Guided Discovery: More circles or more squares?

Let *S* and *T* be finite sets. Recall two facts:

> ***Fact A****: If S properly contains T, then S contains a greater quantity of elements than T.*
>
> ***Fact B:*** *If there is a bijection between S and T, then S and T contain the same quantity of elements.*

Let *U* and *V* be infinite sets. Consider two statements:

> ***Statement A****: If U properly contains V, then U contains a greater quantity of elements than V.*
>
> ***Statement B****: If there is a bijection between U and V, then U and V contain the same quantity of elements.*

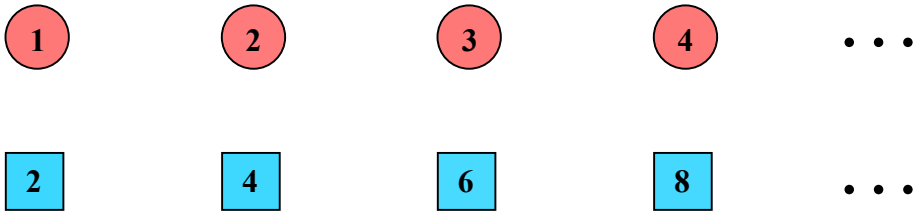
The Question:
Should the quantity of elements in *U* and *V* be compared according to Statement A or Statement B?

Let's consider a thought experiment:

Infinitely many circles and infinitely many squares are lined up on an infinitely long glass window. Each circle and each square is red on one side and blue on the other side, and each side of every circle and every square has a number written on it.

The red sides are numbered 1, 2, 3, 4, 5, . . . by the counting numbers.
Opposite the red side numbered k, is a blue side numbered $2k$.

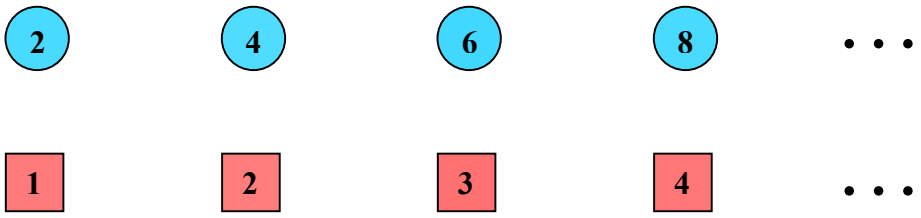
1. Let's look at the line-up from the FRONT of the window:



a) Judging by Statement A and looking at the numbers above, which are more numerous: the circles, the squares, or neither?

b) Judging by Statement B and looking at the numbers above, which are more numerous: the circles, the squares, or neither?

2. Now let's view the line-up from the BACK of the window:



a) Judging by Statement A and looking at the numbers above, which are more numerous: the circles, the squares, or neither?

b) Judging by Statement B and looking at the numbers above, which are more numerous: the circles, the squares, or neither?

Conclusion:

Definition:
Two sets S and T are said to have the ***same cardinality****, or to be* ***equinumerous****, whenever* _____ .
A ***countably infinite****, or* ***denumerable*** *set is a set that* _____ _____. *A* ***countable*** *set is a set that is either* _____ *or* _____ *; otherwise the set is said to be* ***uncountable****.* ■