Math 3330

PRACTICE EXAM II

1. Let $A = \begin{bmatrix} 1 & -2 & 1 & -1 \\ 2 & -4 & 1 & -2 \\ -1 & 2 & 0 & 1 \\ 1 & -2 & 4 & -1 \end{bmatrix}$.

(a) Find a basis for the row space of A.

(b) What is the rank of A?

(c) Find a basis for the column space of A.

(d) Explain why the number of basis vectors for the row space of A equals the number of basis vectors for the column space of A.

2. (a) What is the standard basis for $\mathbb{R}^n$? What is the dimension of $\mathbb{R}^n$?

(b) Determine whether the vectors $\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix}$ span $\mathbb{R}^3$.

(c) Determine whether the vectors $\begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ are linearly independent.

3. (a) State the Rank-Nullity theorem.

(b) What is the dimension of the nullspace of the matrix A from Question 1?

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x - y \\ x + y \end{bmatrix}$.

(a) Show that T is a linear transformation.

(b) Show into what T transforms the unit square.

(c) Find a matrix A such that $TX = AX$ for all $X \in \mathbb{R}^2$.

5. Consider the subspace $\mathcal{W} = \text{Span}(\sin x, \cos x)$ of the space of functions F($\mathbb{R}$). Let T be the transformation of $\mathcal{W}$ given by $T(f(x)) = \frac{d}{dx} f(x)$. Find the matrix A such that $T = T_A$.

6. Show that the image of a linear transformation $T : \mathcal{V} \to \mathcal{W}$ is a subspace of $\mathcal{V}$.

7. (a) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ and $S : \mathbb{R}^3 \to \mathbb{R}^4$ be matrix transformations given by the matrices $B = \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ 2 & -1 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & -1 \\ 2 & -1 & 3 \\ -1 & 2 & -3 \end{bmatrix}$, respectively. Find a matrix which represents the compostion $S \circ T : \mathbb{R}^2 \to \mathbb{R}^4$.

(b) Is the composition $T \circ S$ defined? Why or why not?

8. Determine if the matrix transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -2 & 1 \\ 2 & -1 & 0 \end{bmatrix}$ is

(a) one-to-one;

(b) onto;

(c) invertible. If so, compute its inverse.