981, 5.01 (d) 2/5, 1 (e) 2/5, 5.

31. Find two numbers whose product is a minimum, given that one of the numbers is nine less than one-fifth of the other. Fully justify your answer mathematically.

(a) 5.005, −7.999 (b) $-\sqrt{2}$, $-\frac{1}{5}\sqrt{2} - 9$ (c) 1.1, −8.78 (d) 5.5, −7.9 (e) 22.5, −4.5.

32. [§4.1] A piece of wire 30 cm long is cut into two pieces. One piece is bent into a square and the other piece is shaped into a circle. The total area enclosed by the square and circle is given by the formula $\frac{x^2}{16} + \frac{(30-x)^2}{4\pi}$, where x is the length of the piece that is bent to form the square. To maximize the total area enclosed, the length x should be

(a) 0 cm (b) $\frac{120}{4+\pi}$ cm (c) 30 cm (d) does not exist (e) not enough information given.

You should also look over your old quizzes, worksheet questions, Midterm 1, the practice questions for Midterm 1 and the homework assigned from the book so far.

Also look over supplemental problems and Midterm 1 from Fall 2003 & Spring 2004 which are posted at

www.uta.edu/faculty/retakh/1426/exams.html

I recommend that you time yourself while working on those tests, without access to your notes.