A Scalable Multidimensional Uncertainty Evaluation Approach to Strategic Air Traffic Flow Management

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Convective weather events cause capacity reduction in the National Airspace System (NAS), and lead to traffic congestion. To mitigate congestion, strategic air traffic flow management plans traffic flows at a long look-ahead time (2-15 hour). Planning at this timeframe is challenging, due to the wide possibility of weather events and requirement for real-time management. To conquer these challenges, we need an approach to quickly assess the impact of predicted weather events on the performance of air traffic system. In this paper, we use a scalable multidimensional uncertainty evaluation approach, called M-PCM-OFFD, to address this problem. Simulation studies show the effectiveness of this approach for the performance evaluation of air traffic system. In addition, we investigate further capability of M-PCM-OFFD through exploring higher-level OFFDs. Finally, we introduce an uncertainty-exploiting framework to enable real-time strategic air traffic management under weather uncertainty.

Nomenclature

\[\begin{align*}
l_i[k] &= \text{number of aircraft entering region } i \text{ at time } k \\
N_i[k] &= \text{maximum number of aircraft allowed to pass region } i \text{ at time } k \\
c_i[k] &= \text{number of aircraft passing region } i \text{ at time } k \\
B_i[k] &= \text{number of aircraft delayed in region } i \text{ at time } k \\
b_i[k] &= \text{number of aircraft in region } i \text{ at time } k \\
m &= \text{number of uncertain input parameters} \\
x_i &= i-\text{th uncertain input parameter} \\
y &= \text{system output} \\
f_{X_i}(x_i) &= \text{probability density function of } x_i \\
g(x_1, \ldots, x_m) &= \text{original system mapping} \\
g^*(x_1, \ldots, x_m) &= \text{low-order system mapping constructed using the M-PCM-OFFD approach} \\
\Psi_{a_1, \ldots, a_m} &= \text{coefficient of the term } \prod_{i=1}^{m} x_i^{a_i} \text{ in } g(x_1, \ldots, x_m) \\
\Omega_{a_1, \ldots, a_m} &= \text{coefficient of the term } \prod_{i=1}^{m} x_i^{a_i} \text{ in } g^*(x_1, \ldots, x_m) \\
\tau &= \text{maximal number of parameters in cross-terms of } g(x_1, \ldots, x_m) \\
H_i^{a_i} &= \text{orthogonal polynomial of degree } a_i \text{ for } x_i \\
h_i^{a_i} &= \text{orthonormal polynomial of degree } a_i \text{ for } x_i \\
x_i(f) &= j-\text{th M-PCM point for } x_i \\
x_i^{a_i}(x_i(j)) &= x_i^{a_i} \text{ evaluated at point } x_i(j) \\
l &= \text{number of coefficients in } g^*(x_1, \ldots, x_m) \\
l_{\text{OFFD}} &= \text{number of simulation points selected by OFFD} \\
\gamma &= \text{fraction constant of OFFD} \\
R &= \text{resolution of OFFD} \\
L &= \text{matrix constructed by full set of M-PCM points} \\
L' &= \text{matrix constructed by a subset of M-PCM points} \\
D(\cdot) &= \text{full-column-rank margin of the matrix in parenthesis} \\
e &= \text{perturbation matrix}
\end{align*}\]

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TRATEGIC air traffic flow management (ATFM) plans traffic flows 2-15 hours in advance to resolve capacity-demand imbalances. Managing flows at this time-frame is challenging mainly due to the existence of uncertain weather events. In particular, a wide range of possible weather events modulates the dynamics of traffic systems and complicates the performance prediction of the National Airspace System (NAS) and in turn the design of management initiatives. To conquer this challenge, an effective and scalable uncertainty evaluation method, which is capable of quickly predicting traffic system performance statistics under a large number of uncertain weather parameters, is essential, as a step toward the design of mitigation strategies robust to weather uncertainties. In this paper, we study the feasibility of using a scalable multidimensional uncertainty evaluation approach that we have developed, to address the strategic ATFM problem.

Viewing the dynamic traffic system as a black box, the problem of predicting traffic system performance under weather uncertainties can be formulated as the prediction of system output statistics, with weather uncertainties modeled as input parameters of the system (see Figure 1). In particular, \( E(y) \), the output statistics to predict, captures the mean total delay of aircraft over a time span. The uncertain input parameters \( \{x_1, x_2, ..., x_m\} \) capture properties of convective weather events at geographical distributed regions.

![Figure 1. Black-box view of the uncertainty evaluation problem.](image)

The most widely used method to predict the mean output statistics is the Monte Carlo method. However, Monte Carlo requires a very large number of simulations to converge to meaningful performance estimates, and is thus not suitable for real-time evaluation and decision-making. To reduce the number of simulations to meet the real-time evaluation requirement, our group recently developed an effective and scalable uncertainty evaluation method, called M-PCM-OFFD which integrates the multivariate probabilistic collocation method (M-PCM) and the orthogonal fractional factorial design (OFFD). The method enables correct prediction of system mean output using only a limited number of simulations. In particular, the method smartly selects simulation points based on the statistics of input parameters using M-PCM and then utilizes the procedures of OFFD to further reduce the number of simulations to break the curse of dimensionality with respect to the number of uncertain parameters. The selected simulation points are used to approximate the original system mapping with a low-order mapping, which is proved to have the same mean output as the original mapping. With these properties, the integrated M-PCM-OFFD is able to reduce the number of simulations from \( 2^{2m} \) to \( [2^{2m/m+1}, 2^{m-1}] \) for a system of \( m \) uncertain input parameters. In addition, we also explored robustness of this method to numerical errors and proved that it is the most robust among designs of the same size, making it of practical use.

In this paper, we study the potential of using M-PCM-OFFD for strategic ATFM. In Section II, we review a stochastic modeling framework for the air traffic system, and the fundamentals of M-PCM-OFFD. In Section III, we investigate the practical use of M-PCM-OFFD for ATFM using a simulation study. In Section IV, we discuss further improvement of M-PCM-OFFD by exploring higher-level OFFDs. We also describe an uncertainty-exploiting framework for real-time strategic air traffic management. Finally, we conclude our work in Section V.

II. Preliminaries

In this section, we first review a stochastic modeling framework that captures the dynamics of NAS under weather uncertainties. This model serves as the evaluation foundation for ATFM. We then describe the M-PCM-OFFD method, the design procedures and its properties. This approach can accurately estimate the output statistics of a large-scale system using very limited number of simulations.

A. Modeling of Flow Dynamics Under Weather Uncertainties

In our previous study, we developed an advanced queuing network model for the NAS, where various management actions, including ground-delay programs (GDP), Time-based metering (TBM), rerouting,
MINIT/MIT, and airspace flow programs (AFPs), were captured as restrictions on traffic flows. As air traffic flow dynamics are also modulated by weather uncertainties, we further built an integrated modeling framework to capture the impact of weather uncertainties on the air traffic flows. An analytical approach\(^5\) was then developed to quantitatively analyze weather impact.

Now let us briefly describe this integrated modeling framework\(^5\), which will be used in the simulation study in this paper to investigate the performance of M-PCM-OFFD in ATFM. This modeling framework consists of a demand model, a weather model, and a flow network model. The demand model captures the number of aircraft entering a region \(i\) at time \(k\), denoted as \(l_{i[k]}\). We use a time-dependent Poisson process to capture traffic demands. The weather model captures the impact of weather events\(^8\) on air traffic flows. As weather intensities directly impact the number of aircraft allowed to pass a region, it is natural to use capacity reduction in a region to capture the weather impact on traffic flows. We use \(N_{i[k]}\) to denote capacity, the maximum number of aircraft allowed to pass region \(i\). It is determined by both weather intensity and traffic management restriction at the region. Third, the flow network model captures the traffic flow dynamics, by modeling each region as a queue. Suppose there are \(l_{i[k]}\) number of aircraft entering a queue (or a region) \(i\) at time \(k\). Due to traffic management actions or convective weather (captured by \(N_{i[k]}\)), up to \(N_{i[k]}\) aircraft are able to pass the region, with the remaining aircraft being delayed. We use \(c_{i[k]}\) to capture the number of aircraft passing region \(i\) at time \(k\), \(B_i[k]\) to capture the number of aircraft delayed in the queue, and \(b_{i[k]}\) to represent the length of the queue. Note that backlog \(B_i[k]\) can be used to evaluate the severity of traffic congestion. The integrated modeling framework is expressed by following equations\(^2\).

\[
\begin{align*}
  c_{i[k]} &= \min(b_{i[k] - 1}, N_{i[k]}) \\
  b_{i[k]} &= b_{i[k] - 1} + l_{i[k]} - c_{i[k]} \\
  B_i[k] &= b_{i[k] - 1} - c_{i[k]}
\end{align*}
\]

B. M-PCM-OFFD

The M-PCM-OFFD is an effective multidimensional uncertainty evaluation method, which uses a very limited number of simulation points to estimate the output statistics of a system. It can reduce up to \(2^{2m} - 2^{2\log_2(m+1)}\) number of simulations for a system with \(m\) uncertain input parameters. This method leverages the capability of M-PCM to correctly predict the mean output, and utilizes the OFFDs to make uncertainty evaluation scalable with the number of input parameters. Now let us briefly describe the motivation, key idea, design procedures and properties of M-PCM-OFFD. For more detailed discussions, please refer to our previous papers\(^3\)\(^1\)\(^1\).

Suppose we have a system with \(m\) uncertain input parameters, \(x_i, i = \{1, 2, ..., m\}\). Each parameter \(x_i\) has a degree of up to 3 and follows an independent probabilistic density function, \(f_{x_i}(x_i)\). The M-PCM approach assumes that all combinations of the \(m\) parameters exist in the original system mapping, leading to \(2^m\) terms/coefficients. Therefore, estimating this system mapping requires at least \(2^m\) simulations. To reduce the number of simulations, M-PCM smartly selects simulation points and approximates the original mapping using a low-order mapping, which requires only \(2^m\) simulation runs. However, with the increase of \(m\), the number of simulations required by M-PCM increases exponentially.

To further reduce the number of simulations, we developed the M-PCM-OFFD method for a scalable uncertainty evaluation\(^2\)\(^1\)\(^1\). This approach is motivated by the fact that high-order interactions among parameters usually have very small impacts on system performance in many realistic applications. Therefore, we are able to eliminate high-order crossterms to further reduce the number of coefficients to estimate.

Provided that cross-terms with more than \(\tau\) parameters have no impacts on the system performance, the original system mapping has a form of

\[
g(x_1, ..., x_m) = \sum_{a_1=0}^{3} \sum_{a_2=0}^{3} \cdots \sum_{a_m=0}^{3} \Psi_{a_1,\ldots,a_m} \prod_{i=1}^{m} x_i^{a_i}
\]

where \(\Psi_{a_1,\ldots,a_m} \in R\) are the coefficients, and \(\Psi_{a_1,\ldots,a_m} = 0\) if more than \(\tau\) number of \(a_i\) are non-zero, \(i = \{1,2,\ldots,m\}\). To approximate this system mapping, we select a subset of M-PCM points using OFFDs. After that, we run simulations at the selected M-PCM-OFFD points to obtain corresponding outputs. Finally, the simulation points and the evaluated outputs are used to construct a low-order mapping of the following form

\[
g^*(x_1, ..., x_m) = \sum_{a_1=0}^{1} \sum_{a_2=0}^{1} \cdots \sum_{a_m=0}^{1} \Omega_{a_1,\ldots,a_m} \prod_{i=1}^{m} x_i^{a_i}
\]

where \(\Omega_{a_1,\ldots,a_m} \in R\) are calculated coefficients, and \(\Omega_{a_1,\ldots,a_m} = 0\) if \(\sum_{i=1}^{m} a_i > \tau, a_i = \{0, 1\}\). The output mean can then be calculated. The detailed design procedures are shown in Table 1. An illustrative example is shown in Figure 2. See papers\(^3\)\(^1\)\(^1\) for the complete study.

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Table 1. Algorithm for the M-PCM-OFFD\(^{3,11}\)

### Step 1: Use M-PCM to select \(2^m\) simulation points.

1.1 For each uncertain input parameter \(x_i, i = \{1, 2, \ldots, m\}\)

1.2 Initialize \(H_i^{-1}(x_i) = h_i^{-1}(x_i) = 0,\) and \(H_i^0(x_i) = h_i^0(x_i) = 0.\)

1.3 For \(a_i = 1\) to \(2\)

1.4 \[H_i^{a_i}(x_i) = x_i h_i^{a_i - 1}(x_i) - h_i^{a_i - 1}(x_i)h_i^{a_i - 1}(x_i) - H_i^{a_i - 1}(x_i)h_i^{a_i - 1}(x_i) \cdot \frac{1}{2} H_i^{a_i - 2}(x_i)\]

1.5 \[h_i^{a_i}(x_i) = H_i^{a_i}(x_i) / (H_i^{a_i}(x_i), H_i^0(x_i))^T\]

1.6 End

1.7 Find the roots of \(H_i^2(x_i) = 0, x_i(1)\) and \(x_i(2),\) as the \(2\) M-PCM simulation points for \(x_i.\)

1.8 End

(Note that \(H_i^{a_i}(x_i)\) represents the orthogonal polynomial of degree \(a_i\) for \(x_i.\) \(h_i^{a_i}(x_i)\) is the orthonormal polynomial, obtained by normalizing \(H_i^{a_i}(x_i), (p(x_i), q(x_i)) = \int p(x_i)q(x_i)f(x_i)dx_i.\)

### Step 2: Determine the number of simulations OFFD requires.

2.1 Compute the minimum number of simulations required to uniquely determine all coefficients \(\Psi_{a_1, \ldots, a_m},\)

i.e., \(\ell = \sum_{i=0}^{\ell} \binom{m}{i}.\)

2.2 Determine the minimum number of simulations the 2-level OFFDs require, which is \(2^{\text{log}_2 \ell}.\) (Note that the number of simulations selected by 2-level OFFDs is a power of 2.)

2.3 If \(2^{\text{log}_2 \ell} = 2^m\)

2.4 Go to Step 4, as no further reduction is possible.

2.5 End

2.6 If \(2^m > \ell\) OFFDs exist, where \(0 < \gamma \leq m - \text{log}_2 \ell\) and resolution \(R \geq 2^\gamma + 1\)

2.7 Find \(2^m - \gamma\) OFFD with the largest \(\gamma,\) which can maximally reduce the number of simulations to \(l_{OFFD} = 2^m - \gamma\).

2.8 Go to Step 3 (i.e., we can further reduce the number of simulation points).

2.9 Else

2.10 Go to Step 4.

2.11 End

### Step 3: Use OFFD to select a subset of M-PCM simulation points.

3.1 Generate \(2^m - \gamma\) full factorial design, by listing all \(2^m - \gamma\) combinations for \(m - \gamma\) parameters. (Note that the OFFD picks \(2^m - \gamma\) simulations from the full set of M-PCM points.)

3.2 Specify \(\gamma\) generators that lead to the resolution \(R.\)

3.3 Use obtained generators to determine the values of the other \(\gamma\) parameters.

### Step 4: Run simulations at selected M-PCM-OFFD points.

4.1 For each simulation point, run simulation and find the associated output.

### Step 5: Construct the low-order mapping \(g'(x_1, \ldots, x_m).\)

5.1 If \(l_{OFFD} = l\) \(l_{OFFD} = 2^m - \gamma\) if further reduction is possible, or \(l_{OFFD} = 2^m\) otherwise.

5.2 Apply \(H\)

\[
\begin{bmatrix}
\Omega_{0,0} \\
\vdots \\
\Omega_{0,1} \\
\Omega_{1,1}
\end{bmatrix} = L'^{-1} \begin{bmatrix}
g(x_{1(1)}, \ldots, x_{m(1)}) \\
g(x_{1(1)}, \ldots, x_{m(2)}) \\
\vdots \\
g(x_{1(2)}, \ldots, x_{m(2)})
\end{bmatrix}
\]

5.3 Else

5.4 Apply \(H\)

\[
\begin{bmatrix}
\Omega_{0,0} \\
\vdots \\
\Omega_{0,1} \\
\Omega_{1,1}
\end{bmatrix} = (L'^T L')^{-1} L'^T \begin{bmatrix}
g(x_{1(1)}, \ldots, x_{m(1)}) \\
g(x_{1(1)}, \ldots, x_{m(2)}) \\
\vdots \\
g(x_{1(2)}, \ldots, x_{m(2)})
\end{bmatrix}
\]

where \(L' = PLH^T.\)
\[ L \in \mathbb{R}^{2^n \times 2^m} = \begin{bmatrix} x_1^0(x_1(1)) \cdots x_1^0(x_1(m_1)) & \cdots & x_1^0(x_1(1)) \cdots x_1^0(x_1(m_1)) & \cdots & x_1^0(x_1(1)) \cdots x_1^0(x_1(m_1)) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_1^1(x_1(1)) \cdots x_1^1(x_1(m_2)) & \cdots & x_1^1(x_1(1)) \cdots x_1^1(x_1(m_2)) & \cdots & x_1^1(x_1(1)) \cdots x_1^1(x_1(m_2)) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_1^j(x_1(2)) \cdots x_1^j(x_1(m_2)) & \cdots & x_1^j(x_1(2)) \cdots x_1^j(x_1(m_2)) & \cdots & x_1^j(x_1(2)) \cdots x_1^j(x_1(m_2)) \end{bmatrix} x_1^{a_i}(x_1(j)) \]

is the \( a_i \)-th power of \( x_1 \) evaluated at point \( x_1(j) \). \( H \in \mathbb{R}^{k \times 2^m} \) and \( F \in \mathbb{R}^{k \text{offd} \times 2^m} \) are transformation matrices which aim to eliminate coefficients \( \sum_{i=1}^{m} a_i > \tau \), and pick out selected simulation points respectively. \( L' \) is a sub-matrix of \( L \), which only contains the rows corresponding to the selected points and columns corresponding to the cross-terms with no more than \( \tau \) parameters.

5.5 End

5.6 Construct the reduced-order mapping:

\[ g^*(x_1, \ldots, x_m) = \sum_{a_1=0}^{1} \sum_{a_2=0}^{1} \cdots \sum_{a_m=0}^{1} \Omega_{a_1 \ldots a_m} \prod_{i=1}^{m} x_1^{a_i} \]

Step 6: Compute the mean output \( E[g(x_1, \ldots, x_m)] \).

\[ E[g(x_1, \ldots, x_m)] = \int \cdots \int g^*(x_1, \ldots, x_m) f_{x_1}(x_1) \cdots f_{x_m}(x_m) \, dx_1 \cdots dx_m. \]

Figure 2: An example to illustrate the procedure of M-PCM-OFFD for effective uncertainty evaluation. The knowledge of 3 (\( m = 3 \)) uncertain input parameter distributions (a) lead to the selection of 8 simulation points using M-PCM. (b) The OFFD further selects a subset of 4 simulation points (marked in red in (b)), which produces a reduced-order mapping that predicts the correct mean (c). The number of reduced simulations is shown in (d).

The M-PCM-OFFD has several appealing properties. Two major properties are: 1) it can predict the mean output precisely, i.e., \( E[g(x_1, \ldots, x_m)] = E[g^*(x_1, \ldots, x_m)] \); 2) the subset of M-PCM points selected by OFFD is the most robust to errors compared to other possible selections, i.e., \( \max(D(L)) = D(L'_{\text{offd}}) \), where \( L'_{\text{offd}} \) refers to the matrix constructed by the subset of M-PCM points selected using OFFD, and \( L' \) generally represents a matrix constructed by a subset of M-PCM points of the same size of \( L'_{\text{offd}} \). \( D(L) = \min(\|e\|_F \mid \text{rank}(L' + e) < l) \) is the full-column-rank margin of \( L' \), which quantifies how close matrix \( L' \) is to rank loss, and hence evaluates the robustness of matrix \( L' \) to computational approximation errors. In this equation, \( e \in \mathbb{R}^{k \text{offd} \times l} \) is a perturbation matrix, and \( \|\|_F \) is the Frobenius norm. More detailed discussions about these properties and their proofs are given in our previous paper.

### III. Using M-PCM-OFFD for Strategic ATFM

In this section, we implement the effective multi-dimensional M-PCM-OFFD method to evaluate the weather impact on air traffic system, so as to facilitate the design of strategic ATFM. The procedures are illustrated using a simulation study. Consider that air traffic flows enter 8 regions one by one (see Figure 3 for an illustration), where each region is subject to occasional weather events. For simplicity, we assume that the weather events have only two states: present or not, and the intensities of weather events when present are constants and the same for all regions. Therefore, we can describe the uncertain weather events in each region \( i, i \in \{1, 2, \ldots, 8\} \), using two variables: start time \( S_i \) and duration \( D_i \). As such this system has 16 uncertain input parameters. To evaluate weather impact on the air traffic system, we evaluate the total traffic delay (captured by the backlog \( B_i[k] \)) over a time span \([0, k_p]\), i.e.,

\[ T_B = \sum_{i=1}^{8} \sum_{k=1}^{k_p} B_i[k]. \]

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To obtain system mapping and mean total traffic delay, we need at least $2^{32}$ simulations to calculate all coefficients in the original mapping with all combinations of parameters, each up to a degree of 3. $2^{16}$ simulations are needed to construct the low-order mapping using M-PCM. If a simulation run costs 1 second, then running $2^{32}$ simulations requires about 11.9 hours, and running $2^{16}$ simulations requires about 18.20 hours, which are also very expensive. The M-PCM-OFFD solves this problem. In particular, assume that interaction effects of more than two parameters are negligible and thus cross-terms in the system mapping containing more than 2 input parameters can be eliminated, i.e., $\tau = 2$. Under this condition, the $2^{16-8}$ OFFD can then be used to further reduce the number of simulations from $2^{16}$ to 256, which requires only about 0.07 hour. This large computational saving does not impair the prediction of mean total delay, due to the mean statistics preservation of the M-PCM-OFFD method.

Now let us describe the experimental design and show the simulation results. We assume that each uncertain input parameter, start time $S_i$ and duration $D_i$, follows an independent truncated Gaussian distribution with mean $u_{S_i}$, $u_{D_i}$, and variance $\sigma_{S_i}^2$, $\sigma_{D_i}^2$, in the range of [0, $\infty$]. The values of $u_{S_i}$, $u_{D_i}$, $\sigma_{S_i}^2$, and $\sigma_{D_i}^2$ for each region $i$, $i = \{1, 2, \ldots, 8\}$ are listed in Table 2.

We then follow the procedures described in Table 1 to evaluate the mean total delay over a time span of 24 hours. First, we apply M-PCM to select $M$-PCM simulation points for each uncertain input parameter (see Table 3 for the results), which lead to $2^M$ combinations. Note that these M-PCM points are original values rounded to the nearest integral multiples of a time unit (i.e., $\Delta t = 15\min$). Then, we follow Step 2 to determine the number of simulations that OFFD can further reduce. In particular, the number of coefficients to be estimated is $l = \sum_{i=0}^{2^{16}} \binom{i}{16} = 137$, thus the 2-level OFFD need at least $2^{\log_2 l} = 2^8 = 256$ simulations to estimate these coefficients. Since $2^{16-\gamma}$ OFFD with $0 < \gamma \leq m - \lceil \log_2 l \rceil = 8$ and resolution $R = 5 \geq 2\tau + 1$ exist, we use $2^{16-\gamma_{\max}}$ OFFD, where $\gamma_{\max} = 8$ to further reduce the number of M-PCM points from $2^{16}$ to $2^8$. Third, by following Step 3, we select a subset of M-PCM points using $2^{16-8}$ OFFD.

![Figure 3. Illustration of a traffic flow passing through 8 regions.](image)

**Table 2. Parameters of the distributions for uncertain input parameters**

<table>
<thead>
<tr>
<th>$i$</th>
<th>$u_{S_i}$ (hour)</th>
<th>$u_{D_i}$ (hour)</th>
<th>$\sigma_{S_i}$ (hour)</th>
<th>$\sigma_{D_i}$ (hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>1.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>1.75</td>
<td>0.8</td>
<td>1.2</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
<td>1.2</td>
<td>2.0</td>
<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td>3.5</td>
<td>1.0</td>
<td>0.8</td>
<td>0.7</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.7</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>2.0</td>
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<tr>
<td>7</td>
<td>6.25</td>
<td>1.4</td>
<td>2.5</td>
<td>0.45</td>
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<tr>
<td>8</td>
<td>4</td>
<td>0.6</td>
<td>1.3</td>
<td>0.67</td>
</tr>
</tbody>
</table>

We then follow the procedures described in Table 1 to evaluate the mean total delay over a time span of 24 hours. First, we apply M-PCM to select 2 M-PCM simulation points for each uncertain input parameter (see Table 3 for the results), which lead to 2 combinations. Note that these M-PCM points are original values rounded to the nearest integral multiples of a time unit (i.e., $\Delta t = 15\min$). Then, we follow Step 2 to determine the number of simulations that OFFD can further reduce. In particular, the number of coefficients to be estimated is $l = \sum_{i=0}^{2^{16}} \binom{i}{16} = 137$, thus the 2-level OFFD need at least $2^{\log_2 l} = 2^8 = 256$ simulations to estimate these coefficients. Since $2^{16-\gamma}$ OFFD with $0 < \gamma \leq m - \lceil \log_2 l \rceil = 8$ and resolution $R = 5 \geq 2\tau + 1$ exist, we use $2^{16-\gamma_{\max}}$ OFFD, where $\gamma_{\max} = 8$ to further reduce the number of M-PCM points from $2^{16}$ to $2^8$. Third, by following Step 3, we select a subset of M-PCM points using $2^{16-8}$ OFFD.

**Table 3. M-PCM points selected for each uncertain input parameter**

<table>
<thead>
<tr>
<th>$i$</th>
<th>$S_{i(1)}$ (hour)</th>
<th>$S_{i(2)}$ (hour)</th>
<th>$D_{i(1)}$ (hour)</th>
<th>$D_{i(2)}$ (hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>1.25</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3.25</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1.75</td>
<td>5.25</td>
<td>0.75</td>
<td>1.5</td>
</tr>
<tr>
<td>4</td>
<td>2.75</td>
<td>4.25</td>
<td>0.75</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>4.5</td>
<td>5.5</td>
<td>0.5</td>
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</tr>
<tr>
<td>6</td>
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<td>3.5</td>
<td>1.75</td>
<td>2.25</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>9</td>
<td>1</td>
<td>1.75</td>
</tr>
<tr>
<td>8</td>
<td>2.75</td>
<td>5.25</td>
<td>0.5</td>
<td>1.75</td>
</tr>
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</table>
Next, we implement the air traffic model introduced in Section II.A to run simulations at these M-PCM-OFFD points. A specific experimental design is described as follows. Initially, air traffic flows enter region 1 first, and then pass the other regions successively. This initial incoming flow, \( I_1[k] \), which directly enters region 1 is sampled from a Poisson process with mean of 5 per 15 minutes, with a sequence length of 96 (i.e., 24 hours). The volumes of incoming flows to other regions, \( I_i[k], i = \{2, 3, \ldots, 8\} \), equal to the crossing flows from their previous regions, i.e., \( e_{i-1}(k - t) \), where \( t \) is the flight time for an aircraft to pass from one region to the next. Here, we set \( t = 1 \), which is one unit time (15\( \text{min} \)). When the traffic flows pass through the regions, their dynamics are modulated by convective weather events. To capture weather impacts, we assume that for each region \( i \), its capacity reduces by half in the presence of convective weather. In particular, we set \( N_i[k] = 5 \) at the present of convective weather, and \( N_i[k] = 10 \) otherwise. The dynamics of air traffic flows under weather uncertainties can then be captured by following equations:

\[
\begin{align*}
c_i[k] &= \min\{b_i[k - 1], N_i[k]\} \\
b_i[k] &= b_i[k - 1] + I_i[k] - c_i[k] \\
B_i[k] &= b_i[k - 1] - c_i[k] \\
N_i[k] &= \begin{cases} 5, & S_i \leq k \leq S_i + D_i \\ 10, & \text{else} \end{cases} \\
I_i[k] &= \begin{cases} I[k], & i = 1 \\ e_i(k - t), & i = 2, 3, \ldots, 8 \end{cases}
\end{align*}
\]

where \( I[k] \) represents the initial incoming flow to the chain of regions. Using this simulation model, we are able to find the outputs associated with the selected M-PCM-OFFD points. Given the M-PCM-OFFD points and corresponding outputs, we are now able to construct the low-order system mapping according to Step 5, and approximate the mean total backlog \( E(T_B) = E\left(\sum_{i=1}^{8} \sum_{k=1}^{k_p} B_i[k]\right) \), where \( k_p = 96 \), by following Step 6. The mean total backlog predicted by M-PCM-OFFD is 55.4260. In order to verify the results, we also apply the Monte Carlo method, which estimates the mean total backlog as 50.6810. The Monte Carlo method needs at least \( 10^4 \) simulations to converge to the correct mean total delay. However, the M-PCM-OFFD requires only 256 simulations. Such significant computational reduction realized by M-PCM-OFFD enables a real-time performance evaluation of air traffic flow system. Two reasons may lead to the discrepancy of mean total backlog. First, \( \tau = 2 \) may not capture the correlation of parameters. In fact, the case study of 8 regions in a row represents an extreme case that involves high correlation of all parameters. Increasing \( \tau \) is expected to shorten the gap. Second, the difference may also be caused by the simulation resolution (15\( \text{mins} \)). M-PCM points are rounded according to this resolution. In the following section, we introduce an M-PCM-OFFD-based framework for real-time ATFM.

### IV. Discussions

In this section, we discuss enhancement of the M-PCM-OFFD method by exploring higher-level OFFDs. We also discuss an uncertainty-exploiting framework for real-time strategic air traffic management based on the M-PCM-OFFD method.

#### A. Exploring Higher-level OFFDs

Currently, the M-PCM-OFFD uses the 2-level OFFDs to further reduce the number of simulations from M-PCM. The 2-level OFFD constrains the degrees of uncertain input parameters in the original mapping to at most 3. To generalize the degree of input parameters and broaden the applications of M-PCM-OFFD, we may apply higher-level OFFDs. We leave the detailed development to the future work, but briefly discuss the potential along this direction.

Studies on 3- and mixed-level OFFDs can be found in the literature\(^{12-16}\), which are used when parameters, such as the qualitative parameters (e.g., types of machines), have three or more levels. 3- and mixed-level OFFDs are also suggested when there is a curvilinear relation between the output and quantitative parameters (e.g., weather intensity). The basic idea of 3-level OFFDs is similar to that of 2-level OFFDs. By using 0, 1, and 2 to represent the three levels of a parameter, we first generate the \( 3^{m-\gamma} \) full factorial design, by listing all combinations of \( m-\gamma \) parameters. Then, we use \( \gamma \) generators to determine the levels for the other \( \gamma \) parameters. For mixed-level OFFDs, we can use multiple low-level parameters to represent a high-level parameter. For instance, consider a 2- and 4-level mixed OFFD, with one 2-level and one 4-level parameters. Since two 2-level parameters have four possible

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combinations, we can use each combination to represent one level of the 4-level parameter. Therefore, this mix-level design is converted to a 2-level OFFD with three 2-level parameters.

The 3- and mix-level OFFDs allow us to evaluate a system with higher degrees. In particular, if the 3-level OFFDs are applied, the degree of each parameter in the original mapping is allowed to be up to 5. We can also vary the upbounds for the degrees of parameters by using mix-level OFFDs. Specifically, if \( p \) levels are selected for a parameter, then its highest degree in the original mapping can be \( 2p - 1 \). In our previous studies, we found that the degree of 5 (\( p = 3 \)) is typically enough to obtain good estimation performance\(^\text{3,17} \).

There are mainly two issues need to be solved in order to integrate higher-level OFFDs with M-PCM. The first issue is whether introducing higher-level OFFDs impairs the mean output prediction. In order to address this problem, we need to find whether the matrix \( L \) contructed by selected M-PCM-OFFD points (see Step 5 in Table 1) is full-column-rank, as the coefficients of the system mapping can be computed only under this condition. The second issue is whether the matrix \( L \) constructed by higher-level OFFDs is the most robust to numerical errors compared with other possible selections. Since many system simulators have contraints on the resolution of input parameters, the values of input parameters need to be truncated before simulation. These truncation errors may cause the contructed matrix \( L \) to lose rank, so as to fail the computation of coefficients. Therefore, we require the matrix \( L \) contructed by selected M-PCM points to be robust to such errors. To solve this problem, we need to prove that simulation subset selected by higher-level OFFD has the largest full-column-rank margin compared to all other possible subsets.

B. A Framework of Data-driven Real-time Strategic Air Traffic Management

The NAS is featured by large-scale system dynamics and decision space. To enable an on-line design of management initiatives, some computations need to be processed offline. Based on the assumption that similar weather scenarios lead to similar management initiatives, we develop a management framework that selects representative weather scenarios using M-PCM-OFFD, querying historical data tagged with high-level control solutions using selected weather scenarios, and then fine tuning control parameters in the retrieved reduced decision space to obtain optimal solutions (see Figure 4 for an illustration). More details about each step are illustrated as follows.

![Figure 4. Illustration of the uncertainty-exploiting framework for real-time strategic air traffic management.](image)

1. Selecting representative weather scenarios

To reduce the size of decision space, and quickly develop (near) optimal management solutions under weather uncertainties, we select representative weather scenarios and design control solutions only for these limited number of weather scenarios. The representative weather scenarios are selected in such a way that management solutions optimal for these scenarios are also (near) optimal for the original whole range of possible scenarios. The M-PCM-OFFD method guarantees this near-optimality by preserving the mean output as illustrated below.

Consider the whole ATFM system as a black-box (see Figure 4). The inputs to this system is a set of variables \( \{x_1, x_2, ..., x_m\} \), which capture the features of uncertain weather scenarios. The output \( y \) is the cost of a management strategy corresponding to a weather scenario. M-PCM-OFFD allows us to quickly select a few weather scenarios to correctly predict the mean output of a management strategy. As such, the management strategy optimal to these few weather scenarios is also optimal for a whole range of weather uncertainties.

2. Query of historical data

The query system in the black box aims to quickly find similar weather scenarios from historical data. Assuming that management strategies designed for similar weather scenarios are also similar, we can leverage the historical weather scenarios, which are tagged with high-level control solutions, to help us save computational time on the on-line design of management strategies. In particular, with the representative weather scenarios selected by M-PCM-
OFFD, we use these weather scenarios to query the historical database and find similar scenarios. The high-level management solutions associated with retrieved scenarios are then sent to the multi-scale control module for management strategy design.

3. Fine tuning control parameters
The high-level management solutions retrieved from the historical database significantly reduce the decision space. We then fine tune parameters in these management solutions to find the best management solution for the selected representative weather scenarios. On-line computation time can thus be significantly reduced.

In the above data-driven management framework, the M-PCM-OFFD method is used to significantly reduce the number of weather scenarios to be evaluated, historical control solutions are leveraged to reduce on-line computational cost, and online fine-tuning is used to achieve optimal management solutions.

V. Conclusion
Strategic ATFM requires a fast evaluation of weather impacts on air traffic system performance. However, the vast number of possible weather events make this task challenging. Traditional Monte Carlo simulation method is unsuitable here, as it needs a large number of simulation runs to converge to meaningful performance estimates. To meet the strategic flow management requirement, we use a scalable multi-dimensional uncertainty evaluation approach to effectively estimate the mean statistics of air traffic system under weather uncertainties. This approach first uses M-PCM to select representative weather events according to weather distributions, and then applies OFFDs to further reduce the number of weather events for evaluation. By integrating M-PCM and OFFDs, this approach is able to reduce the number of simulations from \(2^{2m}\) to \([2^{\frac{1}{2}m(m+1)}], 2^{m-1}\), which is a significant reduction especially for systems with large number of parameters. To illustrate the performance of M-PCM-OFFD in evaluating air traffic system dynamics, we use an example of 16 uncertain input parameters and show in detail the evaluation procedures. The simulation results are verified through a comparison with the Monte Carlo method.

We also discuss the enhancement of M-PCM-OFFD by exploring higher-level OFFDs. This development allows us to evaluate systems of higher degrees. In addition, we introduce a data-driven framework for real-time ATFM. This framework uses M-PCM-OFFD and historical data tagged with high-level control solutions to significantly reduce the decision space, and applies multi-scale control to fine tune management parameters in the retrieved control solutions to achieve optimal management solution.

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