Math 5375, Fall 2012:
Constructing Whole Number and Operations

5–8 PM Tuesdays, Pickard Hall Room 305

Instructor: Dr. Theresa Jorgensen
Office: 434 Pickard Hall
Phone: (817)272-1321, fax 272-5802
email: jorgensen@uta.edu
WWW: http://www.uta.edu/faculty/tjorgens/

Prerequisites: Graduate standing and consent of instructor
Text materials: DMI's *Building a System of Tens* and *Making Meaning for Operations* casebooks (henceforth BST and MMO); *Children's Mathematics: Cognitively Guided Instruction* by Carpenter et al. (see bibliography for citations). Additional materials will be provided in class or on the course web page.
Last day for withdrawal: October 31
Class policy on drops, withdrawals, academic honesty, and accommodating disabilities follows the University policy on these matters. Copies can be obtained upon request.

LEARNING OUTCOMES: After completing this course, students should be able to

- identify and use (in teaching) common conceptual models of the four arithmetic operations;
- identify, analyze and explain (justifying in simple terms) both traditional and invented computational algorithms for the four arithmetic operations;
- identify and use appropriate representations of whole numbers and place value in teaching situations, including conceptual, contextual, concrete, pictorial, and symbolic models;
- design and implement research-based lessons to teach whole number concepts, including the development, selection and sequencing of problems, as well as appropriate assessment;
- anticipate and identify common approaches & errors in student learning of number and operation concepts.

FORMAT: This course will study number and operation concepts in several ways: through work on challenging mathematical problems to develop our own mathematical abilities (and communicating the results, to develop our expository abilities); discussing what research has discovered about the learning of number and operation concepts at the K–8 levels; and examining specific instances of K–8 students' computational work, through both case studies and our own classroom practice.

Before class each week you will read articles and/or case studies from K–8 mathematics, and make notes on them in preparation for class discussions. You will also often work on mathematics problems outside of class, to facilitate their discussion in class. We will typically begin class by working on new mathematics problems and discussing their solutions, in both small and large groups. We will follow this up by discussing the assigned readings, as well as other topics related to problem solving. We will typically end class with time for reflection on how the topics we have discussed apply to our own classrooms.

During class discussions we will often refer back to work we have done earlier in the course, so please bring your notes and papers from previous sessions to class.

POLICIES:

- Students who are not classroom teachers will need to make arrangements to interact with K–8 students for many of the assignments (those starred * on the calendar).
- Students are expected to be on time, prepared and ready to work every week. This class meets every Tuesday from 5 PM to 8 PM from August 28 to December 11. Each student is allowed

the equivalent of one week's absence (3 total hours) for whatever reason without penalty. All subsequent absences (including arriving significantly late) will result in the reduction of the final course grade by one-half letter grade (5%) for each absence.

• With the exception of examples of student work, written assignments are expected to be typed and use correct grammar and punctuation.

• Each student is allowed one late submission during the semester. The paper must be submitted before the beginning of the class period following that in which it was due. Papers not submitted by the end of class time on the due date are considered late. Submission of a late paper constitutes the student's agreement that this is the one allowed late assignment.

• Each student is allowed one electronic submission during the semester. Electronic submissions must be complete and not missing any ancillary materials such as student work necessary for grading. (If the electronic submission is made late, then it is both the only late paper allowed and the only electronic submission allowed.) This does not include drafts sent for consultation prior to submission, but consultation must take place in person or via telephone.

GRADES: Your grade for the course will be determined by five elements, each of which has equal weight: (1) journal entries and participation, (2) a written student interview, (3) a short case study, (4) a paper detailing your own mathematical work, and (5) a lesson involving a problem you select to develop whole number and operation concepts in your students. All of these are detailed in the next section.

Assignments

1. Journal

On days in which there is not a major assignment due, you will write a short (about one page) reflection in response to a prompt given below. Many of the prompts are also given in the DMI handouts distributed in class. Some involve "action research" reports in which you will write about your own students' whole number and operation work. You will often use and discuss your responses in class, within your small groups and in large group. These reflections are to be turned in at the end of class; I will respond to them in writing and return them at our next class meeting. Grading will be limited to verifying that responses are appropriate in topic and scope (length).

Your journal entries will serve to document your preparation for class each day (and your growth over time); your preparation and participation grade will be based half on your journal (entries should be complete each day before class) and half on your participation in class discussions (I expect participation in large-group discussion at least ten of the fifteen times we will meet).

J1 *Children's definitions for addition and subtraction.* Ask several students to define or explain what addition and subtraction are (use age-appropriate vocabulary, but be careful not to use terms like putting together and taking away, which already do most of the defining). Report their responses verbatim, and then compare them with the conceptual models for these two operations identified in the readings. Do their definitions cover all the conceptual models?

J2 *Mini-interview.* Interview a single student about a single question related to the meanings and structure of one of the four arithmetic operations or place value (at an appropriate level for the student). Give the student's response and analyze the understanding the student shows. The intent of this assignment is as a dry run for the student interview assignment, so see that portion of the syllabus for the general format, but keep in mind that this mini-interview should cover only a single question and thus be much smaller in scope.

J3 *Division and remainders.* (i) Write six different simple story problems for the division $32 \div 5$, each of which has a different one of the following expressions as answer: (a) 6 remainder 2,

(b) 6 $\frac{2}{5}$, (c) 6.4, (d) 6 or 7, (e) 6, (f) 7. (ii) Write a simple story problem or question that students could use to think about why dividing by zero is meaningless.

J4 *Mini-case study.* This will be a dry run for the case study. The specific prompt for this journal is TBA.

J5 *Defining place value.* First, ask several students to define or explain what we mean by place value. Report their answers, and then give your own definition. What differences do you see?

J6 *Choosing strategies I.* Explain in detail a student's work on a computational problem where the student used a different strategy than the traditional algorithm (preferably from your class, but if not you may use an example from one of the DMI case studies—be sure to explain the strategy anyway, in this case). Then consider the skills required and not required by this computational approach, and speculate on why the student chose that strategy instead of the more traditional approach.

J7 *Choosing strategies II.* Learners develop and choose computational strategies based on features of the problem being solved, as well as their own levels of understanding of various relevant concepts. Investigate whether or not students make these choices *consciously*, by revisiting some students' work with them, and asking them why they chose the approaches they used.

J8 *Article review.* Choose an article from one of the three NCTM practitioner journals which deals with student learning in the number and operation strand, and write a review of it including its implications for teaching. Give a full bibliographic citation. (Back issues of these journals are available at the UTA libraries; each journal also has one sample issue available for free review on www.nctm.org.)

J9 *Analyzing multiplication strategies.* Each of the following three computations uses a nontraditional multiplication algorithm to reach a correct answer. For each computation, answer the following questions:

(1) Is it mathematically sound?

(2) If so, how far can it be extended?

(3) Based upon the skills required and not required (relative to the traditional algorithm), what motivated the approach?

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(a)     2 4   (b)   7 2 5    (c)   1 2 9 0
      x 6 4         x   8          x   4 0 3
      2 5 6         6 6 0        3 6 2 7 0
  + 1 2 8      + 5 1 4 0     + 4 8 3 6 0
    1 5 3 6       5 8 0 0       5 1 9 8 7 0
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J10 *Synthesis.* Looking back on the readings and discussions of the course to date, write a paragraph on each of the following topics:

- the key issues in developing conceptual understanding and computational fluency in operating on whole numbers;
- the problem, discussion or issue that most changed your own mathematical understanding (not your students');
- the problem, discussion or issue that most changed the way you teach (or will teach) number and operation topics;
- the role of invented & traditional algorithms and drill in developing computational fluency.

2. Student interview

In order to develop (or strengthen) the habit of attending to student thinking in detail, you will conduct an interview with a student from your class to determine the extent of her/his understanding of a specific mathematical topic. You may choose the student and topic, but the interview should involve a topic from number and operation. Begin by obtaining all necessary permissions to conduct and record (audio or video) the interview; explain to all interested parties (including the student!) that you need the student's help for a class in which you are studying how students learn, and that this interview will not affect the student's grades; it will just give you a better understanding of how the student thinks. (Recording the interview will keep you from needing to make detailed notes during the interview.)

Before the interview, get a copy of recent written work by the student that shows her/his ability to reason and problem-solve (the work need not be error-free, but the student should have made enough progress that the two of you can discuss the problem). Make sure the student is familiar with the paper, and begin the interview by asking the student to explain her/his work, including what difficulties s/he encountered.

Continue the interview by asking further questions about the mathematical topic involved (I have a separate handout on interviewing tips on the course web site). Remember that *in order to determine the limits of a student's knowledge, you must continue until you reach a question which the student either cannot answer or answers incorrectly for reasons other than a simple careless error*. You should be able to do this without making the student feel badly.

After the interview, use your recording to make a more detailed analysis of the student's thinking, with regard to both problem-solving abilities and knowledge of the particular mathematical topic. Give an overall narration of the interview (e.g., say what specific tasks or problems you asked the student to work on). Use specific details or quotes to support your analysis. Conclude your write-up with an explicit summary of what the student knows, what the student does not know, and what the student is ready (or needs) to work on next (see interview tips handout for more).

3. Case study

During the course we will read and discuss in class several case studies, all describing events in other teachers' classrooms. For this assignment, you are to write a short (roughly 3–5 pages) case study describing a mathematical discussion involving one or more students, similar to these cases. A case is neither a complete transcript of a lesson nor as prefabricated as an interview, although it is very helpful to include direct quotes and dialogue from students.

You must base your case on a conversation for which you were present, and preferably in which you were involved, but it could come out of a lesson you observed, or a conversation among two or more students. You may choose to narrow in on one or two students, or on one small group, or you may describe a whole-class conversation. The most important thing is that the episode illustrate some aspect of children's mathematical thinking.

In writing your case study, begin by describing briefly the class's larger context (including grade level) and the mathematical topic; then describe the relevant parts of the conversation in as much detail as you can manage. Include what you are thinking as you work with the students. Finish up by summarizing your evaluation of the students involved and saying what issues and questions you still have after this conversation. Include an analysis of the students' thinking, and questions the case raises for you.

Your case study must touch on a mathematical topic involving number and operation. We will discuss the writing of cases in more detail before they are due, but of course you are encouraged to begin sooner, especially if you have a good conversation fresh in your mind. I will be glad to work with you one-on-one in helping you write your case.

4. 2-problem paper

In order to understand the concepts underlying number and operation (including teaching it), you must gain experience in explaining its applications. As a summative evaluation of the mathematical portion of this course, you will submit a paper detailing your mathematical work on a *college-level* problem from this course which you solved completely, and a problem from K–8 mathematics which you believe is related. (Please check with me prior to submission to verify that the problem you have selected is appropriate and not from K–8 mathematics!)

For the college-level problem, give a thorough explanation of the original problem (paraphrased), its context, the strategies you used to approach it, what the solution is (and why! that's the tricky part), and what the solution means in context. Distinguish carefully between conjectures and rigorous arguments. Feel free to use drawings, graphs, diagrams, tables, etc. if necessary.

Also select a problem from K–8 mathematics (possibly, and preferably, from your own classroom) which you believe entails number and operation concepts similar to those involved in the college-level problem, and explain the mathematics involved in this problem, clarifying what common ideas the two problems share.

I encourage you to show me a draft of your paper before final submission.

5. Lesson on number and operation

In this course we will study the teaching and learning of ideas related to number and operation in K–8 mathematics. As a summative evaluation of the pedagogical aspects of this course, you will develop or select a lesson which fosters the learning of these concepts, teach and document the lesson, and give a short (10-minute) presentation to the class on how it went. The parts of this assignment are as follows:

1. Select or develop a problem that is intended for use with the students you teach, which involves some aspect of number and operation. You may use or adapt a problem from class materials, but be sure it is appropriate for the target audience. (Say where you got it from, and, if you have used it before, in what capacity, and what you learned from it.) The best lessons tend either to integrate multiple strands of mathematics to illustrate connections, or to address significant conceptual issues within a single strand as a summative activity following multiple experiences in developing and exploring a concept.
2. Write a paragraph explaining what number and operation concepts are entailed in this problem. (You may use deconstruction if it helps.)
3. Add to the above written descriptions a short sketch of how you plan to use the problem in a lesson, and meet with your instructor to discuss your progress. (This is the lesson draft checkpoint. The above items will also form part of your final paper.)
4. Write a lesson plan that uses the modified problem as a significant problem-solving opportunity with your students.
5. Teach the lesson to your students (see me if this is problematic).
6. Write a one-page reflection on how the lesson went, including what strategies students used to approach the problem, what ideas were raised in its discussion, and to what extent your students' understanding of the underlying number and operation concepts—or ability to apply them—changed as a result of the lesson. Be specific.
7. Make a one-page handout (you may use front and back if necessary, but it *must* fit on one sheet) summarizing your lesson for the class. Include modified problem, grade level, mathematical topics addressed, and anything your colleagues would need to know in order to use the lesson, including (briefly) any difficulties the students tended to encounter. The handout should *not* be the same as your lesson plan, and must be turned in at Session 15.
8. Give a brief (10-minute) presentation to the class on this lesson, using the handout, at our last class meeting.

You are encouraged to discuss this project with me as often as you like, throughout the semester. A preliminary draft of the modified problem and lesson idea (not [necessarily] yet taught) is due at Session 11 (see step 3 above). Final documentation is due at Session 15 , including a handout, with the presentations to be given at Session 16.

Bibliography

Ambrose, R., Baek, J., and Carpenter, T. (2003). Children's invention of multidigit multiplication and division algorithms, in A. J. Baroody and A. Dowker (Eds.), *The development of arithmetic concepts and skills: constructing adaptive expertise.* Lawrence Erlbaum Associates, Mahwah, N.J. pp. 305–336.

James K. Bidwell. (1991). Susan's personal algorithm. *Arithmetic Teacher* 39(3): 1.

Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L., Empson, S.B. (1999). *Children's Mathematics: Cognitively Guided Instruction.* Heinemann/NCTM, Portsmouth, NH.

Carpenter, T. P., Franke, M. L., Jacobs, V., & Fennema, E. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education* 29: 3–20.

Guershon Harel and Merlyn Behr. (1991). Ed's strategy for solving division problems. *Arithmetic Teacher* 39(3): 38-40.

Deborah Schifter, Virginia Bastable, and Susan Jo Russell, *Building a System of Tens (Number and Operation, Part 1) Casebook.* Parsippany, NJ: Dale Seymour/Pearson, 1999.

Deborah Schifter, Virginia Bastable, and Susan Jo Russell, *Making Meaning for Operations (Number and Operation, Part 2) Casebook.* Parsippany, NJ: Dale Seymour/Pearson, 1999.

Calendar

A tentative schedule with topics is given below (subject to updating).

Sess.	Date	Topic	Readings/Cases Due	Assignments Due
1	8/28	Early concepts of operations (MMO1)	MMO1, CGI1	—
2	9/04	Models for addition and subtraction (MMO2)	MMO2, CGI2,3	J1* defn. add/sub
3	9/11	Models for multiplication (MMO3)	MMO3, CGI4	J2* mini-interview
4	9/18	Models for division	CGI5	J3 divisions
5	9/25	Numeration systems	BST Case 14, [num read]	Interview*
6	10/02	Place value and Mayan numeration (BST3)	BST3	J4* mini-case
7	10/09	Bases in place value	—	J5* defn. place value
8	10/16	Algorithms for multidigit add. and sub. I (BST1)	BST1	Case study*
9	10/23	Algorithms for multidigit add. and sub. II (BST2)	BST2	J6 choosing I
10	10/30	Grouping and regrouping in add. and sub.	CGI6	Lesson draft, J7*
11	11/06	Algorithms for mult. of multidigit numbers	Ambrose et al.	J8 article review
12	11/13	Partial factors in multiplication (BST5)	BST5, CGI7	J9 mult. algs.
13	11/20	Algorithms for div. of multidigit numbers (BST6)	BST6	J10 synthesis
14	11/27	Division by multidigit numbers	CGI8, [alt div]	2-problem paper
15	12/04	Invented and traditional algorithms in learning	BST8, MMO8(half), CGI Appendix	Lesson paper*
16	12/11	Final presentations	—	Give presentations

{BST/MMO/CGI}n means Chapter n of the given book.
See bibliography for further details of readings (nonelectronic readings are available at the UTA Libraries).