

Due March 25 to be turned in:

1. **Final Draft of Assignment 6:** Area of a triangle on a sphere
2. **Reflection:** In what ways does what we have studied thus far in Math 5341 have value and usefulness to you as a teacher (or potential teacher) of mathematics?

Today we will find the formula for the area of a triangle on a hyperbolic plane of radius ρ . (See pp 91-95 in text. Problem 7.2.)

- a. Show that all $2/3$ -ideal triangles with the same angle θ are congruent.
- b. Show that the area function A for a $2/3$ -ideal triangle is additive, i.e. $A(\alpha + \beta) = A(\alpha) + A(\beta)$, where α, β are exterior angles.
- c. We will use the fact that a *continuous* additive function must have the form $A(\alpha) = \text{constant} \cdot \alpha$.
- d. Now let the finite vertex of a $2/3$ -ideal triangle go to infinity. The interior angle will go to zero and the exterior angle will go to π . So $A(\pi)$ is the area of an ideal triangle.
- e. So we now know that all ideal triangles on the same hyperbolic plane have the same area. We will call it I .
- f. Put e. together with c. to get that $A(\alpha) = (I/\pi) \cdot \alpha$.
- g. (From chapter 17, we will use the fact that $I = \pi\rho^2$.)
- h. Now look at figure 7.3 on page 92, and put the pieces together to figure out a formula for the area of a hyperbolic triangle. Note that the interior angles of the triangle are the exterior angles of the $2/3$ -ideal triangles.

What does this say about the sum of the interior angles of a hyperbolic triangle?