

1. A rectangle is bounded by the x -axis, the y -axis and the graph of $y = (6 - x)/2$. What length and width should the rectangle have so that its area is a maximum?
2. A piece of wire 30 cm long is cut into two pieces; one piece is bent into a square and the other piece is shaped into a circle. Assuming the wire is cut so that the total area enclosed by the square and circle is a maximum, how long should be the piece that is cut to form the square?
3. The following is INCORRECT: $\lim_{x \rightarrow 0^+} \left(\frac{1}{x^2} - \frac{1}{x} \right) = \infty - \infty = 0$.

The following, using L'Hôpital's Rule, is also INCORRECT:

$$\lim_{x \rightarrow 0^+} \left(\frac{1}{x^2} - \frac{1}{x} \right) = \lim_{x \rightarrow 0^+} \left(\frac{1-x}{x^2} \right) = \lim_{x \rightarrow 0^+} \left(\frac{-1}{2x} \right) = -\infty.$$

The limit is, in fact, $+\infty$. Find what is wrong with the previous arguments and give a correct argument to compute the limit.

4. (a) What is the logarithmic rule for $\ln x^p$?
(b) Find $\ln x^{a/(\ln x)}$.
(c) We will now see that a limit of type ∞^0 can be any value. Let $a \in \mathbb{R}$ be a nonzero constant. Write $L = \lim_{x \rightarrow +\infty} x^{a/(\ln x)}$. Using (b), find L .
(d) In (c), what value should a be to force $L = 2$?
(e) In (c), what value should a be to force $L = 5$?
(f) Write down your favorite number. In (c), what value should a be to force $L =$ your favorite number?
5. (a) Find the value of the constant k such that $\lim_{x \rightarrow \infty} \frac{3kx^2}{(x+k)(x-2k)} = 1$.
(b) Find the value of the constant k such that $\lim_{x \rightarrow \infty} \left(\frac{x+k}{x-2k} \right)^x = e$.
(Hint: the formula in (a) should show up at some stage of your solution.)