Before we begin investigating the problem below, let's create a **cylinder**, **90-degree cone**, a **180-degree cone**, and a **450-degree cone**.

- Cone angle: generally defined as the angle measured at the point of the cone on the surface
- One definition of cone: Take any simple (non-intersecting) closed curve **a** on a sphere and consider a point *P* in the center of the sphere. A cone is the union of the rays that start at *P* and go through each point on **a**. The cone angle is then equal to $\frac{\text{arclength of } a}{\text{radius of sphere}}$ in radians.

We eventually want to understand why the following is true.

Theorem: If a surface is smooth then a straight line on the surface is always the shortest path between "nearby" points. If the surface is smooth and complete (every geodesic on it can be extended indefinitely) then the shortest path between any two points is always straight.

Assignment 4 – First version due February 12

Problem 4.1. Straightness on Cylinders and Cones[1]

a. What curves are straight with respect to the surface of a cylinder or a cone? Why? Why not? [It is essential that you create models for your investigations – think intrinsically, not extrinsically. Be the bug on the surface.]
b. Examine:
    - Can geodesics (intrinsically straight curves) intersect themselves on cylinders and cones?
    - Can there be more than one geodesic joining two points on cylinders and cones?
    - What happens on cones with varying cone angles, including cone angles greater than 360 degrees?
    - Look up Euclid's definition of a right angle, and Euclid's 4th Postulate. On cylinders and cones, are right angles always equal?

Additional Questions:

i. What happens to triangles and lines on a 450° cone?
ii. Is the shortest path straight on a 450° cone?
iii. On a 450° cone, does every pair of points determine a straight line?

Dates to keep in mind:

Final version of Assignment 2 due February 12
Quiz 1 will be February 19
We will need your model of the hyperbolic plane on February 19 (in two weeks)
Assignment 3 will not be handed in

[1] Page 44 in your textbook.