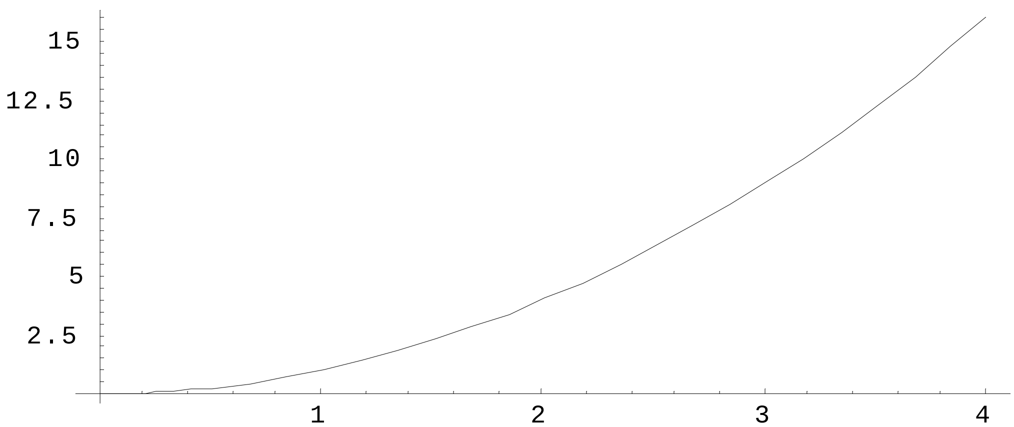
Math 1426 **Lab 12**
Spring 2006 **Week of 4-24 to 4-28**

When $f(x) \geq 0$ on the interval [a, b], the integral represents the area under the curve. If we can find an antiderivative for f, we can use the fundamental theorem of calculus to find the area. However, if no antiderivative can be found, then we can approximate the area using rectangles. Another approximation can be obtained using a trapezoid, a four-sided figure with exactly 2 parallel sides. If b_1 and b_2 are the lengths of the parallel sides and h is the distance between those sides, the area of a trapezoid is $\frac{1}{2}h(b_1 + b_2)$.

Consider the function $f(x) = x^2$ on the interval [1,2]; the graph is below.



1. We will divide the interval [1,2] into four equal subintervals. The first subinterval will be $\left[1, \frac{5}{4}\right]$. This will form one side of the first trapezoid. One of the parallel sides will be the vertical line segments connecting (1,0) and $(1, f(1))$; the other will connect $\left(\frac{5}{4}, 0\right)$ and $\left(\frac{5}{4}, f\left(\frac{5}{4}\right)\right)$. Draw the first trapezoid on the above graph and <u>find its exact area.</u>

2. Draw three more trapezoids in the same manner and find their <u>exact areas.</u>

3. Add the areas of the 4 trapezoids to get an approximate area of __________________.

4. Approximate the same area, using 4 rectangles whose heights are the midpoints of each subinterval.

5. Find the exact area (you may use the fundamental theorem). Which approximation method was closest to the exact answer?

6. We haven't learned how to find an antiderivative for the following integrand, but you can use your knowledge of geometry to explain why $\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}$.
7. Estimate π correct to 1 decimal place by approximating the integral in #6, first using rectangles (use the midpoint of each subinterval for the height) and then using trapezoids.

Another important reason for studying methods for approximating integrals is because sometimes we don't have a formula for the function we are trying to integrate. In the physical and biological sciences and engineering, often all we know about a function has been obtained by measurements or observations made at a finite number of points.

8. Estimate $\int_0^1 f(x)\,dx$ where the values of the unknown function *f* is given in the table below. Do this in two ways: first use rectangles, then use trapezoids.

x	$f(x)$
0.0	1.0
0.25	0.8
0.5	1.3
0.75	1.1
1.0	1.6