Two dimensional signals - Quaternary Amplitude Modulation (QAM)

General format of QAM signal:

\[ U_{m,n}(t) = A_m g(t) \cdot \cos(2\pi f_c t + \phi_n) \]

\[ = A_m g(t) \left[ \cos(\omega_0 f_c t) \cdot \cos(\phi_n) - \sin(\omega_0 f_c t) \cdot \sin(\phi_n) \right] \]

\[ = A_m \cos(\phi_n) \cdot [g(t) \cdot \cos(2\pi f_c t)] + A_m \sin(\phi_n) \cdot [-g(t) \cdot \sin(2\pi f_c t)] \]

Basic channels:

\[ \Psi_1(t) = \sqrt{\frac{2}{E_g}} \cdot g(t) \cdot \cos(2\pi f_c t) \quad E_g = \int_0^T g^2(t) dt \]

\[ \Psi_2(t) = -\sqrt{\frac{2}{E_g}} \cdot g(t) \cdot \sin(2\pi f_c t) \]

\[ U_{m,n}(t) = A_m \cos(\phi_n) \Psi_1(t) + A_m \sin(\phi_n) \Psi_2(t) \]

\[ \equiv S_{m,n} = \left[ A_m \cos(\phi_n), \sqrt{\frac{E_g}{2}}, -\sqrt{\frac{E_g}{2}} \cdot A_m \sin(\phi_n) \right] \]

Notes:

* Total number of points in Constellation is \( M = M_1 \times M_2 \)
* Number of bits per symbol \( K = \log_2(M_1 \times M_2) \)

Some Examples: (4-PAM, 8-PSK)

4-PAM \uparrow \Psi_1 \quad (also \ 4-PSK) \quad \Psi_2 \uparrow 8-PSK
Block Diagram

Bit Sequence -> Amplitude

Am. cos (θx) -> g(x(t))

Am. sin (θx) -> g(x(t))

−sin (θx(t))

Received of Digitally Modulated Signals in AWGN Channel

TX

R(t)

Rx

Sm(t) - one of M possible waveforms selected by TX (N = 2^k)

h(t) - Gaussian noise

\text{h(t)} \sim N\left(0, \frac{\sigma^2}{\text{signal}}\right)

\text{T} : \text{Symbol period} \quad \left(\text{T} : \text{Symbol duration} \quad \frac{K}{T} : \text{bit rate}\right)

\text{W} : \text{Symbol rate}
PSD of noise

\[ S_n(f) = N_0 \]

Vector representation of Tx/RX signal:

![Vector diagram](image)

The receiver performs two tasks:

1) Demodulation: Transforms the received waveform \( R(t) \) into a \( N \)-entry vector \( \mathbf{r} = [r_1, r_2, \ldots, r_N] \).

   The receiver decomposes the signal along the axes associated with the basis functions.

2) Detection: Decision made by the RX on which of the \( M \) possible waveforms was sent by the transmitter.

Demodulation - Correlation type:

\[ r(t) = \left( S_m(t) + h(t) \right) \text{ received signal} \]
\[ y(t) \xrightarrow{\text{De modulation}} r(t) \Rightarrow r = \begin{bmatrix} r_1 & \cdots & r_n \end{bmatrix} \]

\( k = 1, \ldots, N \)

\[ y_k = \langle y(t), \psi_k(t) \rangle = \int_0^T y(t) \cdot \psi_k(t) \, dt = \int_0^T (S_{m}(t) + n(t)) \psi_k(t) \, dt = \int_0^T S_{m}(t) \psi_k(t) \, dt + \int_0^T n(t) \psi_k(t) \, dt \]

\[ r = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = S_{m} + n \]

\( S_{m} : \) Coordinate of the transmitted waveform in the transmission vector space

\( n : \) Coordinate of the noise signal in the transmission vector space

Outline of the Correlation Receiver:

\[ y(t) \xrightarrow{\text{Cross}} \int_0^T () \ dt \]

\[ r_1 = S_{m} + n_1 \]

\[ r_2 = S_{m} + n_2 \]

\( \Rightarrow \) to the detector

\[ r_N = S_{m} + n_N \]

\[ \Rightarrow \]

*Fundamental problem is the synchronization of phase between the TX and RX oscillator*
\[ y_k = \int_0^T r(t) \psi_k(t) dt \]
\[ y_x(t) = \int_0^T r_\tau(t) \cdot h_x(t - \tau) \, d\tau \]

2) Supposing \( t = T \)
\[ y_x(T) = \int_0^T r_\tau(t) \cdot h_x(t - \tau) \, d\tau \]
\[ = \int_0^T r_\tau(t) \cdot \psi_x(t - \tau) \, d\tau \]
\[ = \int_0^T \psi_x(t - \tau) \, d\tau \]

Example: \( x(t) \)

**Original Signal**

**Matched Filter**

\[ h_x(\tau) = \psi_x(\tau - T) \]

**Matched Filter Response**

\[ \psi_x(\tau - T) = h_x(\tau) \]

**Advantages:** Matched filter are linear devices
- No need for multipliers – nonlinear processing devices
- No phase or angle synchronization
Properties of matched filter

1) If signal $S(t)$ is convolved with AWGN, the filter with impulse response matched to $S(t)$ maximizes the output SNR.

$$Y(t) = S(t) + n(t)$$

Output Signal

Output Signal

Convolution

$$y(t) = \int_0^T r(t) \cdot h(t) \, dt$$

$$= \int_0^T (s(t) + n(t)) \cdot h(t) \, dt$$

$$= \int_0^T s(t) \cdot h(t) \, dt + \int_0^T n(t) \cdot h(t) \, dt$$

Signal term

Noise term

Sampling the output as $t = T$

$$y(T) = \int_0^T s(t) \cdot h(T-t) \, dt + \int_0^T n(T-t) \cdot h(t) \, dt$$

The output SNR is given.
$$\text{SNR} = \frac{\text{Signal power}}{\text{Noise power}} = \frac{\left(\frac{1}{t_2-t_1} \int_{t_2-t_1} Y_S(t) \, dt\right)^2}{E\left[Y_n^2(t)\right]}$$

$$Y_S(t) = \left[ \int_{t_2-t_1}^{t_2} s(2)h(T-t) \, d\tau \right]^2$$

$$E\left[Y_n^2(t)\right] = E\left[Y_n(t) \cdot Y_n(T)\right]$$

$$= E\left[\int_{t_2-t_1}^{t_2} h(\tau_1) \cdot h(T-t_1) \, d\tau_1\right] \times E\left[\int_{t_2-t_1}^{t_2} h(\tau_2) \cdot h(T-t_2) \, d\tau_2\right]$$