Math 3330

PRACTICE EXAM I

1. (a) Let $S = \{A_1, A_2, \ldots, A_k\}$ be a set of mxn matrices. What does it mean for S to be linearly independent?

(b) Is the set of polynomials $S = \{x+1, x^2 - x, x^2 + 1\}$ linearly independent in the vector space of all polynomials P? Justify your answer.

(c) Is the set of vectors $S = \{\cos x, \sin x\}$ linearly independent in $\mathcal{F}(\mathbb{R})$? Justify your answer.

2. Let $X = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$ and $Y = \begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix}$. Exhibit three different vectors in the span of X and Y. Find a vector in $M(1,3)$ which is not in the span of X and Y, and justify why it is not in the span.

3. Suppose that X and Y are elements of some vector space. Suppose that V and W belong to the span of X and Y. Show that all linear combinations of V and W also belong to the span of X and Y.

4. Consider the system of linear equations

$$\begin{aligned} x + y + z + w &= 1 \\ 2x - 2y + z + 2w &= 3 \\ 2x + 6y + 3z + 2w &= 1 \\ 5x - 3y + 3z + 5w &= 7 \end{aligned}$$

(a) Use Gaussian elimination to find all solutions (if any), keeping track of your elementary row operations. Express your answer in parametric form and give the translation vector and the spanning vectors.

(b) Find matrices A, X, and B such that the system can be written as the matrix equation $AX = B$.

5. Determine which of the following matrices is in row reduced echelon form, echelon form, or neither.
$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}, F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}, G = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, H = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}, I = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}.$

6. For the matrix $A = \begin{bmatrix} 1 & 3 & 2 & 0 \\ -1 & 3 & 1 & -2 \\ 2 & 0 & 1 & 2 \end{bmatrix}$ and the vector $X = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 2 \end{bmatrix}$, compute AX.

7. Find the nullspace of the matrix A from Question 6.

8. Note that the vector $T = \begin{bmatrix} -1/2 \\ 1/2 \\ 0 \\ 0 \end{bmatrix}$ is a solution to the system

$$\begin{aligned} &x + 3y + 2z + 0 = 1 \\ &-x + 3y + z - 2w = 2 \\ &2x + z + 2w = -1. \end{aligned}$$

Find *all* solutions to the system.

9. Find the basis of the column space of the matrix $A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 4 & 6 \end{bmatrix}$.

10. Are the following vectors linearly independent: $V = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$, $U = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, and $W = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$?