Due February 26: Draft 1.5 of Assignment 4/Assignment 5 (This will count for two assignments, but you will turn in one monster write-up.) You will address:

a) **What curves are straight on a cylinder or cone?**
b) **Can geodesics intersect themselves on cylinders or cones? If so, how many times can they intersect? (Optional: At what angle do they intersect?)**
c) **Is there always a geodesic joining two points on cylinders or cones? Can there be more than one geodesic joining two points? How many are there?**
d) **Are right angles always equal on cylinders or cones?**

Let's use what we know about geodesics on the plane, sphere, cylinder, and cones to examine some questions about parallel lines.

<u>Euclid's Postulates</u>

1. A unique straight line can be drawn from any point to any point
2. A finite straight line can be extended continuously in a straight line.
3. A circle may be described with any point as center and any distance as radius.
4. All right angles are equal to one another.
5. If a transversal falls on two lines in such a way that the interior angle on one side of the transversal are less than two right angles, then the lines meet on that side on which the angles are less than two right angles.

Playfair's axiom: Through a point not on a given line, exactly one line can be drawn in the plane parallel to the given line. (The word *parallel* as used here means not intersecting or having no Euclidean point in common.)

1. Does Euclid's 5^{th} postulate hold for these spaces? If it fails, how does it fail?
2. Does Playfair's Axiom hold for these spaces? If it fails, how does it fail?

<u>*Characteristic Postulate of Hyperbolic Geometry:*</u> Through a given point C, not on a given line AB, passes **more than one** line in the plane not intersecting the given line.

Examine what is straight in a hyperbolic plane (see page 66 of text). Some properties you should convince yourself of using your model:

- Every pair of points is joined by a unique geodesic with reflection-in-itself symmetry
- Two geodesics intersect no more than once
- Two non-intersecting geodesics are either asymptotic or diverge from each other in *both* directions
- As two radial geodesics cross an annulus, their distance apart will increase [or decrease] in the ratio $\frac{(\rho+\delta)}{\rho}$ [or $\frac{\rho}{(\rho+\delta)}$] and thus they will converge to [or diverge from] each other at an exponential rate.
- As a non-radial geodesic goes to infinity (in either direction) it becomes more and more nearly perpendicular to the annuli.