Math 1426 Problem Solving Lab 3 Spring 2006

(1) Sketch the graph of a function f that satisfies: $\lim_{x \to 3} f(x)$ exists, but f is not continuous at $x = 3$.

(2) Sketch the graph of a function f that satisfies: f is continuous on $(-\infty, 0) \bigcup (0, +\infty)$, but f is not continuous on $(-\infty, +\infty)$.

(3) Sketch the graph of a function f that satisfies: f is continuous every where except at $x = 3$, at which point it is continuous from the right.

(4) Sketch the graph of a function f that satisfies: f is not continuous at $x = 3$, but if $f(3) = 0$, then f becomes continuous at $x = 3$.

(5) Find constants a and b so that $f(x) = \begin{cases} \frac{\sin a x}{x} & \text{if } x < 0 \\ 8 & \text{if } x = 0 \\ 3x + b & \text{if } x > 0 \end{cases}$ will be continuous for all real numbers x. Give a coherent mathematical argument to justify your values for a and b.

(6) Prove that the function $f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$.