Measuring the Persistence of Deviations from Purchasing Power Parity with a Fractionally Integrated STAR Model

Aaron D. Smallwood*
Department of Economics
University of Texas at Arlington
701 South West Street
Campus Box 19479
Arlington, TX 76019

October 2007

Abstract:

Empirically, elements of both fractional long memory and threshold non-linearity are present in the real exchange rates of the G-7 countries against the US, notably in the EU countries. Estimated half lives of deviations from PPP using median unbiased corrections to conventional linear autoregressive models corroborate existing evidence related to the PPP paradox as half lives range from at least four years to an infinite number of years. In contrast, for each EU country, accounting for threshold non-linearity results in estimated half lives that can be less than 3 years even with the allowance for fractional long memory.

JEL classifications: C22, C52, F31

Key words: Purchasing Power Parity Paradox; STAR non-linearity; long memory, FI-ESTAR.

---

* Phone: 817-272-3062, Fax: 817-272-3145. Email: smallwood@uta.edu.

1. Introduction

International macroeconomists are now intimately aware of the purchasing power parity (PPP) paradox, the inability to theoretically reconcile empirically volatile and persistent real exchange rates. The puzzle, as initially elucidated by Rogoff (1996), points to slow mean reversion in the real exchange rate with half lives of deviations from PPP of at least 3-5 years. In the presence of financial shocks, sticky prices and wages can theoretically generate deviations from PPP with half lives that are only a fraction of these estimates. Even more confounding is recent evidence that suggests the measured half lives based on autoregressive models may be downward biased. In fact, Murray and Papell (2002, 2005) present evidence that shows the half lives could be infinite once the downward bias is corrected.

A large literature has been devoted to the PPP puzzle, largely in an attempt to revive the theory. Recently, the theoretical literature has attempted to incorporate habit persistence and additional rigidities such as asset market frictions to produce more persistent real exchange rates with limited success (c.f. Chari, Kehoe, and McGrattan, 2002). Empirically, others have considered the extent to which time and sectoral aggregation may produce a bias in estimated half lives. For example, Imbs et al. (2005) argue that half lives of deviations from PPP are significantly reduced when one accounts for heterogeneous dynamics in a panel context, while Chen and Engel (2005) argue that, among other issues, small sample bias still remains in Imbs et al.'s estimates. When corrected, the PPP puzzle remains firmly in place.

Alternatively, researchers have attempted to reconcile both the demonstrably long half lives and the empirical finding of a unit root in the real exchange rate by using various time series models that deviate from the standard linear autoregressive paradigm. At least two plausible alternatives have arisen. On one hand, researchers have questioned the assumption of an integer order of integration for the real exchange rate and have argued that a fractional order of integration may be present (Diebold, Husted, and Rush, 1991; Cheung and Lai, 1993; Cheung and Lai, 2001; Achy, 2003). On the other hand, a number of researchers have found threshold non-linear mean reversion in the real exchange rate and have argued that previous estimates of half lives based on autoregressive linear specifications are biased upward (Baum, Barkoulas, and Caglayan, 2001; Taylor, Peel, and Sarno, 2001; Paya, Venetis, and Peel, 2003).

It is thus clear that there has been considerable interest in determining the extent to which fractional long memory or threshold non-linearity are present in the dynamics associated with PPP. Yet little is known about the relative importance of these two modeling procedures and the implications for tests related to PPP. More generally, the dichotomous modeling approach in the PPP literature highlights the emerging debate among researchers employing macroeconometric techniques. The possibility of confusing regime switching models and other general forms of non-linearity with fractional integration and vice-versa is now established in the literature (Diebold and Inoue, 2001; Kramer and Sibbertsen, 2002). It would thus be of interest to consider an encompassing modeling technique that allowed for non-linear dynamics and fractional long memory simultaneously to determine their relative importance.

A parametric non-linear model that has received extensive attention in the PPP literature is the exponential smooth transition autoregressive (ESTAR) model, which allows for smooth transitions between regimes. In this paper, I employ an encompassing model and testing procedure applied to the post Bretton Woods real exchange rates of the G-7 countries vis-à-vis the United States that allows for both fractional long memory and ESTAR non-linearity. There is evidence of both ESTAR non-linearity with fractional long memory for each of the EU countries in my sample, especially for the euro-zone subset. There is little evidence of either

ESTAR non-linearity or mean reversion for the real exchange rates of Canada or Japan. Median unbiased estimates of half lives based on linear autoregressive models reconfirm the existing paradox, with half lives ranging from a minimum of four years to an infinite number of years. In contrast, the use of generalized impulse response functions for estimated fractionally integrated non-linear models for the EU countries reveals that the half lives can be less than 3 years in duration. The results show that the use of fractional integration coupled with threshold non-linearity could be an important avenue for studying the PPP paradox.

The rest of the paper is organized as follows. In section 2, I review the literature on the PPP puzzle, highlighting the empirical research related to long memory and non-linearity. In section 3, I present the autoregressive fractionally integrated moving average (ARFIMA), ESTAR, and fractionally integrated ESTAR (FI-ESTAR) models and review the testing procedures for linearity. In section 4, I present the estimated models for the real exchange rates, while section 5 presents estimates of the impulse response functions and accompanying half lives. Finally, section 6 concludes.

2. Relevant Literature

Empirical studies have emerged that contest the standard autoregressive modeling approach to the real exchange rate. As pointed out by Taylor (2001), the selected model can have important implications for the measured half lives. One modeling alternative is based on the idea that integer orders of integration may be unduly restrictive. Cheung and Lai (2001) assert that fractional integration is likely for the real exchange rate, since econometrically, it accommodates the long swings that are prototypical of exchange rates. Cheung and Lai also argue that the use of fractional integration can be important in testing PPP as it allows one to separate low frequency components from high frequency components easily. Further, long memory is plausible given the aggregation of price indices and exchange rates themselves. A number of researchers have employed fractional integration to model the dynamics associated with PPP, including Diebold, Husted, and Rush (1991), Cheung and Lai (1993), Cheung and Lai (2001), and Achy (2003). In particular, Cheung and Lai (2001) assert that half lives are reduced by considering fractional integration for yen based real exchange rates.

Alternatively, researchers have employed nonlinear models that allow the persistence of the real exchange rate to vary depending on the amount of the deviation from an equilibrium threshold. For example, for small deviations from PPP, the real exchange rate may behave like a random walk, whereas for substantial deviations, the process becomes means reverting. Nonlinear threshold models are theoretically well motivated from the perspective of impediments to arbitrage. For example, as in Sercu, Uppal, and Van Hulle (1995), PPP may be expected to hold only after deviations are large enough to make arbitrage profitable because of impediments to trade such as transactions costs, trade restrictions, and transportation costs. In this case, there are two regimes, one in which the exchange rate is allowed to deviate from its equilibrium value, and one in which arbitrage is activated. Existing non-linear models, therefore, extend readily to this important problem. Recently, Michael, Nobay, and Peel (1997), Baum, Barkoulas, and Caglayan (2001), Taylor, Peel, and Sarno (2001), and Paya, Venetis, and Peel (2003) have presented evidence of threshold non-linearity in the real exchange rate. In particular, Taylor, Peel, and Sarno (2001) show that half lives of deviations from PPP with dollar based real exchange rates can be reduced through the use of a restricted non-linear ESTAR model. Paya, Venetis, and Peel (2003) add exogenous variables including a time trend and a proxy for a relative measure of productivity to the standard ESTAR model and demonstrate that the speed of adjustment to deviations from PPP can be extremely quick, with half lives of deviations that are typically less than two years.

A natural extension of the dichotomous methods discussed above allows for both fractional integration and non-linearity or regime-switching. A limited number of studies have attempted to use such models. Baum, Barkoulas, and Caglayan (1999) consider a model that allows for a double mean shift with long memory for the real exchange rate, while Baillie and Kapetanios (2006) present evidence that DM based real exchange rates are subject to long memory and a general form of non-linearity. Smallwood (2005) performs preliminary estimation of the FI-ESTAR model for a number of countries. None of these papers employ impulse response analysis, nor do they consider how the modeling choice affects half lives of deviations from PPP.

3. The FI-ESTAR Model

The FI-ESTAR model extends the standard ESTAR model to allow for fractional integration. Following Granger and Joyeux (1980) and Hosking (1981), a fractionally integrated univariate time series process, y_t, can be represented as

$$(1-L)^d (y_t - \mu) = z_t , \qquad (1)$$

where z_t is a short memory I(0) process and where the differencing operator $(1-L)^d$ is defined as,

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)}{\Gamma(-d)\Gamma(k+1)} L^k , \qquad (2)$$

where $\Gamma()$ is the gamma function. If z_t is an ARMA(p,q) process, then y_t becomes an ARFIMA(p,d,q) process. Let $\phi(L)$ and $\theta(L)$ denote p and q order polynomials with all roots outside the unit circle, and let ε_t denote a martingale difference sequence, with $E(\varepsilon_t^2)=\sigma^2$. The ARFIMA process is then defined as

$$\phi(L)(1-L)^d (y_t - \mu) = \theta(L)\varepsilon_t . \qquad (3)$$

The process in Eq. (3) is stationary and invertible if -1/2<d<1/2 and for stationary representations has an autocorrelation function that decays at a hyperbolic rate. Alternative specifications are possible for z_t. For example, to allow for non-linearity, one could let z_t be a STAR process rather a simple ARMA process.

The general STAR model, which was made operational by Granger and Teräsvirta (1993) and Teräsvirta (1994), has proven to be an especially useful non-linear model. The two-regime STAR(p) model with transition function $G(s_t;\gamma,c)$ is defined as

$$y_t = \phi_{1,0} + \phi_{1,1} y_{t-1} + \ldots + \phi_{1,p} y_{t-p} + (\phi_{2,0} + \phi_{2,1} y_{t-1} + \ldots + \phi_{2,p} y_{t-p}) G(s_t;\gamma,c) + \varepsilon_t \,, \qquad (4)$$

where ε_t is as above, c is the equilibrium threshold, γ controls the degree of curvature of G, and s_t is the transition variable governing the non-linear behavior of the process. Although a number of choices are plausible for $G(s_t;\gamma,c)$, I concentrate on the exponential function,[1]

$$G(s_t;\gamma,c) = 1 - \exp[-\gamma(s_t - c)^2] \,. \qquad (5)$$

Clearly, as $\gamma \rightarrow 0$ the function collapses to 0, whereas if $\gamma \rightarrow \infty$ the function converges to unity. In either case, the model in Eq. (4) becomes a linear AR(p) model. The inner regime corresponds to the case where γ=0, whereas the outer regime corresponds to the case where $\gamma \rightarrow \infty$.

The FI-STAR model, which was developed by van Dijk, Franses, and Paap (2002), is nothing more than a simple combination of a fractional long memory model and a STAR model. Let x_t be the fractional difference of y_t, $x_t \equiv (1-L)^d y_t$. The FI-ESTAR(p) model is then defined as

$$x_t = \phi_{1,0} + \phi_{1,1} x_{t-1} + \ldots + \phi_{1,p} x_{t-p} + (\phi_{2,0} + \phi_{2,1} x_{t-1} + \ldots + \phi_{2,p} x_{t-p})(1 - \exp[-\gamma(s_t - c)^2]) + \varepsilon_t \,, \qquad (6)$$

---

[1] Alternatively, one could employ a logistic or simple threshold function. Baum, Barkoulas, and Caglayan (2001) argue that the most appropriate choice in the current context is the exponential function, a choice I follow.

with ε_t defined as above.[2] The FI-ESTAR model generalizes most of the existing models that have been used to study the dynamics associated with PPP. If d=0, the model in Eq. (6) reduces to the standard ESTAR model without fractional long memory. In contrast, if γ=0 or if $\phi_{2,j}$=0 for j=0,…,p, the result is an ARFIMA(p,d,0) model. Of course, without non-linearity, the ARFIMA(p,d,0) process becomes a stationary AR(p) process when d=0 and a unit root when d=1. With known d, if we assume that all roots to $1-\phi_{1,1}L-\ldots-\phi_{1,p}L^p=0$ lie outside the unit circle, then the process defined in Eq. (6) is stationary and ergodic under linearity as in Teräsvirta (1994). Here, for tests of linearity, it is necessary to use a consistent estimator of d.

To construct tests for linearity based on the FI-ESTAR model, one can extend the testing procedure outlined by Teräsvirta (1994) for the ESTAR model. The null hypothesis of linearity for the standard ESTAR model, with d assumed to be zero, can be built in more than one way (γ=0, $\phi_{2,0}=\phi_{2,1}=\ldots=\phi_{2,p}=0$). Further, the nuisance parameters are not restricted under the null. The original linearity tests were developed by Teräsvirta to avoid this issue. In particular, one can take an appropriate Taylor series expansion of the transition function about γ=0. Under the null hypothesis, the test can be carried out through a standard OLS regression with restrictions on a subset of parameters. The extension to the FI-ESTAR model is straightforward. As discussed by Smallwood (2005), a first order Taylor series expansion of Eq. (6) about γ=0 results in the following auxiliary regression,

---

[2] Building on the earlier asymptotic results of Teräsvirta (1994), the testing and estimation methods established by van Dijk, Franses, and Paap (2002), which are used here, are developed without MA terms, which is unlikely to be a concern for the real exchange rate. Typically, as the diagnostic tests reveal, the inclusion of only one AR term is enough to ensure serially uncorrelated residuals in the estimated models below. The use of models lacking MA components is well established for studying the real exchange rate as the literature reviewed above shows.

$$x_t = \beta_{1,0} + \sum_{j=1}^{p} \beta_{1,j} x_{t-j} + \sum_{j=1}^{p} \beta_{2,j} x_{t-j} s_t + \sum_{j=1}^{p} \beta_{3,j} x_{t-j} s_t^{\ 2} + u_t \ . \qquad (7)$$

The null hypothesis for linearity is then given by $\beta_{2,j}=\beta_{3,j}=0$. The test must account for the change in the likelihood function which results from estimating the value of d. Let $\hat{\varepsilon}_t$ denote the residuals from a consistently estimated ARFIMA(p,d,0) model. Under the hypothesis of linearity, and with a Gaussian assumption, the gradient of the likelihood function for the t^{th} observation for the model above with respect to d is given by:

$$\frac{\partial \ell_t}{\partial d} = -\frac{\hat{\varepsilon}_t}{\sigma^2} \sum_{j=1}^{t-1} \frac{\hat{\varepsilon}_{t-j}}{j} \ . \qquad (8)$$

The regression under the alternative hypothesis controls for the expression in Eq. (8). Then, one can obtain the restricted sum of squared errors (SSE_R) and the unrestricted sum of squared errors (SSE_{UR}). The χ^2 and F test statistics are given by,

$$LM_{\chi^2} = \frac{T(SSE_R - SSE_{UR})}{SSE_R}, LM_F = \frac{(SSE_R - SSE_{UR})/2p}{(SSE_{UR})/(T-3p-1)} \ . \qquad (9)$$

The test statistics tests are distributed as $\chi^2(2p)$ and $F(2p,T\text{-}3p\text{-}1)$ statistics (see Teräsvirta, 1994, van Dijk, Franses, and Paap, 2002, and especially Smallwood, 2005 for details).

In formulating the tests, it is important to note that the possibility exists that the integration parameter, d, may differ under the null and alternative hypotheses as pointed out by Baillie and Kapetanios (2006). For example, van Dijk et al. (2002) fit a fractionally integrated STAR model to the US unemployment rate and find that the degree of persistence as measured by the differencing parameter decreases once non-linearity is considered. Similarly, Taylor, Peel, and Sarno (2001) suggest, without reporting estimates from various models, that the persistence of the real exchange rate is expected to fall once the researcher extends their analysis beyond linear models and allows for non-linearity. Therefore, below, d is estimated both under the hypothesis of linearity as above, and under the alternative hypothesis. The χ^2 version of the test and the minimized p-value are reported in each instance.[3]

A number of estimation methods are available for the models presented here. For the FI-ESTAR model, following van Dijk, Franses, and Paap (2002), I choose to use a modification of the time domain estimator of Beran (1995) to jointly estimate all of the model parameters.[4] Under the assumption that the pre-sample observations are non-stochastic, the estimator is asymptotically equivalent to MLE. Following Beran, I set $y_0=y_{-1}=\ldots=\varepsilon_0=\varepsilon_{-1}=\ldots=0$. The estimates of all of the FI-ESTAR model parameters are the set of parameters that maximize the following function:

$$\begin{aligned} &AMLE(c, \gamma, \phi', d, \sigma^2) = -\frac{T}{2}\log(2\pi) - \frac{T}{2}\log\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=2}^{T}\varepsilon_t^2 \\ &\varepsilon_t = x_t - \phi_{1,0} - \sum_{j=1}^{p}\phi_{1,j}x_{t-j} - (\phi_{2,0} + \sum_{j=1}^{p}\phi_{2,j}x_{t-j})(1-\exp[-\gamma(s_t-c)^2]) \end{aligned}, \qquad (10)$$

with $x_t=(1-L)^d y_t$. To estimate an ARFIMA(p,d,0) model, which is necessary for testing linearity under the null, Beran's (1995) estimator is used. This can essentially be accomplished through maximization of the function in Eq. (10) with $c=\gamma=\phi_{2,j}=0$, $j=0,\ldots,p$, imposed.

4. Estimation Results

In this section, I consider the tests and estimation results for the monthly real exchange rates of the G-7 countries vis-à-vis the US. All data employed in this study were obtained from the IFS-CD ROM. The nominal exchange rate data are monthly averages of the foreign currency

---

[3] Under the alternative hypothesis, the true model is the FI-ESTAR model. To obtain the value of d under the alternative, I estimate the full model, and then use this estimated value of d in place of $\hat{d}$ above.

[4] Beran (1995) shows that for the ARFIMA model, the estimator is consistent and asymptotically normal provided $d>-1/2$.

price of the US dollar. For price indices, both the consumer price index (CPI) and a measure of the wholesale price index (WPI) are used. For the euro-zone countries, the data extend from January 1973-December 1998. For Japan, Canada, and the UK, the data extend from January 1973-June 2004. As in Taylor, Peel, and Sarno (2001), the real exchange rate in January 1973 is normalized to be 1.

The analysis conducted in this paper is primarily concerned with the log real exchange rate, r_t, defined as $r_t=\log(S_t)-\log(P_t^*)+\log(P_t)$, where S_t is the nominal foreign currency price of the US dollar, P_t^* is the price level in the foreign country, and P_t is the US price level. However, some researchers have asserted a relationship between exchange rates and price indices may exist even if PPP fails to hold in the presence of measurement errors in prices (see Cheung and Lai, 1993). These researchers contend that a very weak version of PPP holds if $\log(S_t)=\alpha_1\log(P_t^*)+\alpha_2\log(P_t)$, with $\alpha_1,\alpha_2\neq 0$. The parameters, α_1 and α_2, can be estimated using OLS, and the resulting residuals are taken to be the *estimated* real exchange rate. I conducted analysis based on the estimated real exchange rate, along with the real exchange rate based on the wholesale price index (WPI). The estimates for the WPI based real exchange rates, which are available upon request, are consistent with the estimates reported here, and so to conserve space I concentrate on the CPI based real exchange rates. There are only a couple of minor differences for the analysis conducted below between the estimated real exchange rate and the real exchange rate based on conventional PPP, which is discussed whenever relevant.

For the non-linear models, s_t is set to be a lag of the real exchange rate, with $s_t=r_{t-\tau}$, where τ is the delay parameter. Here, it is sensible to allow the degree of non-linearity to be controlled by the extent to which the real exchange rate deviates from the equilibrium threshold c. Following Baum, Barkoulas, and Caglayan (2001), τ ranges from 1 to 12.

Table 1 presents the test results for linearity based on the FI-ESTAR model associated with Eq. (6) for the CPI based real exchange rates. The delay parameter, τ, is obtained by minimizing the probability value for the linearity test for τ between 1 and 12. When d is estimated under the alternative hypothesis, linearity is rejected at the 5% level for each of the EU countries. In contrast, we fail to reject linearity for Canada and Japan regardless of whether d is estimated under the null or alternative hypothesis. Further, the test results from the estimated real exchange rate using OLS indicate that non-linearity is present for each of the EU countries, but again, not for Canada and Japan.

[INSERT TABLE 1 HERE]

Table 2 presents the estimation results based on the preferred model for each of the real exchange rates.[5] Beneath the parameter estimates, I record the numerical standard errors, which have been calculated using the outer product of the numerical gradient. For Canada and Japan, I present the results for the estimated linear ARFIMA model. For the UK, I am unable to reject the hypothesis that d=0 and thus present estimates from the ESTAR model. For Canada and Japan, the estimated linear ARFIMA models provide little support for the theory of PPP. For Japan, using the numerical standard errors, the estimated value of d is within two standard deviations of unity. For Canada, the estimated value of d is 0.1445, although the estimated autoregressive parameter is very close to unity. As a further robustness check of these results, I estimated an over-parameterized FI-ESTAR model for Canada and Japan. The resulting models are approximately linear and strongly support the parameter estimates in Table 2 for these countries. For example, the estimated value of γ is 269.5053 for Canada and thus implies that

---

[5] Teräsvirta (1994) and van Dijk, Franses, and Paap (2002) suggest multiplying γ by the inverse of the variance of the transition variable, which I follow.

the transition function in Eq. (6) is about one for all observations. The estimated value of *d* for the FI-ESTAR model for Canada is 0.1227, while the sum of the autoregressive parameters in the two regimes, $\phi_{1,1}+\phi_{2,1}$, is 0.9872. In other words, the resulting FI-ESTAR model is approximately an ARFIMA(1,0.1227,0) model with an AR(1) parameter equal to 0.9872.

[INSERT TABLE 2 HERE]

Turning to the countries where evidence of ESTAR non-linearity is detected, we see that there is strong evidence of fractional dynamics amongst the euro-zone sub-sample, since *d* is significantly different from either zero or unity for these countries at the 10% level. For the estimated real exchange rates, the results are highly comparable, except that there is evidence of both fractional long memory and non-linearity for the estimated real exchange rate of the UK. In particular, the value of *d* for the UK's estimated real exchange rate is 0.3549 and given a standard error of 0.1249, is easily more than two standard deviations away from 0 and 1.

Finally, in regards to diagnostics, I calculate the Ljung-Box Q statistics for serial correlation in the residuals and the squared residuals. The results of the Table show that the residuals are generally free of serial correlation but that GARCH effects may be present for the real exchange rates of Canada, the UK, and possibly Japan.[6] For the countries where there is evidence of ESTAR non-linearity, I apply the tests for no remaining non-linearity as discussed by Eitrheim and Teräsvirta (1996) for the ESTAR model and van Dijk, Franses, and Paap (2002) for the FI-STAR model. In each case, the tests show that I am unable to reject the null hypothesis, implying that the estimated models adequately account for all non-linearity.

---

[6] Although outside the scope of this paper, one could consider heteroskedastic effects directly, by replacing σ^2 with σ^2_t in Eq. (10) and allowing σ^2_t to follow a GARCH specification.

To get a sense of the estimated non-linear models, Figure 1 depicts the transition function for the estimated FI-ESTAR model for France. Referring to the results in Table 2, we see that the inner regime (with *G*=0) is affiliated with more persistence, since the autoregressive parameter in this regime is 1.0356. During the appreciation of the US dollar in the beginning of the 1980's, the French real exchange rate appears to be near the more persistent regime. As the deviation from equilibrium becomes large, market participants and potentially active intervention after the Plaza and Louvre Accords (1985, 1987) begin to bring the real exchange rate back toward equilibrium. In this case, the real exchange rate has deviated substantially from its equilibrium value and approaches the less persistent regime.

[INSERT FIGURE 1 HERE]

5. Impulse Response Analysis and Estimated Half Lives

Given the evidence of both fractional long memory and threshold non-linearity in the dynamics associated with purchasing power parity, it would be of interest to consider the implications for the PPP puzzle. A substantial volume of research has been devoted to calculating impulse response functions and associated half lives in the context of PPP. However, no research has considered impulse response analysis using a modeling technique that allows for elements of both fractional long memory and threshold non-linearity. The standard approach for measuring half lives of deviations from PPP is the use of an autoregressive model (see Rogoff, 1996). An AR(*p*) model for the real exchange rate is given by:

$$r_t = c + \phi_1 r_{t-1} + \ldots + \phi_p r_{t-p} + \varepsilon_t . \tag{11}$$

The half-lives can be estimated using an augmented Dickey-Fuller type regression,

$$r_t = c + \alpha r_{t-1} + \zeta_1 \Delta r_{t-1} + \ldots + \zeta_{p-1} \Delta r_{t-p+1} + \varepsilon_t , \tag{12}$$

where $\alpha=\phi_1+\phi_2+\ldots+\phi_p$, and $\zeta_j=-(\phi_{j+1}+\ldots+\phi_p)$, $j=1,\ldots,p-1$. The half life can be calculated from the impulse response function of the estimated model in Eq. (12) by determining the number of periods it takes for the initial shock to dissipate by half. However, it is now well known, going back to at least Hurwicz (1950) that estimates of autoregressive parameters are downward biased in small samples. As such, estimated half lives are also biased downward. As pointed out by Andrews (1993) and Andrews and Chen (1994), the problem can be corrected using median unbiased estimators. Suppose $\hat{\alpha}_{\mathrm{LS}}$ is the estimated OLS parameter for α from Eq. (12). Then, $\hat{\alpha}_{\mathrm{LS}}$ is median unbiased for α, if it has as its median value the true population parameter α for all values in the sample space.[7] To put the results in this section in the context of analysis associated with the PPP paradox, I report the OLS estimates of the parameters from Eq. (12) and the associated median unbiased estimates of the model parameters along with the estimated half lives in Table 3. When uncorrected, the estimated half lives based on OLS range from 2.750 years to 9.833 years. The median unbiased estimates yield half lives ranging from 4.083 years to an infinite half life. These results are similar to those of Murray and Pappell (2002, 2005) who show that median unbiased corrections yield half lives that are longer than previously thought.

[INSERT TABLE 3 HERE]

As documented above, there is an extensive literature that has attempted to use fractional long memory to explain the finding of a unit root in the real exchange rate, while potentially mitigating estimates of half lives of deviations from PPP. The impulse response function for the ARFIMA(p,d,0) model is based on the calculation of the moving average (MA) representation using Eq. (2) and Eq. (3), which is given by:

---

[7] I use the procedure outlined in Andrews and Chen (1994) to obtain the estimated median unbiased estimates of the model parameters and associated half lives.

$$(y_t - \mu) = \frac{\sum_{i=0}^{\infty} c_i \varepsilon_{t-i}}{1 - \phi_1 L - \ldots - \phi_p L^p}, \qquad (13)$$

with $c_0=1$, $c_i=\frac{\Gamma(i+d)}{\Gamma(d)\Gamma(i+1)}$. The MA coefficients can be calculated through recursion. For example, for an ARFIMA(1,d,0) model the j^{th} coefficient is, $\psi_j = \phi^j + \phi^{j-1}c_1 + \phi^{j-2}c_2 + \ldots + c_j$.

Threshold non-linearity has also been used in an attempt to shorten estimated half lives. Calculation of the impulse response function for non-linear models is more complex, since the function is not independent of the history of the process, size of the shock, or future innovations of the process (see Koop, Pesaran, and Potter, 1996). Typically, therefore, researchers have utilized the generalized impulse response function (GIRF). The GIRF for a horizon h of a time series process Y_t with a shock of size δ and conditional on the history, ω_{t-1}, is given by:

$$GIRF_Y(h, \varepsilon_t, \omega_{t-1}) = E[Y_{t+h} \mid \varepsilon_t = \delta, \omega_{t-1}] - E[Y_{t+h} \mid \varepsilon_t = 0, \omega_{t-1}], \qquad (14)$$

where ε_t is the disturbance sequence associated with the process. The computation of the impulse response function involves the calculation of the conditional expectation of the process contingent on a given history. As discussed by Koop, Pesaran, and Potter (1996), calculation of the GIRF is accomplished by averaging out future shocks through numerical integration.

I closely follow Taylor, Peel, and Sarno (2001), and van Dijk, Franses, and Paap (2002) in the calculation of the GIRF for the non-linear models. For the FI-ESTAR model, it is necessary to forecast the future values of a process that has both long memory and non-linearity. This can be accomplished through recursion noting that for h=1, we have the following expression for the forecast of y_{t+1} conditional on information known at time t, where again $x_t \equiv (1-L)^d y_t$:

$$\hat{y}_{t+1|t} = -\sum_{k=1}^{\infty} c_k y_{t-k+1} + \phi_{1,0} + \sum_{j=1}^{p} \phi_{1,j} x_{t-j+1} + (\phi_{2,0} + \sum_{j=1}^{p} \phi_{2,j} x_{t-j+1})(1 - \exp[-\gamma(y_{t-\tau+1} - c)^2]. \qquad (15)$$

The above expression must be truncated, and similar to van Dijk, Franses, and Paap, the first 100 values are excluded. To calculate the GIRF, I start with the observation in April 1981 and condition on the information up to and including that point. Following the forecasting procedure outlined in Granger and Teräsvirta (1993), I then forecast the process h periods into the future, both with the initial shock equal to δ and with the initial shock equal to 0. For a given history, this procedure is repeated 200 times, where I randomly sample with replacement from the estimated residuals of the FI-ESTAR model. The sample is updated one period and the same procedure continues until the end of the sample is reached. The disparity between the forecasted series with $\varepsilon_t=\delta$ and $\varepsilon_t=0$ is recorded for $i=1,\ldots,h$ each point in time as in Eq. (14). For the euro-zone countries, this results in the calculation of 42,400 simulations of the quantity in Eq. (14). The estimated GIRF is the mean of these values for each $i=1,\ldots,h$.[8]

Here, perturbations to the initial conditions of 1%, 10%, 25%, 40%, and 50% are considered, which results in setting $\delta=\log(1+k/100)$, for k=1,10,25,40, and 50, where k is the percentage size of the shock. To make the quantity comparable to linear models, the half lives are calculated as the number of periods for the GIRF to fall below $\delta/2$.

Table 4 reports the estimates of the half lives of deviations from PPP based on the preferred model for each country selected in Section 4.[9] The half lives calculated for the linear ARFIMA

---

[8] To be consistent, I chose to truncate the first 100 observations even when I calculate the impulse response functions for the ESTAR model. The estimated GIRF is calculated in precisely the same way for the ESTAR model as the FI-ESTAR model with the first quantity on the right hand side of Eq. (15) set equal to 0.

[9] Given that I estimate different models for different countries, I do not restrict the parameter space for any of the models. Previous authors (e.g. Taylor, Peel, and Sarno, 2001) using STAR models impose the reasonable restriction

models are independent of the size of the shock and are thus constant. As documented above, I am unable to reject the hypothesis of linearity for Canada and Japan. Based on the estimated ARFIMA models for these countries, we see that the half lives range from over 11 years (for Canada) to over 5,793 years (for Japan).[10] A fully parameterized FI-ESTAR model was also estimated for these countries as discussed above. The estimated half lives based on these models are always in excess of three years, and thus provide little relief for the PPP puzzle.

[INSERT TABLE 4 HERE]

The remaining countries have significant threshold non-linear dynamics. The estimated non-linear STAR models generate half lives less than 3 years for France and the UK for at least one shock size. The estimated half lives in Table 4 are almost always smaller than the half lives obtained using median unbiased estimates based on AR models. I also estimated the non-linear models and associated half lives with the restriction that the constants in both regimes are zero. Note that for the FI-ESTAR model, a non-zero constant with fractional differencing implies trending behavior, which may not be expected in the real exchange rate. Further, for France and Germany, I am unable to reject the hypothesis that these parameters are zero. The estimated half lives based on the restricted non-linear models are reported in Table 5. Although the results are similar to those reported in Table 4, it should be pointed out that the estimated half lives were

---

that the inner regime is characterized by a unit root, and is more persistent. Without this restriction, it is possible to get half lives that increase with the size of the shock.

[10] The demonstrably long half lives are obtained because the estimated ARFIMA models are arbitrarily close to a unit root. These results are robust to the estimation method for *d*. I also estimated *d* using Andrews and Guggenberger's (2003) bias reduced log periodogram regression based estimator. The results, which are available upon request, show that the estimates of *d* range from 0.7877 to 1.2277 with strong concentration near the unit root boundary.

generally smaller with this restriction. In fact, for at least one shock size, we are able to generate half lives that are less than or equal to three years for each EU country. With the exception of Italy, half lives of about two years are common.

[INSERT TABLE 5]

6. Conclusion

Independently, evidence of both fractional long memory and threshold non-linearity has been uncovered in the dynamics related to purchasing power parity. To date, little empirical work has been done to disentangle the relative importance of these modeling choices in the context of the PPP puzzle. In particular, no paper has used impulse response analysis or estimated half lives while allowing for elements of both modeling techniques. This paper addresses these issues by using a procedure that allows one to jointly model elements of fractional integration and exponential smooth transition autoregressive non-linearity.

For the post Bretton Woods sample of G-7 countries vis-à-vis the US, the analysis in this paper shows that there is evidence of both ESTAR non-linearity and fractional long memory in the dynamics associated with PPP for the EU countries in my data set. There is little evidence of ESTAR non-linearity in the real exchange rates of Canada and Japan.

Estimated half lives using median unbiased corrections for linear autoregressive models reconfirm existing evidence related to the PPP paradox, as half lives range from 4.08 years to an infinite number of years. For Canada and Japan, the estimated half lives based on the linear ARFIMA models are at least 11 years in duration. Conversely, the estimated FI-ESTAR model can produce half lives of less than or equal to 3 years for each EU country in the sample. The results suggest that the use of a model with both fractional long memory and threshold non-linearity can be important for analyzing the dynamics associated with PPP.

Acknowledgements: The paper has improved thanks to comments and suggestions from an anonymous referee. I would also like to thank seminar participants at the University of Texas-Arlington, Kevin Grier, Robin Grier, and Stefan Norrbin for very helpful comments. Any errors are my own.

References

Achy, L., 2003. Parity reversion in the real exchange rates: middle income country base. Applied Economics 35, 541-53.

Andrews, D.W.K., 1993. Exactly median-unbiased estimation of first order autoregressive/unit root models. Econometrica 61, 139-165.

Andrews, D.W.K., Chen, H-Y., 1994. Approximately median-unbiased estimation of autoregressive models. Journal of Business and Economic Statistics 12, 187-204.

Andrews, D.W.K, Guggenberger, P., 2003. A biased reduced log-periodogram regression estimator for the long memory parameter. Econometrica 71, 675-712.

Baillie, R., Kapetanios, G., 2006. Testing for neglected nonlinearity in long memory models. Journal of Business and Economic Statistics (forthcoming).

Baum, C., Barkoulas, J.T., Caglayan, M., 1999. Long memory or structural breaks: can either explain nonstationary real exchange rates under the current float? Journal of International Financial Markets, Institutions, and Money 9, 359-376.

Baum, C., Barkoulas, J.T., Caglayan, M., 2001. Nonlinear adjustment to purchasing power parity in the post-Bretton Woods era. Journal of International Money and Finance 20, 379-399.

Beran, J., 1995. Maximum likelihood estimation of the differencing parameter for invertible short and long memory autoregressive integrated moving average models. Journal of the Royal Statistical Society B 57, 659-672.

Chari V.V., Kehoe, P., McGrattan, E., 2002. Can sticky price models generate volatile and persistent real exchange rates? Review of Economic Studies 69, 533-563.

Chen, S-S., Engel, C., 2005. Does ‘aggregation bias’ explain the PPP puzzle? Pacific Economic Review 10, 49-72.

Cheung, Y.W., Lai, K.S., 1993. Long run purchasing power parity during the recent float. Journal of International Economics 34, 181-192.

Cheung, Y.W., Lai, K.S., 2001. Long memory and nonlinear mean reversion in Japanese yen-based real exchange rates. Journal of International Money and Finance 20, 115-132.

Diebold, F.X., Husted, S., Rush, M., 1991. Real exchange rates under the Gold Standard. Journal of Political Economy 99, 1252-71.

Diebold, F.X., Inoue, A., 2001. Long memory and regime switching. Journal of Econometrics 105, 131-159.

Eitrheim, O., Teräsvirta, T., 1996. Testing the adequacy of smooth transition autoregressive models. Journal of Econometrics 74, 59-76.

Granger, C.W.J., Joyeux, R., 1980. An introduction to long-memory time series. Journal of Time Series Analysis 1, 15-30.

Granger, C.W.J., Teräsvirta, T., (1993). Modelling Nonlinear Economic Relationships. Oxford University Press, New York.

Hoskng, J.R.M., 1981. Fractional differencing. Biometrika 65, 165-176.

Hurwicz, L., 1950. Least squares bias in time series. In: Koopmans, T.C. (Ed.) Statistical Inference in Dynamic Economic Models. John Wiley, New York, pp. 365-383.

Imbs, J., Mumtaz, H., Ravn, M., Ray, H., 2005. PPP strikes back: Aggregation and the real exchange rate. Quarterly Journal of Economics 70, 1-43.

Koop, G., Pesaran, M.H., Potter, S., 1996. Impulse response analysis in nonlinear multivariate models. Journal of Econometrics 74, 119-147.

Kramer, W., Sibbertsen, P., 2002. Testing for structural changes in the presence of long memory. International Journal of Business and Economics 1, 235-242.

Michael, P., Nobay, A.R., Peel, D.A., 1997. Transactions costs and nonlinear adjustment in real exchange rates: an empirical investigation. Journal of Political Economy 105, 862-879.

Murray, C., Papell, D., 2002. The purchasing power parity persistence paradigm. Journal of International Economics 56, 1-19.

Murray, C., Papell, D., 2005. The purchasing power parity puzzle is worse than you think. Empirical Economics 30, 783-790.

Paya, I., Venetis, I., Peel, D., 2003. Further evidence of PPP adjustment speeds: The case of effective real exchange rates and the EMS. Oxford Bulletin of Economics and Statistics 65, 421-437.

Rogoff, K., 1996. The purchasing power parity puzzle. Journal of Economic Literature 34, 647-668.

Sercu, P., Uppal, R., Van Hulle, C., 1995. The exchange rate in the presence of transactions costs: implications for tests of purchasing power parity. Journal of Finance 50, 1309-1319.

Smallwood, A.D., 2005. Joint tests for long memory and non-linearity: The case of purchasing power parity. Studies in Nonlinear Dynamics and Econometrics 9, article 7, 1-28.

Taylor, A.M., 2001. Potential pitfalls for the purchasing-power parity puzzle? Sampling and specification biases in mean reversion tests of the law of one price. Econometrica 69, 473-498.

Taylor, M.P., Peel, D., Sarno, L., 2001. Nonlinear mean reversion in real exchange rates: toward a solution to the purchasing power parity puzzles. International Economic Review 42, 1015-1042.

Teräsvirta, T., 1994. Specification, estimation, and evaluation of smooth transition autoregressive models. Journal of the American Statistical Association 89, 208-218.

van Dijk, D., Franses, P.H., Paap, R., 2002. A nonlinear long memory model for US unemployment. Journal of Econometrics 102, 135-165.

FIGURES

Figure 1 Caption (No Legend):

“Estimated Transition Function from the FI-ESTAR Model

CPI Based French Real Exchange Rate”

Table 1
Test Results for Linearity for Fractionally Differenced Series
Real Exchange Rates

	Differencing Parameter Estimated Under the Null		Differencing Parameter Estimated Under the Alternative		
	LM χ^2	p value χ^2	LM χ^2	p value χ^2	Selected Delay
Canada	2.5401	0.2808	2.8278	0.2432	5
France	1.7172	0.4237	6.4786	0.0392	6
Germany	8.4053	0.0150	11.9704	0.0025	1
Italy	4.4460	0.1083	6.8969	0.0318	6
Japan	1.8446	0.3976	1.7617	0.4144	12
UK	3.5212	0.4747	11.4613	0.0218	11

Notes: This table reports the results of the χ^2 version of the LM tests for linearity against the fractionally differenced ESTAR model. The differencing parameter has been estimated in two ways. First, it is estimated under the null using an ARFIMA model as discussed by van Dijk, Franses and Paap (2002) and extended by Smallwood (2005). Alternatively, it has been estimated using the full FI-ESTAR model under the alternative. The selected delay parameter has been obtained under the null.

Table 2

Estimation Results Based on the Preferred Model for the Real Exchange Rate

Coefficient	Canada (ARFIMA)	Japan (ARFIMA)	Coefficient	France (FI-ESTAR)	Germany (FI-ESTAR)	Italy (FI-ESTAR)	UK (ESTAR)
μ	0.2029	-0.1433	$\phi_0^{(1)}$	0.0014	-0.0292	-0.2905	-0.0128
	[0.1474]	[0.0752]		[0.0055]	[0.0035]	[0.0171]	[0.0674]
ϕ_1	0.9868	0.4141	$\phi_1^{(1)}$	1.0356	0.3494	-2.5870	0.2918
	[0.0130]	[0.1564]		[0.2381]	[0.1828]	[0.2805]	[0.3051]
d	0.1445	0.8961	$\phi_2^{(1)}$	N/A	N/A		0.5340
	[0.0667]	[0.1361]					[0.2804]
			$\phi_3^{(1)}$	N/A	N/A		0.6004
							[0.5874]
			$\phi_0^{(2)}$	-0.0059	0.0270	0.2898	0.0107
				[0.0063]	[1.0711]	[0.0170]	[0.0681]
			$\phi_1^{(2)}$	-0.1538	0.5731	3.5453	1.0755
				[0.2386]	[59.0080]	[0.2625]	[0.1927]
			$\phi_2^{(2)}$	N/A	N/A	N/A	-0.9143
							[0.2052]
			$\phi_3^{(2)}$	N/A	N/A	N/A	-0.4788
							[0.6027]
			γ	1.2744	3.9135	326.5277	5.4990
			$1/\sigma^2_s$	43.5493	38.4246	46.0501	60.3479
				[5.2323]	[0.6716]	[38.6125]	[5.6230]
			c	0.1400	-0.3513	-0.2576	0.1072
				[0.0324]	[0.0548]	[0.0155]	[0.0244]
			d	0.2559	0.3965	0.2559	0.0000
				[0.1208]	[0.1385]	[0.1395]	
Pval-Q (5)	0.6310	0.6014	Pval-Q (5)	0.1618	0.5470	0.0476	0.2623
Pval-Q (10)	0.0734	0.4613	Pval-Q (10)	0.4082	0.4640	0.1350	0.4101
Pval-Q (20)	0.0179	0.1389	Pval-Q (20)	0.5114	0.7546	0.2213	0.0451
Pval-Q^2(1)	0.5900	0.4017	Pval-Q^2(1)	0.8302	0.9127	0.3014	0.0054
Pval-Q^2(5)	0.0040	0.0225	Pval-Q^2(5)	0.9681	0.6207	0.4184	0.0001
Pval-Q^2(10)	0.0006	0.0618	Pval-Q^2(10)	0.5762	0.6322	0.3273	0.0004
Pval No Rem	N/A	N/A	Pval No Rem	0.3940	0.5695	0.0684	0.1333

Notes: The results here yield estimates of the preferred time series model. For example, the FI-ESTAR model is:

$$(1-L)^d r_t=\phi_{1,0}+\phi_{1,1}(1-L)^d r_{t-1}+\ldots+\phi_{1,p}(1-L)^d r_{t-p}+(\phi_{2,0}+\phi_{2,1}(1-L)^d r_{t-1}+\ldots+\phi_{2,p}(1-L)^d r_{t-p})(1-\gamma[r_{t-\tau}-c]^2)+\varepsilon_t,$$

where r_t denotes the log of the real exchange rate. The estimated model is based on the test results in Table 1. The ESTAR model is the same as above with d=0. The ARFIMA(p,d,0) model results when γ=0 or $\phi_{2,j}$=0, j=0,…,p. I fail to reject linearity for Canada and Japan and thus report estimates for the linear ARFIMA model given by:

$$(1-\phi_1 L-\ldots-\phi_p L^p)(1-L)^d(r_t-\mu)=\varepsilon_t.$$

P-valQ and Pval-Q^2 are the p-values for the Ljung-Box Q-statistics for the hypothesis of serial correlation in the residuals and squared residuals, respectively. P-val No Rem records the probability value for the hypothesis of no remaining non-linearity as discussed by van Dijk, Franses, and Paap (2002). The numbers appearing in brackets beneath the parameter estimates are numerical standard errors.

Table 3

Results for Estimation of the ADF Model with Implied Median Unbiased Parameter Estimates

Real Exchange Rates

Country	Canada	France	Germany	Italy	Japan	UK
α_{LS}	0.9938	0.9779	0.9799	0.9792	0.9855	0.9775
	[0.0047]	[0.0098]	[0.0100]	[0.0094]	[0.0066]	[0.0099]
c	0.0016	-0.0022	-0.0023	-0.0003	-0.0076	-0.0045
	[0.0010]	[0.0018]	[0.0019]	[0.0014]	[0.0034]	[0.0021]
ζ_{1LS}	0.1810	0.3009	0.2996	0.3502	0.3099	0.3678
	[0.0510]	[0.0565]	[0.0540]	[0.0534]	[0.0485]	[0.0513]
ζ_{2LS}	N/A	-0.0931	N/A	N/A	N/A	-0.1114
		[0.0587]				[0.0517]
ζ_{3LS}	N/A	0.1531	N/A	N/A	N/A	N/A
		[0.0564]				
Half Lives$_{\text{LS}}$ (in years)	**9.833**	**2.833**	**3.083**	**3.000**	**4.250**	**2.750**
α_{MU}	1.0000	0.9860	0.9890	0.9870	0.9930	0.9850
ζ_{1MU}	0.1787	0.2947	0.2952	0.3463	0.3084	0.3662
ζ_{2MU}	N/A	-0.1017	N/A	N/A	N/A	-0.1171
ζ_{3MU}	N/A	0.1494	N/A	N/A	N/A	N/A
Half Lives$_{\text{MU}}$ (in years)	∞	**4.417**	**5.583**	**4.750**	**8.750**	**4.083**
Pval- Q (5)	0.9310	0.8686	0.8398	0.8947	0.5766	0.6573
Pval- Q (10)	0.0934	0.8580	0.6760	0.8440	0.4794	0.8208
Pval- Q (20)	0.0193	0.9506	0.9235	0.8396	0.1408	0.1958
Pval-Q^2(1)	0.6586	0.6633	0.0607	0.7860	0.3463	0.0061
Pval-Q^2(5)	0.0094	0.8702	0.0129	0.8199	0.0379	0.0000
Pval-Q^2(10)	0.0865	0.4104	0.0412	0.5163	0.0805	0.0000
SSE	0.0504	0.1976	0.2243	0.1781	0.2887	0.2198
Implied AR(p) Model in Levels						
	Canada	France	Germany	Italy	Japan	UK
ϕ_1	1.1787	1.2807	1.2842	1.3333	1.3014	1.3512
ϕ_2	-0.1787	-0.3964	-0.2952	-0.3463	-0.3084	-0.4833
ϕ_3	N/A	0.2511	N/A	N/A	N/A	0.1171
ϕ_4	N/A	-0.1494	N/A	N/A	N/A	N/A

Notes: The specification used here is given by:

$$r_t = c + \alpha_{LS} r_{t-1} + \zeta_1 \Delta r_{t-1} + \ldots + \zeta_{p-1} \Delta r_{t-p+1} + u_t.$$

The table yields the parameter estimates for an ADF regression with a constant. P-valQ and P-valQ^2 are the p-values for the Ljung-Box Q-statistics for the hypothesis of serial correlation in the residuals and squared residuals. Half Lives$_{\text{LS}}$ denotes the half-lives based on the least squares estimate of α (α_{LS}), which is given by the number of years for the initial shock to decrease by 1/2. Half Lives$_{\text{MU}}$ denotes the same quantity for the median unbiased estimates of α(α_{MU}). The median unbiased parameters have been calculated computationally where α ranges along a grid from -0.999 to 1 with a step size of 0.001. See Andrews and Chen (1994) for details. Standard errors appear in brackets beneath parameter estimates.

Table 4
Estimated Half-Lives for Unrestricted Models
Real Exchange Rates

Estimated Half Lives (in years) for Countries where the ARFIMA Model is Preferred

Shock Size	1%	10%	25%	40%	50%
Country					
Canada	11.5833	11.5833	11.5833	11.5833	11.5833
Japan	5793.4167	5793.4167	5793.4167	5793.4167	5793.4167

Estimated Half Lives (in years) for Countries where the ESTAR Model (with $d=0$) Model is Preferred

Shock Size	1%	10%	25%	40%	50%
Country					
UK	4.5833	4.1667	3.0000	2.0000	1.9167

Estimated Half Lives (in years) for Countries where ESTAR Model (with $d=0$) Model is Preferred

Country	1%	10%	25%	40%	50%
France	2.7500	4.0833	4.1667	3.1667	2.4167
Germany	3.3333	3.7500	3.9167	4.2500	4.4167
Italy	5.6667	4.8333	4.6667	4.6667	4.6667

Notes: The table reports the estimated half lives of shocks to the CPI based real exchange rates for the estimated ARFIMA, ESTAR, and FI-ESTAR models reported in Table 2. The half lives are calculated using the estimated impulse response functions (see Eqs. (13)-(15) in the text).

Table 5
Estimated Half-Lives for Models with Restricted Constants in Inner and Outer Regimes
Real Exchange Rates

Estimated Half Lives (in years) for Countries where the ESTAR Model (with $d=0$) Model is Preferred

Country	Shock Size	1%	10%	25%	40%	50%
UK		2.0833	4.1667	3.1667	2.1667	2.0833

Estimated Half Lives (in years) for Countries where the FI-ESTAR Model is Preferred

Country	Shock Size	1%	10%	25%	40%	50%
France		2.0000	2.5000	2.5833	2.3333	2.0000
Germany		1.8333	2.1667	2.6667	2.9167	3.0833
Italy		3.2500	3.0000	3.5000	3.5000	3.6667

Notes: The table reports the estimated half lives of shocks to the CPI based real exchange rates for the estimated ESTAR and FI-ESTAR models with the constants, $\phi_{1,0}$ and $\phi_{2,0}$, restricted to be zero (see Eq.5). The half lives are calculated using the estimated impulse response functions (see Eq. (14) and Eq. (15) in the text).