Are the properties of real interest rates affected by measurement approaches?

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Abstract

The time series properties of real interest rates has been a controversial topic in the past literature. Authors find the distribution of the real interest rate to range from a stationary process to a unit root, or any process in between. We argue that the confusion may stem from the method used in constructing the real interest rates, and consider a variety of methods of calculating the real rate. The results of the unit root tests present different conclusions for the various real rates, seemingly adding to the existing confusion. However, estimation of the integration order using long memory fractional techniques is much more robust, suggesting that the real interest rate is distributed as a persistent mean reverting process.

Keywords: Real interest rates; measuring expected inflation; unit roots; long memory.

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1 Introduction

The behavior of the real interest rate has always been a concern of economists due to the fact that the real interest rate is a key variable that affects saving and investment decisions. However, the empirical findings of the distribution of real interest rates are mixed and inconclusive. For example, Fama (1975) Garcia and Perron (1996), and Mishkin (1984) found the real rate to be stationary, whereas Rose (1988) and Rapach and Weber (2004) find the U.S. real interest rate to be distributed as a unit root process. Recently authors have allowed the potential distribution to be long memory. For example, Lai (1997), Tsay (2000), and Sun and Phillips (2004) show that the U.S. real interest rate appears to be a long memory process.

The real interest rate has been used in testing the Fisher hypothesis (Huizinga and Mishkin, 1984, Hoffman and Schlagenhauf, 1985, Kandel, Ofer, and Sarig, 1996, Garcia and Perron, 1996) and real interest rate parity (Cumby and Mishkin, 1986, Nelson, 1985, Goodwin and Grennes, 1994, Gagnon and Unferth, 1995). Researchers employ a wide variety of approaches for measuring the expected real returns on assets when attempting to test these hypotheses. In this paper we examine various methodologies used in the literature and document the difference in time series processes that result from the assumptions of agents' forecasts of inflation. We do not attempt to identify the "correct" specification. Instead, the paper focuses on determining whether different methods of deriving the real interest rates yield different conclusions about the distribution of the time series process. In particular, we aim to investigate whether the question of the real interest rates being stationary or not is sensitive to the underlying approach of deriving the rates.

We select five methodologies of constructing real interest rates: (*i*) the *ex post* real interest rate, (*ii*) autoregressive inflation forecasts, (*iii*) Mishkin's linear projection, (*iv*) a rolling regression, and (*v*) a regime-switching technique. Methods (*ii*)-(*iv*) are based on the linear regression model under different specifications. For example, Mishkin's approach is the extended version of the AR(4) model where additional macroeconomic variables are added into the regression model for lagged inflation. In contrast, the regime-switching method estimates a potential non-linear pattern for the real interest rate. In addition to using different methodologies researchers have used different methods for annualizing inflation rates. We use both annualized month-to-month and year-to year changes in prices. Thus a total of ten different real interest rates are calculated.

We use five different unit root tests to investigate whether the real interest rates from different approaches yield different results regarding stationarity. In addition, we measure persistence for each of the constructed real rates through fractional integration and obtain estimates of the memory parameter using the exact Whittle estimator of Shimotsu and Phillips (2005). Because most of the past research only employs one type of calculation of the real interest rate authors implicitly assume that the type of real interest rate used is irrelevant to the test. If that is the case, then all ten real interest rates should have similar time series properties. If that is not the case, then this study may shed some light on which type of assumptions lead to similar times series processes, and whether any type of real interest rate calculation is different from the others.

The following section discusses the different methods of computing real interest rates in the past literature, whereas section three discussed the different econometric methodologies used to determine the times series properties of the real interest rates. The fourth section compares the calculated real interest rates, while the fifth section discusses the results of the unit root and long memory tests, and the last section provides some conclusions.

2 Methods of Constructing Real Interest Rates

Studying the real interest rate is difficult in the sense that the *ex ante* real interest rate is unobservable. Based on the Fisher equation, the nominal rate of interest can be thought of as the equilibrium desired real return plus the market's assessment of the expected rate of inflation, given by

$$i_t = r_t^e + \pi_{t+1}^e \tag{1}$$

where i_t is the nominal interest rate from holding the one-period bond from t to $t+1$, r_t^e is the one-period real rate of interest expected for the bond maturing at time $t+1$; and π_t^e is the time t expected inflation rate from t to $t+1$. Methods for calculating π_t^e differ among economists. This section provides a brief summary of approaches researchers have taken in estimating the *ex ante* real rate of interest.

2.1 *Ex Post* Real Rates

By assuming rationality of inflationary expectations, researchers use the *ex post* real rate to study the behavior of the *ex ante* real rate. Under rationality, the *ex post* real interest rate, where the expected inflation in equation (1) with actual inflation (π_{t+1}), will differ from the *ex ante* real interest rate by a random inflation forecast error which is well-behaved with zero mean and orthogonal to the available information set. Kugler and Neusser (1993) , Gagnon and Unferth (1995), and Goodwin and Grennes (1994) are examples of papers that use the *ex post* rates for the empirical methodology.

2.2 Linear *Ex Ante* Real Rates

Linear methods differ with respect to the information set used by agents. The basic autoregressive representations are appealing to researchers because they link the present observable data to the past history of the data so that one can extrapolate to form a forecast of future values based on present and past observations. The expected inflation rate is obtained by fitting an AR model to the actual inflation rate and forming a one-period ahead forecast from the estimated AR coefficients as follows

$$\pi_{t+1}^e \equiv \hat{\pi}_{t+1} = \hat{\varphi}_1 \pi_{t-1} + \hat{\varphi}_2 \pi_{t-2} + ... + \hat{\varphi}_p \pi_{t-p} \tag{2}$$

where $\{\hat{\varphi}_1, \hat{\varphi}_2, ..., \hat{\varphi}_p\}$ are the estimated coefficients of $\{\varphi_1, \varphi_2, ..., \varphi_p\}$. An example of authors using the AR specification is Baharumshah et al. (2005) who use an AR(1) specification for expected inflation.

Mishkin (1984), Cumby and Mishkin (1986), Huizinga and Mishkin (1984) add relevant macroeconomic variables to an ARMA model of expected inflation. This approach, which we refer to as the "Mishkin approach," implies that the *ex ante* real rate equals the expected real return on a one-period bond, conditional on available information at time t:

$$r_t^e = X_t \beta + u_t \tag{3}$$

where $u_t = r_t^e - P(r_t^e | X_t)$ are the projection errors and u_t is orthogonal to X_t. Mishkin's choice of X_t includes four lags of the inflation rate, one lag of money growth (M1), the nominal eurodollar interest rate, and a fourth-order time polynomial.

The above methods assume that agents have the full data sample to estimate the coefficients even in the beginning of the forecasting period. To remedy this problem Juntilla (2001) uses a rolling regression technique to allow agents only the information that was available at the time of the forecast. The rolling regression technique requires a fixed sample size, which is accomplished by changing the starting and end-point of the data. This assumes that agents are not concerned about the information that is too far in the past. In our estimation we use a 5-year range of data on inflation to predict next period's inflation rate.

2.3 Non-linear *Ex Ante* Real Rates

Prior research has found some evidence of different inflation regimes. For example, Huizinga and Mishkin (1984) find that a significant shift in the stochastic process of real rates occurs around October 1979 when the Fed changed its policy procedure. Garcia and Perron (1996) considered such shifts in the behavior of the U.S. real interest rate, by allowing three possible regimes affecting both the mean and the variance. The three-state Markov-switching mean-variance model accounts for regime shifts in an autoregressive model of the *ex post* real rate in the following way:

$$(y_t - \mu_{s_t}) = \phi_1(y_{t-1} - \mu_{s_{t-1}}) + \phi_2(y_{t-2} - \mu_{s_{t-2}}) + e_t, \tag{4}$$

where e_t is normally distributed with a zero mean and a variance of $\sigma^2_{s_t}$, y_t is an AR(2) process of the *ex post* real rate calculated by subtracting the inflation rate from the nominal interest rate and μ_{s_t}, and $\sigma^2_{s_t}$ is the mean and variance switching parameters when state S_{jt} is realized for $j = 1,2,3$, respectively. The state variable S_t is unknown *a priori*, that is, the dates of structural breaks are unobservable. Therefore, to determine the log likelihood function, one needs to consider the joint density of y_t and the unobserved S_t variable and then integrate the S_t variable out of the joint density to obtain the marginal density of y_t. However, the integrals of the joint density is difficult to solve, therefore we use the Gibbs-sampling procedure that generates a sample from the marginal density without requiring the marginal density distribution itself.[1]

3 Testing Methodology

Due to the low power of unit root tests we present results using five different tests, namely: the augmented Dickey-Fuller test (ADF), the Phillips-Perron unit root test (PP), the Dickey-Fuller test with GLS detrending (DF-GLS) introduced by Elliot, Rothenberg and Stock (1996), the Ng-Perron test, and the Kwiatkowski, Phillips, Schmidt, and Shin test (KPSS).

The most commonly used method to test for unit roots is the ADF test. For a time series process y_t, the ADF test is carried out by estimating

$$\Delta y_t = a + \alpha y_{t-1} + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + \varepsilon_t \tag{5}$$

The augmented terms, Δy_t, are included in equation (5) to correct the serial correlation of the disturbances ε_t. The PP test is an alternative non-parametric approach to deal with autocorrela-

[1]See Casella and George (1992) for examples and applications of the Gibbs-sampling algorithm.

tion in the error term and allows for heterogeneity of the variance. Phillips and Perron (1988) proposed the nonparametric test statistic as follows:

$$Z_t = \frac{s_e}{s} t_{\hat{\alpha}} - \frac{1}{2} \frac{(s^2 - s_e^2)}{s(T^{-2} \sum_1^T y_{t-1}^2)^{1/2}} \tag{6}$$

where $\hat{\alpha}$ is the OLS estimate of α for the regression in (5) without the augmented differencing terms, $t_{\hat{\alpha}}$ is the t-ratio of α, s_e is the coefficient standard error, and s^2 is a consistent estimate of the error variance.[2]

The DF-GLS and Ng-Perron unit root tests purport to solve the problem of low power and serious size distortions in the ADF and PP tests when the moving-average component in the series y_t is significant and negative. Therefore we include these potentially more powerful tests as a comparison to the more commonly used ADF and PP tests.[3] Lastly, we use the KPSS test to test the null of difference stationarity against the alternative hypothesis of a stochastic trend. Switching the null to that of stationarity provides a useful comparison to the four other test that have nonstationarity as the null hypothesis. We use the Bartlett spectral window kernel-based estimator to obtain a consistent estimate of the variance and select the bandwidth by using the Newey-West method. [4]

The preponderance of the past research using real interest rates tests for the persistence of the time series using techniques that explicitly restrict the order of integration to be either zero or one. However, such an assumption may be unduly restrictive. A long memory model relaxes the assumption by allowing the order of integration to be any real number. A long memory fractional model for y_t is defined as follows,

$$(1 - L)^d y_t = u_t, \tag{7}$$

[2]We use a kernel sum-of-covariances estimator with Bartlett weights in combination with the Newey-West bandwidth selection method to obtain estimators of the residual spectrum at frequency zero.

[3]See Elliott et al. (1996) for details on the DF-GLS test, and Perron and Ng (1996) for Monte Carlo simulations of the size and power of the Ng-Perron test.

[4]Despite the KPSS test's low power (see, Maddala and Kim, 1998), we report results from this test for the sake of completeness and because it is often used in empirical studies.

where u_t is assumed to be a general short memory process and where d can take on any real value. Following Sun and Phillips (2004), we model y_t as a Type II fractional process, where

$$y_t = (1-L)^{-d} u_t = \sum_{k=0}^{t-1} \frac{\Gamma(d+k)}{\Gamma(d)\Gamma(k+1)} u_{t-k}. \tag{8}$$

Granger and Joyeux (1980) show that the process in (8) is stationary when $d < 1/2$ and mean reverting when $d < 1$. A unit root is detected when $d = 1$.

To estimate the persistence parameter d, for the real interest rate, we follow Sun and Phillips (2004) and use the exact Whittle estimator of Shimotsu and Phillips (2005). The estimator is practical as it does not require one to explicitly model the short run dynamics for u_t and has an asymptotic theory that is applicable for all d. The estimate of d is the value that maximizes the Whittle likelihood function, which is given as:

$$Q_m(G,d) = \frac{1}{m} \left(\log G \omega_j^{-2d} + \frac{1}{G} I_{(1-L)^d y_t}(\omega_j) \right), \tag{9}$$

where G is a constant typically associated with short run dynamics such as ARMA components, $I_{(1-L)^d y_t}(\omega_j)$ is the periodogram of the fractional difference of y_t and where ω_j are the set of Fourier frequencies, $2\pi j/T, j = 1, ..., m$ with T denoting the sample size. Finally, m is the bandwidth parameter, which must satisfy $\frac{m}{T} + \frac{1}{m} \longrightarrow 0$ as $T \longrightarrow \infty$. For hypothesis testing, Shimotsu and Phillips (2005) provide the following limiting distribution for the exact Whittle estimate of d, $\hat{d}$, as $T \rightarrow \infty$.[5]

$$\sqrt{m}(\hat{d} - d) \xrightarrow{d} N(0, 1/4). \tag{10}$$

[5]Optimization of the Whittle likelihood function requires numerical optimization techniques, which were carried out utilizing MATLAB code written by the authors, which is available upon request. For additional details concerning the exact Whittle estimator, see Shimotsu and Phillips (2005).

4 Examining Real Interest Rates

The data are monthly data from 1971:1 to 2003:12. The interest rate is a eurodollar interest rate, and prices are proxied by the seasonally adjusted Consumer Price Index. Both variables are from the Federal Reserve Bank of St. Louis. Since the rolling regression requires a starting period, we lose the first five years of data. Consequently, the real interest rate series begins in 1975:1. Moreover, we lose the last year of observations in the procedure of computing the year-to-year inflation rate. To facilitate comparison across methods, we limit our analysis of the time series properties of the constructed real rates to the common sample period of 1975:1 to 2002:12 for monthly data.

There are two common approaches used in calculating the annual rate of inflation. Most of the literature that mentions how the rate of inflation is obtained uses the period-to-period approach to calculate the inflation rates. [6] For monthly data the inflation rate can be calculated as:

$$\pi_t = \ln\left(\frac{P_t}{P_{t-1}}\right)^{12}. \tag{11}$$

In contrast, some other authors use a year-to-year approach (e.g. Gagnon and Unferth, 1995). Based on monthly data, the year-to-year approach implies:

$$\pi_t = \ln\left(\frac{P_t}{P_{t-12}}\right). \tag{12}$$

We use both methods in calculating the inflation rate to determine if differences exist in the real interest rate processes using the two different ways of computing inflation.

The plots of the real interest rates are exhibited in Figures 1 and 2. The dynamic patterns of these constructed real interest rates appear very similar. In the 1970s, the real eurodollar rates of returns fell and fluctuated around zero. There was a sudden jump of the real interest rate in the beginning of the 1980s. These estimated real rates seem to fluctuate persistently around six percent in the early eighties. The sudden shift in the real rates was identified by Huizinga and

[6] For example, Chen (2001) calculates the quarter-to-quarter inflation

Mishkin (1984) as the well-documented changes in the Fed's operating procedures in October 1979 and October 1982. Since 1986, the rates have decreased to around three percent and oscillate around this level until the end of sample period. The impression that the different ways of computing real rates are similar is confirmed by statistical tests in that the hypothesis of equal means cannot be rejected for either type of inflation calculation.[7] In contrast, the hypothesis of equal variances can be rejected for both inflation calculations.[8]

5 Time Series Properties of Real Interest Rates

The above results indicate that the real rates appear very similar. Tables 1 and 2 show the unit root tests for the different real interest rate calculations with annualized monthly inflation and year-to-year inflation, respectively. In all cases the null hypothesis is that the real interest rate is distributed as a unit root, except in the case of the KPSS test where the null is one of stationarity. To facilitate interpretation of the results the last column in Table 1 and 2 indicate a summary of the unit root tests. If the method of calculation does not matter then the summary should be consistent across the different methods.

Although the descriptive evidence indicates that the different measurement approaches result in similar time series processes, the results of the unit root tests are extremely mixed. The results for the different types of real interest rate approaches in Table 1 indicate that about half of the time (13/25 cases) one would find a rate to be a unit root process. Only the results of the Ng-Perron test and the KPSS tests are uniform across the methods of computing the real interest rate, but with the opposite conclusion. The Ng-Perron tests indicate that all rates are stationary, whereas the KPSS tests indicate that all are unit roots. All other tests are mixed across methods of computations. Table 1 also points out that not only are the results mixed for the different real rates, but also for the different unit root tests. Table 2 shows similar results with exactly the

[7]The p-value of an ANOVA test of equal means are 0.152 and 0.294 for month-to-month and year-to-year, respectively.

[8]The p-value of a Brown-Forsythe test of equal variances between the different real interest rates are 0.001 for both types of inflation calculations.

same probability of finding a unit root (13/25 cases), but with remarkable difference across unit root tests. In this table the only tests that are uniform across the real rates are the PP test and the KPSS test. Again the two tests are conflicting in conclusion.[9] In addition to mixed findings across unit root tests and real interest approaches, the results also differ across inflationary calculations. Comparing the results in Table 1 and 2 one can observe the the same unit root test and real interest rate calculation may have a different result of the unit root test. For example, the *ex post* real rate in Table 1 seems mostly to be stationary when using the month-to-month inflation rate, but the evidence in Table 2 indicates that the *ex post* real rate likely has a unit root. [10]

The results from the unit root tests corroborate previous research detailing the difficulty in detecting whether or not the real interest rate has a unit root. These tests restrict the order of integration of a variable to be either zero or unity. Therefore, in Table 3 and 4, we report estimates of the integration order using the exact Whittle estimator of Shimotsu and Phillips (2005) for the various real interest rates using a month-to-month and a year-to-year calculation for inflation, respectively. Following Sun and Phillips (2004), we utilize a range of bandwidths, with $m = T^{\alpha}$, where $\alpha = 0.60, 0.65, 0.70, 0.75, 0.80$. This range allows us to determine how the bandwidth choice affects the estimated persistence of the real interest rate. Beneath the point estimates for d, we report the upper band of the 95% confidence interval for the Whittle estimate of d based on equation (10).

The results in Table 3 differ substantially in interpretation from the unit root tests. First, the upper band of the 95% confidence interval contains unity for only one of the 30 reported estimates of d, for the rolling regression with m=$T^{0.60}$, and in no case does the estimated d parameter include zero. Further, the different methods result in remarkably close estimates of the order of integration, with the estimates generally exceeding 0.50 indicating that the fractional

[9]Throughout, the KPSS test failed to reject the null, possibly as a result of the low power of the test as documented by Maddala and Kim (1998).

[10]Sun and Phillips (2004) note that the additional noise in the *ex-post* real rate may cause the unit root tests to be more likely to reject nonstationarity. This conjecture is supported by the monthly results as this real rate would be more likely to have a high degree of noise, while the year-to-year calculation of inflation would be expected to be smoother.

process is likely to be nonstationary. The *ex post* real rate and the regime-switching process have slightly lower orders of integration which supports Sun and Phillips (2004) assertion that the order of integration of the *ex post* rate may be underestimated due to the additional noise in the computation.

The results in Table 4, using annual changes in the inflation rate, are equally compelling. Again, there is only one case where we fail to reject d=1. Interestingly, this occurs for the ex post real rate when $m = T^{0.80}$. Thus, of the 60 estimates of d in Tables 3 and 4, 58 indicate a rejection of the unit root hypothesis for the real interest rate with these results being highly robust to the methods used in constructing the real rate. These results are obviously in stark contrast to those in Tables 1-2. Further, in Table 4, we see that the point estimates are strikingly close to each other, and the results are largely robust in that they are not sensitive to the selected bandwidth. For every real rate, d attains its largest value for a bandwidth of $m = T^{0.80}$. The persistence parameter d attains its lowest value for every real rate when $m = T^{0.65}$. At the bottom of the table, we report the average value of d across all five bandwidths. Again, the results are telling and we see virtually no disparity between the estimated values. The average values range from 0.7480 to 0.7951, indicating that each real interest rate appears to be a very persistent series that is significantly different from a unit root process.

6 Conclusions

Since the time series properties of the expected real interest rate are important in many economic theories, we review the different approaches of constructing the real interest rates that have been used in prior literature. We construct the real interest rate using different methods to calculate the inflation rate including using: (*i*) actual realized inflation (*ii*) the AR(4) inflation forecasts, (*iii*) Mishkin's linear projection, (*iv*) a rolling regression, and (*v*) the regime-switching technique. Our aim was to investigate if the time series properties of real interest rates constructed differ across a variety of models.

Our findings indicate that the real interest rates from different approaches yield similar time series processes in appearance, but with very different conclusions about the potential nonstationarity of the processes when using a variety of unit root tests. In particular, the inconsistent results obtained by previous authors concerning the stationarity of the real interest rate appears to depend on both the type of method used to construct the real rate of interest and the type of unit root test employed. Thus authors that conclude that the real interest rate they are testing is stationary or a unit root might reach a different conclusion in hypothesis testing if a different method of computing the real interest rate was used. In contrast, the results using long memory models appear much more consistent across the real interest rate types. Although this paper is not attempting to find the "true" time series process for the U.S. real interest rate, the consistency of the long memory findings for the U.S. real interest rate, combined with the fact that the long memory tests include both stationarity and a unit root as potential results, leads to the conclusion that the U.S. real interest rate is likely to be a very persistent mean reverting process that is easily confused as a unit root.

References

[1] Baharumshah, Ahmad, Chan T. Haw and Stilianos Fountas. 'A panel study on real interest rate parity in East-Asian countries: Pre and post liberalization era.' *Global Finance Journal*, 1, (2005) 69-85.

[2] Casella, George and Edward George. 'Explaining the Gibbs Sampler.' *American Statistician*, 46(3), (1992) 167-174.

[3] Chen, Li-Hsueh. 'A model for ex ante real interest rates.' *Applied Economics Letters*, 8, (2001) 713-718.

[4] Cumby, Robert and Frederic Mishkin. 'The international linkage of real interest rates: the European-US connection.' *Journal of International Money and Finance*, 5, (1986) 5-23.

[5] Elliott, Graham, Thomas J. Rothenberg and James H. Stock. 'Efficient tests for an autoregressive unit root', *Econometrica*, 64, (1996) 813-836.

[6] Fama, Eugene. 'Short-term interest rates as predictors of inflation.' *American Economic Review*, 65(3), (1975) 269-282.

[7] Gagnon, Joseph E. and Mark D. Unferth. 'Is there a world real interest rate?' *Journal of International Money and Finance*, 14(6), (1995) 845-855.

[8] Garcia, Rene, and Pierre Perron. 'An analysis of real interest under regime shift.' *Review of Economics and Statistics*, 78, (1996) 111-125.

[9] Goodwin, Barry and Thomas Grennes. 'Real interest rate equalization and the integration of international financial markets.' *Journal of International Money and Finance*, 13, (1994) 107-124.

[10] Granger, Clive, and Roselyne Joyeux. 'An introduction to long memory time series models and fractional differencing.' *Journal of Time Series Analysis*, 1, (1980) 15-29.

[11] Hoffman, Dennis and Don E. Schlagenhauf 'Real interest rates, anticipated inflation, and unanticipated money: a multi-country study.' *Review of Economics and Statistics*, 67(2), (1985) 284-296.

[12] Huizinga, John and Frederic Mishkin. 'Inflation and real interest rates on assets with different risk characteristics.' *Journal of Finance*, 39(3) , (1984) 699-712.

[13] Junttila, Juha 'Structural breaks ARIMA model and Finnish inflation forecasts.' *International Journal of Forecasting*, 17, (2001) 203-230.

[14] Kandel, Shmuel, Aharon R. Ofer and Oded Sarig 'Real interest rates and inflation: an ex-ante empirical analysis.' *Journal of Finance*, 51(1), (1996) 205-225.

[15] Kugler, Peter and Klaus Neusser. 'International real interest rate equalization.' *Journal of Applied Econometrics*, 8, (1993) 163-174.

[16] Lai, Kon S. 'Long-term persistence in the real interest rate: Some evidence of fractional unit root.' *International Journal of Finance and Economics*, 2, (1997) 225-235.

[17] Maddala, G. S. and In-Moo Kim. Unit Roots, Cointegration, and Structural Change. Cambridge University Press, Cambridge (1998).

[18] Mishkin, Frederic. 'Are real interest rates equal across countries? An empirical investigation of international parity conditions.' *Journal of Finance*, 39(5), (1984) 1345-1357.

[19] Nelson, Mark. 'A note on international real interest rate differentials.' *Review of Economics and Statistics*, 67(4), (1985) 681-684.

[20] Perron, Pierre and Serena Ng. 'Useful modifications to some unit root tests with dependent errors and their local asymptotic properties.' *Reviews of Economic Studies*, 63, (1996) 435-463.

[21] Phillips, Peter C. B. and Pierre Perron. 'Testing for a Unit Root in Time Series Regression.' *Biometrika*, 75(2), (1988) 335-346.

[22] Rapach, David E. and Christian W. Weber. 'Are Real Interest Rates Really Nonstationary? New Evidence from Tests with Good Size and Power.' *Journal of Macroeconomics* 26(3), (2004) 409-430.

[23] Rose, Andrew. 'Is the real interest rate stable?' *Journal of Finance*, 43(5), (1988) 1095-1112.

[24] Shimotsu, Katsumi and Peter C. B. Phillips. 'Exact local Whittle estimation of fractional integration.' *Annals of Statistics*, 33, (2005) 1890-1933.

[25] Sun, Yixiao and Peter C. B. Phillips. 'Understanding the Fisher Equation.' *Journal of Applied Econometrics*, 19, (2004) 869-886.

[26] Tsay, Wen-Jen. 'The long memory story of the real interest rate.' *Economics Letters*, 67, (2000) 325-330.

Table 1: Unit Root Tests Using $\ln(\frac{p_t}{p_{t-1}})^{12}$ Inflation Rate Calculation.

Statistics	Ex Post	AR(4)	Mishkin	Rolling Reg.	Switching	$I(0)/I(1)$
ADF test:	-3.216*	-1.787	-1.243	-2.301*	-1.408	2/3
Lagged terms	2	4	6	3	3	
PP test:	-5.390*	-2.319*	-2.204*	-3.445*	-1.777	4/1
Bandwidth	8	16	46	5	6	
DF-GLS test:	-2.027*	-1.515	-1.225	-1.444	-1.312	1/4
Lagged terms	14	7	7	10	12	
Ng-Perron test:	-1.636	-1.173	-1.099	-1.395	-1.638	0/5
Lagged terms	14	7	7	10	12	
KPSS test:	0.231	0.363	0.308	0.245	0.333	5/0
Bandwidth	14	14	14	14	14	

The sample period is 1975:12-2002:12. Switching stands for the Markov regime-switching model. The critical values of the unit root tests are reported in column 2-6. Column 7 shows the summary of the acceptance of the hypotheses $I(0)$ or $I(1)$. The lag length for the ADF test is based on SIC criterion. Bandwidths in the PP and KPSS unit root tests are determined by the Newey-West statistic using the Bartlett kernel. Lag lengths of the DF-GLS and Ng-Perron tests are based on the modified AIC criterion. * indicates rejection of the null hypothesis at a 5% significance level.

Table 2: Unit Root Test Using $\ln(\frac{p_t}{p_{t-12}})$ Inflation Rate Calculation.

Statistics	Ex Post	AR(4)	Mishkin	Rolling Reg.	Switching	$I(0)/I(1)$
ADF test:	-1.893	-1.924	-2.111*	-2.316*	-1.952*	3/2
Lagged terms	2	2	2	1	2	
PP test:	-1.586	-1.588	-1.718	-1.883	-1.703	0/5
Bandwidth	11	12	19	10	10	
DF-GLS test:	-0.770	-2.507*	-0.805	-2.021*	-2.155*	3/2
Lagged terms	12	16	12	13	16	
Ng-Perron test:	-0.953	-1.410	-0.877	-2.173*	-1.449	1/4
Lagged terms	12	16	12	13	16	
KPSS test:	0.272	0.279	0.288	0.200	0.261	5/0
Bandwidth	14	14	14	14	14	

The sample period is 1975:12-2002:12. Switching stands for the Markov regime-switching model. The critical values of the unit root tests are reported in columns 2-6. Column 7 shows the summary of the acceptance of the hypotheses $I(0)$ or $I(1)$. The lag length for the ADF test is chosen based on the SIC criterion. Bandwidths in the PP and KPSS unit root tests are determined by the Newey-West statistic using the Bartlett kernel. The lag length of the DF-GLS and Ng-Perron tests are chosen based on the modified AIC criterion. * indicates rejection of the null hypothesis at a 5% significance level.

Table 3: Estimation Results for Long Memory Using $\ln(\frac{p_t}{p_{t-1}})^{12}$ Inflation Rate Calculation.

Bandwidth	Ex Post	AR(4)	Mishkin	Rolling Reg.	Switching
$m = T^{0.60}$	0.767	0.766	0.790	0.861	0.757
	[0.938]	[0.937]	[0.961]	[1.031]	[0.928]
$m = T^{0.65}$	0.539	0.654	0.671	0.644	0.563
	[0.687]	[0.801]	[0.819]	[0.792]	[0.711]
$m = T^{0.70}$	0.538	0.709	0.742	0.677	0.566
	[0.666]	[0.837]	[0.869]	[0.800]	[0.694]
$m = T^{0.75}$	0.478	0.706	0.792	0.581	0.529
	[0.589]	[0.817]	[0.903]	[0.691]	[0.639]
$m = T^{0.80}$	0.480	0.728	0.842	0.594	0.569
	[0.575]	[0.824]	[0.938]	[0.689]	[0.665]
Mean Est. of d	0.561	0.713	0.768	0.671	0.597

The sample period is 1975:12-2002:12. Switching stands for the Markov regime-switching model. The estimate of d has been calculated using the exact Whittle estimator of Shimotsu and Phillips (2002). We allow the bandwidth, m, to vary from $m = T^{0.60}$ to $m = T^{0.80}$ with a step size of 0.05. "Mean est. of d" refers to the average estimate of d across these five selected bandwidth parameters. The number in brackets is the upper value in the 95% confidence interval for the estimate of d.

Table 4: Estimation Results for Long Memory Using $\ln(\frac{p_t}{p_{t-12}})$ Inflation Rate Calculation.

Bandwidth	Ex Post	AR(4)	Mishkin	Rolling Reg.	Switching
$m = T^{0.60}$	0.758	0.742	0.721	0.747	0.750
	[0.929]	[0.913]	[0.892]	[0.917]	[0.921]
$m = T^{0.65}$	0.718	0.698	0.651	0.711	0.704
	[0.866]	[0.845]	[0.798]	[0.858]	[0.851]
$m = T^{0.70}$	0.792	0.772	0.726	0.787	0.772
	[0.920]	[0.900]	[0.854]	[0.914]	[0.900]
$m = T^{0.75}$	0.777	0.754	0.742	0.765	0.739
	[0.887]	[0.864]	[0.852]	[0.875]	[0.849]
$m = T^{0.80}$	0.930	0.889	0.900	0.904	0.872
	[1.025]	[0.984]	[0.995]	[0.999]	[0.967]
Mean Est. of d	0.795	0.771	0.748	0.783	0.767

The sample period is 1975:12-2002:12. Switching stands for the Markov regime-switching model. The estimate of d has been calculated using the exact Whittle estimator of Shimotsu and Phillips (2002). We allow the bandwidth, m, to vary from $m = T^{0.60}$ to $m = T^{0.80}$ with a step size of 0.05. "Mean est. of d" refers to the average estimate of d across these five selected bandwidth parameters. The number in brackets is the upper value in the 95% confidence interval for the estimate of d.

Figure 1: Real Interest Rates Using $\ln(\frac{p_t}{p_{t-1}})^{12}$ Inflation Rate

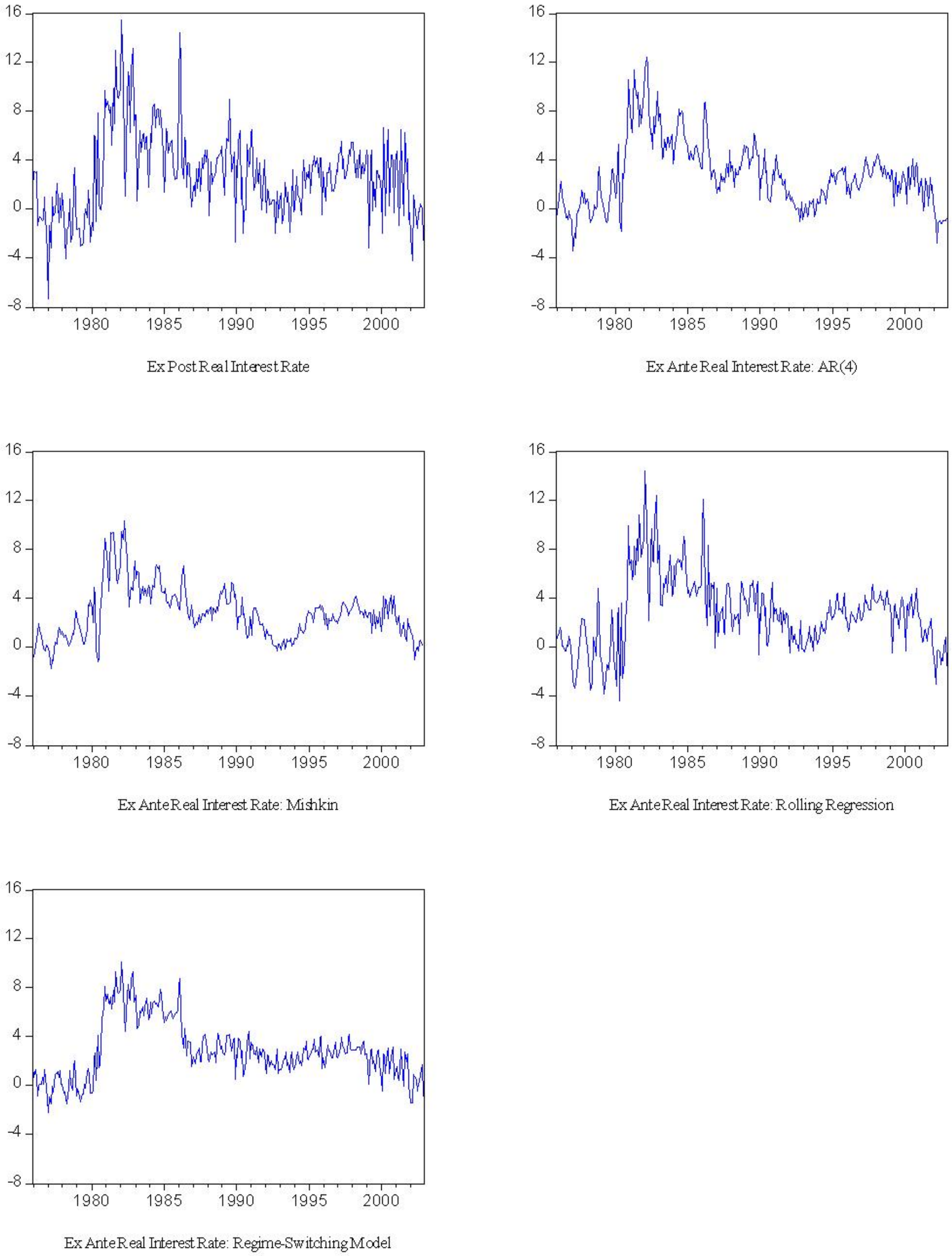

Figure 2: Real Interest Rates Using $\ln(\frac{p_t}{p_{t-12}})$ Inflation Rate