Math 1426 **Lab 1**
Spring 2006 **Week of 1-16-06**

When properly used, calculators are great tools for discovering and understanding the concepts of calculus. ***However, in some instances, the answers produced by your calculator will be incorrect***. In this lab, we will investigate some of the situations where you should be skeptical of your calculator's output.

In general, there are two types of errors that can arise when performing computations with a calculator or a computer: ***round-off errors*** and ***loss-of-significance errors***. You should be aware that calculators perform most computations only approximately. Usually, this causes little difficulty because the computations perfomed by today's calculators are accurate to a very high degree. Occasionally, however, the results of round-off errors in a string of computations can be disastrous (think of what happens in the game of Gossip).

Loss-of-significance errors occur when your calculator performs arithmetic operations on very large or very small numbers. For example, most calculators will give the answer to

$$\left(10.0\times10^{50}\right)+911.0-\left(10.0\times10^{50}\right)$$

as 0 when you can clearly see that the answer is 911. The reason for this incorrect answer lies in the computer representation of real numbers.

To conserve memory and to make computations fasters, computers store positive real numbers internally in scientific notation, i.e., in the form $a\times10^{b}$ where a is a decimal number between 1 and 10 (not including 10) and b is an integer; a is called the ***mantissa*** and b is called the ***exponent.*** The number of decimals used for the mantissa and the largest possible magnitude of the exponent are machine dependent. Most scientific calculators today have at least a 14-digit mantissa and a 3-digit exponent. ***You should read your calculator manual to find out your calculator's specifications.***

1. Determine how $\frac{1}{3}$ is stored internally on a 14-digit calculator. _______________

2. Determine how $\frac{2}{3}$ is stored internally on a 10-digit calculator. _______________

3. Compute the following sum ***by hand*** (factor first and observe that each mantissa has the same number of zeros):

$$\left(1.00000000000004\times10^{18}\right)-\left(1.00000000000001\times10^{18}\right)$$

   Write your answer in scientific notation _________________________________
   and in the usual form ____________________.

4. Note that each mantissa in #3 has 15 digits. How will a calculator that retains 14 digits (or less) in a mantissa store each of the numbers? __________________
What is the ***incorrect*** answer such a calculator will display? ______________
What is the magnitude of the error? ________________________________

5. Let $f(x)=\dfrac{\left(x^3+4\right)^2-x^6}{x^3}$. Compute ***by hand*** the evaluation of $f\left(5\times10^4\right)$ as it would be performed by a 14-digit calculator. Is the answer correct? If not, point out the error. What answer does ***your*** calculator produce?

6. Go back to our first example: $\left(10.0\times10^{50}\right)+911.0-\left(10.0\times10^{50}\right)$. Now you understand why your calculator produced an incorrect answer. How could you rewrite the problem to insure that your calculator will produce the correct answer?

7. How can you rewrite the function in #5 to avoid the loss-of-significance error?

8. The defining concept of calculus is that of a ***limit.*** In coming lectures, we will discuss the meaning of the symbols

$$\lim_{x \to c} f(x) = L.$$

For now, we will say that the above means: as x gets close to c, $f(x)$ gets close to L.

Use your calculator to help you fill in the table below.

x	$\frac{1-\cos x^2}{x^4}$
0.1	
0.01	
0.001	
0.0001	
0.00001	
-0.1	
-0.01	
-0.001	
-0.0001	
-0.00001	

Based on this table, you might conjecture that as x gets close to 0, $\frac{1-\cos x^2}{x^4}$ gets close to _____, i.e., $\lim_{x \to 0} \frac{1-\cos x^2}{x^4} =$ ______. However, a loss of significance error has occurred, caused by the subtracting of nearly equal values in the numerator.

The error can be eliminated: multiply the expression $\frac{1-\cos x^2}{x^4}$ by the conjugate of the numerator and simplify. Use this equivalent expression in the second column of the next table to see that as x gets close to 0, $\frac{1-\cos x^2}{x^4}$ gets close to _____, i.e., $\lim_{x \to 0} \frac{1-\cos x^2}{x^4} =$ ______.

x	
0.1	
0.01	
0.001	
0.0001	
0.00001	
-0.1	
-0.01	
-0.001	
-0.0001	
-0.00001	

9. Graph $y = \frac{1 - \cos x^2}{x^4}$. Is the graph produced by your calculator a correct one? Explain.